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Total mean cordiality of some derived graphs

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Abstract. A total mean cordial labeling of a graph $G = (V, E)$ is a function $f : V(G) \to \{0, 1, 2\}$ such that for each edge xy assign the label $\left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$ and $\left|ev_f(i) - ev_f(j)\right| \leqslant 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x $(x = 0, 1, 2)$. If there exists a total mean cordial labeling on a graph G, we will call G is total mean cordial. In this paper, we investigate the total mean cordial labeling behavior of some derived graphs.

1. Introduction

By a graph $G = (V, E)$ we mean a finite, undirected graph with neither loops nor multiple edges. The number of vertices of G is called order of G and it is denoted by p. Similarly the number of edges of G is called size of G and it is denoted by q. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Labeled graphs are used in several areas of science and technology such as astronomy, radar, circuit design and database management [**3**]. The origin of graph labeling is graceful labeling which was introduced by Rosa [**12**] in the year 1967. In 1980, Cahit [**1**] introduced the cordial labeling of graphs. Product cordial set (PC set) of a graph was studied by Ebrahim Salehi, Yaroslav Mukhin [**2**], Harris Kwong, Sin Min Lee and Ho Kuen NG [**5**], W. C. Shiu, Harris Kwong [**13**] etc. Ponraj, Sathish Narayanan and Ramasamy [**6**] introduced the concept of total mean cordial labeling of graphs and studied about the total mean cordial labeling behavior of path, cycle, wheel and some more standard graphs. Also they investigate the total mean cordiality of olive tree, P_n^2, $S(P_n \odot K_1)$, $S(K_{1,n})$ in [**8, 11**]. In [**7, 9, 10**] Ponraj and Sathish Narayanan proved that $K_n^c + 2K_2$ is total mean cordial if and only if $n = 1, 2, 4, 6, 8$ and they studied about the total mean cordial labeling of some more graphs. In this paper we investigate the total mean

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cordial labeling behavior of $P_n \odot K_2$, $C_n \odot K_2$, dragon and some derived graphs. If x is any real number. Then the symbol $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are follow from Harary [**4**].

2. Total mean cordial labeling

Definition 2.1. A total mean cordial labeling of a graph $G = (V, E)$ is a function $f : V(G) \to \{0, 1, 2\}$ such that for each edge xy assign the label $\left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$ and $|ev_f(i) - ev_f(j)| \leqslant 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x ($x = 0, 1, 2$). If there exists a total mean cordial labeling on a graph G, we will call G is total mean cordial.

The following results are frequently used in the subsequent section.

Theorem 2.1. [**6**] Any Path P_n is total mean cordial.

Theorem 2.2. [**6**] The Cycle C_n is total mean cordial if and only if $n \neq 3$.

Theorem 2.3. [**6**] The star $K_{1,n}$ is total mean cordial.

3. Main Results

Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Theorem 3.1. Let G be a (p, q) graph. If G satisfies any one of the following then $G \odot 2K_1$ is total mean cordial.

(1) G is a tree.
(2) G is a unicycle.
(3) $q = p + 1$.

Proof. Assign the label 0 to the vertices of G and 2 to the pendent vertices. If G is a tree then $ev_f(0) = 2p - 1$, $ev_f(1) = ev_f(2) = 2p$. If G is a unicyclic graph then $ev_f(0) = ev_f(1) = ev_f(2) = 2p$. If G is a graph with $q = p + 1$ then $ev_f(0) = 2p + 1$, $ev_f(1) = ev_f(2) = 2p$. □

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G, G' and G'' and joining each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''.

Theorem 3.2. Let G be a (p, q) graph. If G satisfies any one of the following then $D_2(G)$ is total mean cordial.

(1) G is a tree.
(2) G is a unicycle.
(3) $q = p + 1$.

