BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Vol. 5(2015), 125-132

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

Grading Wild Blocks via Stable Equivalences

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Abstract. In this paper we show how to construct a non-trivial $\mathbb{Z}$-grading on a wild block of group algebras by using stable equivalences between categories of graded modules.

1. Introduction and preliminaries

Let A and B be algebras over a field k, and let us assume that there is a stable equivalence between A and B, and that the algebra B is graded. A natural question to ask is if there is a graded structure on A that is compatible with the stable equivalence between A and B, i.e. if there is a grading on A such that there is a stable equivalence between categories of graded A-modules and graded B-modules. In other words, is there a way to non-trivially grade algebras by using transfer of gradings between algebras via stable equivalences? In [8], Rouquier introduced the idea of transfer of gradings via stable equivalences, and proved that it is possible to construct non-trivial gradings in such a way.

The question of how to effectively transfer gradings from B to A arises, because it is very difficult to conduct these computations in concrete situations. This is the main topic of this paper. We show how to recover the graded quiver of A and effectively transfer gradings between algebras via stable equivalences in the case when the stable equivalence between A and B is of