PROOF. Assign the label 0 to one copy of G and 2 to another copy of G. If G is a tree then $ev_f(0) = ev_f(2) = 2p-1$, $ev_f(1) = 2p-2$. If G is a unicyclic graph then $ev_f(0) = ev_f(1) = ev_f(2) = 2p$. If $q = p+1$ then $ev_f(0) = ev_f(2) = 2p+1$ and $ev_f(1) = 2p+2$. □

Now we look into the graph dragon.

A dragon is a graph formed by joining an end vertex of a path P_n to a vertex of the cycle C_m. It is denoted by $C_m@P_n$. Let C_m be the cycle $u_1u_2\dots u_mu_1$ and P_n be the path $v_1v_2\dots v_n$.

THEOREM 3.3. All dragons $C_m@P_n$ are total mean cordial.

PROOF. Without loss generality unify the vertices u_1 and v_1.

Case 1. $n \leqslant 8$.

Subcase 1. $m \equiv 0 \pmod 3$, $m > 3$.

Let $m = 3t$. Let f be a total mean cordial labeling defined in theorem 2.2. Then $ev_f(0) = ev_f(1) = ev_f(2) = 2t$. The vertex labeling given in table 1 together with the labeling f forms a total mean cordial labeling h of $C_m@P_n$ ($n \leqslant 8$).

n	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
2	0	2							$2t$	$2t+1$	$2t+1$
3	0	0	2						$2t+2$	$2t+1$	$2t+1$
4	0	0	1	2					$2t+2$	$2t+2$	$2t+2$
5	0	0	1	2	0				$2t+3$	$2t+3$	$2t+2$
6	0	0	0	1	2	1			$2t+4$	$2t+3$	$2t+4$
7	0	0	0	2	1	1	2		$2t+4$	$2t+4$	$2t+4$
8	0	0	0	2	1	1	2	1	$2t+4$	$2t+5$	$2t+5$

TABLE 1

Subcase 2. $m \equiv 1 \pmod 3$.

Let $m = 3t+1$. For $n = 3$ and $t > 2$ define a map $f : V(C_m@P_3) \to \{0,1,2\}$ by

$$\begin{array}{lcl} f(u_i) & = & 0, \quad 1 \leqslant i \leqslant t-1 \\ f(u_{t+1+i}) & = & 2, \quad 1 \leqslant i \leqslant t+1 \\ f(u_{2t+2+i}) & = & 1, \quad 1 \leqslant i \leqslant t-1 \end{array}$$

$f(u_t) = 1$, $f(u_{t+1}) = 0$, $f(v_2) = f(v_3) = 0$. Here, $ev_f(0) = ev_f(1) = ev_f(2) = 2t+2$. For $C_4@P_3$, assign the labels 1, 2, 2, 0, 0, 0 to the vertices u_1, u_2, u_3, u_4, v_2, v_3 respectively. For $C_7@P_3$, assign the labels 0, 2, 2, 2, 0, 2, 0, 0, 0 to the vertices u_1, u_2, u_3, u_4, u_5, u_6, u_7, v_2, v_3 respectively.

Let f be a total mean cordial labeling defined in theorem 2.2. Then $ev_f(0) = ev_f(1) = 2t+1$, $ev_f(2) = 2t$. The vertex labeling given in table 2 together with the labeling f forms a total mean cordial labeling h of $C_m@P_n$ ($n \leqslant 8$).

Subcase 3. $m \equiv 2 \pmod 3$.

Let $m = 3t+2$. Let f be a total mean cordial labeling defined in theorem 2.2. Then $ev_f(0) = ev_f(2) = 2t+1$, $ev_f(1) = 2t+2$. The vertex labeling given

n	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
2	0	2							$2t+1$	$2t+2$	$2t+1$
4	0	2	2	0					$2t+3$	$2t+3$	$2t+3$
5	0	0	2	2	1				$2t+3$	$2t+3$	$2t+4$
6	0	0	2	2	1	0			$2t+4$	$2t+4$	$2t+4$
7	0	0	2	2	1	0	2		$2t+4$	$2t+5$	$2t+5$
8	0	0	2	2	1	0	2	0	$2t+5$	$2t+6$	$2t+5$

TABLE 2

n	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
3	0	0	2						$2t+3$	$2t+3$	$2t+2$
4	0	0	2	2					$2t+3$	$2t+3$	$2t+4$
5	0	0	2	2	0				$2t+4$	$2t+4$	$2t+4$
6	0	0	2	2	0	2			$2t+4$	$2t+5$	$2t+5$
7	0	0	2	2	0	2	0		$2t+5$	$2t+6$	$2t+5$
8	0	0	0	1	2	2	1	0	$2t+6$	$2t+6$	$2t+6$

TABLE 3

in table 3 together with the labeling f forms a total mean cordial labeling h of $C_m@P_n$ $(n \leqslant 8)$.

For $n = 2$ and $t \geqslant 2$ define a map $f : V(C_m@P_2) \to \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= 0, \quad 1 \leqslant i \leqslant t \\ f(u_{t+2+i}) &= 2, \quad 1 \leqslant i \leqslant t+1 \\ f(u_{2t+3+i}) &= 1, \quad 1 \leqslant i \leqslant t-1 \end{aligned}$$

$f(u_{t+1}) = 1$, $f(u_{t+2}) = 0$, $f(v_2) = 0$. Here, $ev_f(0) = ev_f(1) = ev_f(2) = 2t + 2$. For $C_5@P_2$ assign the labels 0, 2, 2, 0, 2, 0 to the vertices u_1, u_2, u_3, u_4, u_5, v_2 respectively.

Case 2. $m = 3$.

Without loss of generality we may assume that the vertex u_1 is unified with v_1. By theorem 2.1, P_n is total mean cordial. Let g be the total mean cordial labeling of P_n as defined in theorem 2.1.

Subcase 1. $n \equiv 0 \pmod 3$.

Put $n = 3t$. Define a function $f : V(C_m@P_n) \to \{0, 1, 2\}$ by $f(u_1) = f(u_2) = 0$, $f(u_3) = 2$ and $f(v_i) = g(v_i)$, $1 \leqslant i \leqslant n$. In this case $ev_f(0) = ev_f(2) = 2t + 1$ and $ev_f(1) = 2t + 2$.

Subcase 2. $n \equiv 1 \pmod 3$.

Take $n = 3t+1$. Define a function $f : V(C_m@P_n) \to \{0, 1, 2\}$ by $f(u_2) = f(u_3) = 0$ and $f(v_i) = g(v_i)$, $1 \leqslant i \leqslant n$. Then relabel the vertex $v_1 (= u_1)$ by 2. Since $ev_f(0) = ev_f(1) = 2t + 2$ and $ev_f(2) = 2t + 1$, f is a total mean cordial labeling.

Subcase 3. $n \equiv 2 \pmod 3$.

Put $n = 3t + 2$. Define a labeling f as in subcase 1. Here $ev_f(0) = ev_f(1) = 2t + 3$ and $ev_f(2) = 2t + 2$.

Case 3. $n \geqslant 8$.
Suppose the graph $C_m@P_n$ is obtained by unifying the cycle vertex u_i with the path vertex v_1. Treating u_i as u_1, u_{i+1} as u_2 and so on. Then assign the labels to the vertices of C_m as in theorem 2.2. Similarly assign the labels to the vertices of the path P_n as in theorem 2.1. Let f be the vertex labeling of $C_m@P_n$ described above.
Subcase 1. $m \equiv 0 \pmod 3$ and $n \equiv 0 \pmod 3$.
Let $m = 3t_1$ and $n = 3t_2$. Now relabel the vertex v_{t_2+2} by 0. Here $ev_f(0) = ev_f(1) = 2t_1 + 2t_2 - 1$ and $ev_f(2) = 2t_1 + 2t_2$.
Subcase 2. $m \equiv 0 \pmod 3$ and $n \equiv 1 \pmod 3$.
Let $m = 3t_1$ and $n = 3t_2 + 1$. In this case $ev_f(0) = ev_f(1) = ev_f(2) = 2t_1 + 2t_2$.
Subcase 3. $m \equiv 0 \pmod 3$ and $n \equiv 2 \pmod 3$.
Let $m = 3t_1$ and $n = 3t_2+2$. Here $ev_f(0) = 2t_1+2t_2$, $ev_f(1) = ev_f(2) = 2t_1+2t_2+1$.
Subcase 4. $m \equiv 1 \pmod 3$ and $n \equiv 0 \pmod 3$.
Let $m = 3t_1 + 1$ and $n = 3t_2$. Now relabel the vertex v_{t_2+2} by 0. Here $ev_f(0) = ev_f(1) = ev_f(2) = 2t_1 + 2t_2$.
Subcase 5. $m \equiv 1 \pmod 3$ and $n \equiv 1 \pmod 3$.
Let $m = 3t_1 + 1$ and $n = 3t_2 + 1$. In this case $ev_f(0) = ev_f(1) = 2t_1 + 2t_2 + 1$ and $ev_f(2) = 2t_1 + 2t_2$.
Subcase 6. $m \equiv 1 \pmod 3$ and $n \equiv 2 \pmod 3$.
Let $m = 3t_1 + 1$ and $n = 3t_2 + 2$. Here $ev_f(0) = ev_f(2) = 2t_1 + 2t_2 + 1$ and $ev_f(1) = 2t_1 + 2t_2 + 2$.
Subcase 7. $m \equiv 2 \pmod 3$ and $n \equiv 0 \pmod 3$.
Let $m = 3t_1 + 2$ and $n = 3t_2$. Now relabel the vertex v_{t_2+2} by 0. Then $ev_f(0) = 2t_1 = 2t_2$, $ev_f(1) = ev_f(2) = 2t_1 + 2t_2 + 1$.
Subcase 8. $m \equiv 2 \pmod 3$ and $n \equiv 1 \pmod 3$.
Take $m = 3t_1 + 2$ and $n = 3t_2 + 1$. Here $ev_f(0) = ev_f(2) = 2t_1 + 2t_2 + 1$ and $ev_f(1) = 2t_1 + 2t_2 + 2$.
Subcase 9. $m \equiv 2 \pmod 3$ and $n \equiv 2 \pmod 3$.
Let $m = 3t_1 + 2$ and $n = 3t_2 + 2$. Now relabel the vertex v_{t_2+3} by 0. Here $ev_f(0) = ev_f(1) = ev_f(2) = 2t_1 + 2t_2 + 2$. Hence $C_m@P_n$ is total mean cordial. □

Next investigation is about $P_n \odot K_2$ and $C_n \odot K_2$.

THEOREM 3.4. $P_n \odot K_2$ is total mean cordial iff $n \neq 1$.

PROOF. Let $P_n : u_1u_2 \dots u_n$ be the path. Let $V(P_n \odot K_2) = V(P_n) \cup \{v_i, w_i : 1 \leqslant i \leqslant n\}$ and $E(P_n \odot K_2) = E(P_n) \cup \{u_iv_i, v_iw_i, w_iu_i : 1 \leqslant i \leqslant n\}$. The order and size of $P_n \odot K_2$ are $3n$ and $4n - 1$ respectively.
Case 1. $n \equiv 0 \pmod 3$.
Let $n = 3t$ and $t > 0$. Define a map $f : V(P_n \odot K_2) \to \{0, 1, 2\}$ as follows:

$$\begin{array}{llll} f(u_i) & = f(v_i) & = f(w_i) & = 0, \quad 1 \leqslant i \leqslant t \\ f(u_{t+i}) & = f(v_{t+i}) & = f(w_{t+i}) & = 1, \quad 1 \leqslant i \leqslant t \\ f(u_{2t+i}) & = f(v_{2t+i}) & = f(w_{2t+i}) & = 2, \quad 1 \leqslant i \leqslant t \end{array}$$

In this case $ev_f(0) = 7t - 1$, $ev_f(1) = ev_f(2) = 7t$.
Case 2. $n \equiv 1 \pmod 3$.

Clearly $P_n \odot K_2$ is not total mean cordial. Assume $n > 1$. Take $n = 3t+1$ and $t > 0$. Define a function $f : V(P_n \odot K_2) \to \{0,1,2\}$ as follows:

$$\begin{array}{llll} f(u_i) & = f(v_i) & = f(w_i) & = 0, \quad 1 \leqslant i \leqslant t \\ f(u_{t+1+i}) & = f(v_{t+1+i}) & = f(w_{t+1+i}) & = 1, \quad 1 \leqslant i \leqslant t \\ f(u_{2t+1+i}) & = f(v_{2t+1+i}) & = f(w_{2t+1+i}) & = 2, \quad 1 \leqslant i \leqslant t \end{array}$$

$f(u_{t+1}) = 2$, $f(v_{t+1}) = f(w_{t+1}) = 0$. Here $ev_f(0) = ev_f(1) = ev_f(2) = 7t+2$.

Case 3. $n \equiv 2 \pmod 3$.

For $P_2 \odot K_2$, assign the label 0, 0, 0, 2, 2, 2 to the vertices u_1, u_2, v_1, v_2, w_1, w_2 respectively.

Let $n = 3t+2$ and $t > 0$. Define $f : V(P_n \odot K_2) \to \{0,1,2\}$ by

$$\begin{array}{llll} f(u_i) & = f(v_i) & = f(w_i) & = 0, \quad 1 \leqslant i \leqslant t \\ f(u_{t+2+i}) & = f(v_{t+2+i}) & = f(w_{t+2+i}) & = 1, \quad 1 \leqslant i \leqslant t \\ f(u_{2t+2+i}) & = f(v_{2t+2+i}) & = f(w_{2t+2+i}) & = 2, \quad 1 \leqslant i \leqslant t \end{array}$$

$f(u_{t+1}) = f(u_{t+2}) = 2$, $f(v_{t+1}) = f(v_{t+2}) = f(w_{t+1}) = f(w_{t+2}) = 2$. In this case $ev_f(0) = 7t+5$, $ev_f(1) = ev_f(2) = 7t+4$. □

THEOREM 3.5. $C_n \odot K_2$ is total mean cordial.

PROOF. Let $V(C_n \odot K_2)$ be taken as in that of $P_n \odot K_2$ and $E(C_n \odot K_2) = E(P_n \odot K_2) \cup \{u_n u_1\}$, see theorem 3.4. The order and size of $C_n \odot K_2$ are $3n$ and $4n$ respectively.

Case 1. $n \equiv 0 \pmod 3$.

Let $n = 3t$. Define a map $f : V(C_n \odot K_2) \to \{0,1,2\}$ by

$$\begin{array}{rcll} f(u_i) & = & 2, & 1 \leqslant i \leqslant 3t \\ f(v_i) & = & 0, & 1 \leqslant i \leqslant 3t \\ f(w_i) & = & 0, & 1 \leqslant i \leqslant 2t \\ f(w_{2t+i}) & = & 1, & 1 \leqslant i \leqslant t. \end{array}$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 7t$.

Case 2. $n \equiv 1 \pmod 3$.

Label the vertices of $C_n \odot K_2$ as in case 2 of theorem 3.4. It is easy from the fact that $ev_f(0) = ev_f(2) = 7t+2$ and $ev_f(1) = 7t+3$, f is a total mean cordial labeling.

Case 3. $n \equiv 2 \pmod 3$.

Assign the labels to the vertices of $C_n \odot K_2$ as in case 2 of theorem 3.4. Here we have $ev_f(0) = ev_f(1) = 7t+5$ and $ev_f(2) = 7t+4$ and hence $C_n \odot K_2$ is a total mean cordial graph. □

Finally we investigate the total mean cordiality of splitting graph of star and comb.

For a graph G, the splitting graph of G, $S'(G)$, is obtained from G by adding for each vertex v of G a new vertex v' so that v' is adjacent to every vertex that is adjacent to v. Note that if G is a (p,q) graph then $S'(G)$ is a $(2p, 3q)$ graph.

THEOREM 3.6. Splitting graph of a star, $S'(K_{1,n})$ is total mean cordial.

PROOF. Let $V(S'(K_{1,n})) = \{u, v, u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(S'(K_{1,n})) = \{uu_i, vu_i, vv_i : 1 \leqslant i \leqslant n\}$. Clearly, $p + q = 5n + 2$.

Case 1. $n \equiv 0 \pmod 6$.

Let $n = 6t$ and $t > 0$. Define a function $f : V(S^{'}(K_{1,n})) \to \{0, 1, 2\}$ by $f(u) = 0$ and $f(v) = 1$,

$$\begin{array}{lcll} f(u_i) & = & 0 & 1 \leqslant i \leqslant 5t \\ f(u_{5t+i}) & = & 1 & 1 \leqslant i \leqslant t \\ f(v_i) & = & 2 & 1 \leqslant i \leqslant 5t \\ f(v_{5t+i}) & = & 1 & 1 \leqslant i \leqslant t \end{array}$$

In this case $ev_f(0) = ev_f(1) = 10t + 1$, $ev_f(2) = 10t$.

Case 2. $n \equiv 1 \pmod 6$.

Let $n = 6t-5$ and $t > 0$. Define a function $f : V(S'(K_{1,n})) \to \{0, 1, 2\}$ by $f(u) = 0$ and $f(v) = 1$,

$$\begin{array}{lcll} f(u_i) & = & 0 & 1 \leqslant i \leqslant 5t-4 \\ f(u_{5t-4+i}) & = & 1 & 1 \leqslant i \leqslant t-1 \\ f(v_i) & = & 2 & 1 \leqslant i \leqslant 5t-4 \\ f(v_{5t-4+i}) & = & 1 & 1 \leqslant i \leqslant t \end{array}$$

Here $ev_f(0) = 10t - 7$, $ev_f(1) = ev_f(2) = 10t - 8$.

Case 3. $n \equiv 2 \pmod 6$.

For $S'(K_{1,2})$, asign the labels 0, 1, 0, 2, 2, 0 to the vertices u_1, u_2, v_1, v_2, u, v respectively.

Let $n = 6t + 2$ and $t > 0$. Define a map $f : V(S^{'}(K_{1,n})) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(v) = 1$ and $f(v_1) = 0$,

$$\begin{array}{lcll} f(u_i) & = & 0 & 1 \leqslant i \leqslant 5t+1 \\ f(u_{5t+1+i}) & = & 1 & 1 \leqslant i \leqslant t+1 \\ f(v_{i+1}) & = & 2 & 1 \leqslant i \leqslant 5t+2 \\ f(v_{5t+3+i}) & = & 1 & 1 \leqslant i \leqslant t-1 \end{array}$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 10t + 4$.

Case 4. $n \equiv 3 \pmod 6$.

Let $n = 6t-3$ and $t > 0$. Define a function $f : V(S^{'}(K_{1,n})) \to \{0, 1, 2\}$ by $f(u) = 0$ and $f(v) = 1$,

$$\begin{array}{lcll} f(u_i) & = & 0 & 1 \leqslant i \leqslant 5t-3 \\ f(u_{5t-3+i}) & = & 1 & 1 \leqslant i \leqslant t \\ f(v_i) & = & 2 & 1 \leqslant i \leqslant 5t-2 \\ f(v_{5t-2+i}) & = & 1 & 1 \leqslant i \leqslant t-1 \end{array}$$

Here $ev_f(0) = 10t - 5$, $ev_f(1) = ev_f(2) = 10t - 4$.

Case 5. $n \equiv 4 \pmod 6$.

Let $n = 6t-2$ and $t > 0$. Define a function $f : V(S^{'}(K_{1,n})) \to \{0, 1, 2\}$ by $f(u) = 0$ and $f(v) = 1$,

$$\begin{array}{lcll} f(u_i) & = & 0 & 1 \leqslant i \leqslant 5t-2 \\ f(u_{5t-2+i}) & = & 1 & 1 \leqslant i \leqslant t \\ f(v_i) & = & 2 & 1 \leqslant i \leqslant 5t-1 \\ f(v_{5t-1+i}) & = & 1 & 1 \leqslant i \leqslant t-1 \end{array}$$

In this case $ev_f(0) = ev_f(1) = 10t - 3$, $ev_f(2) = 10t - 2$.
Case 6. $n \equiv 5 \pmod 6$.
Let $n = 6t-1$ and $t > 0$. Define a function $f : V(S^{'}(K_{1,n})) \to \{0, 1, 2\}$ by $f(u) = 0$ and $f(v) = 2$,

$$\begin{array}{lcll} f(u_i) & = & 0 & 1 \leqslant i \leqslant 5t-1 \\ f(u_{5t-1+i}) & = & 1 & 1 \leqslant i \leqslant t \\ f(v_i) & = & 1 & 1 \leqslant i \leqslant 3t \\ f(v_{3t+i}) & = & 2 & 1 \leqslant i \leqslant 3t-1 \end{array}$$

In this case $ev_f(0) = ev_f(1) = ev_f(2) = 10t - 1$. □

THEOREM 3.7. $S'(P_n \odot K_1)$ is total mean cordial.

PROOF. Let $V(P_n \odot K_1) = \{u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(P_n \odot K_1) = \{u_iu_{i+1} : 1 \leqslant i \leqslant n-1\} \cup \{u_iv_i : 1 \leqslant i \leqslant n\}$. Let $u_i^{'}$ $(1 \leqslant i \leqslant n)$ be the vertex corresponding to u_i $(1 \leqslant i \leqslant n)$ and $v_i^{'}$ $(1 \leqslant i \leqslant n)$ be the vertex corresponding to v_i $(1 \leqslant i \leqslant n)$. Define a map $f : V(S'(P_n \odot K_1)) \to \{0, 1, 2\}$ by

$$\begin{array}{lcll} f(u_i) & = & 0, & 1 \leqslant i \leqslant n \\ f(u_i^{'}) & = & 2, & 1 \leqslant i \leqslant n \\ f(v_i) & = & 2 & 1, \leqslant i \leqslant n \\ f(v_i^{'}) & = & 0, & 1 \leqslant i \leqslant \left\lceil \frac{2n}{3} \right\rceil \quad \text{if} \quad n \equiv 0, 1 \pmod 3 \\ & & & 1 \leqslant i \leqslant \left\lfloor \frac{2n}{3} \right\rfloor \quad \text{if} \quad n \equiv 2 \pmod 3 \\ f(v^{'}_{\lceil \frac{2n}{3} \rceil + i}) & = & 2, & 1 \leqslant i \leqslant \left\lfloor \frac{n-3}{3} \right\rfloor \quad \text{if} \quad n \equiv 0, 1 \pmod 3 \\ f(v^{'}_{\lfloor \frac{2n}{3} \rfloor + i}) & = & 2, & 1 \leqslant i \leqslant \left\lceil \frac{n-3}{3} \right\rceil \quad \text{if} \quad n \equiv 2 \pmod 3 \end{array}$$

and $f(v_n^{'}) = 1$. The following table 4 shows that f is a total mean cordial labeling.

Nature of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0 \pmod 3$	$\frac{10n-3}{3}$	$\frac{10n-3}{3}$	$\frac{10n-3}{3}$
$n \equiv 1 \pmod 3$	$\frac{10n-1}{3}$	$\frac{10n-4}{3}$	$\frac{10n-4}{3}$
$n \equiv 2 \pmod 3$	$\frac{10n-5}{3}$	$\frac{10n-2}{3}$	$\frac{10n-2}{3}$

TABLE 4

□

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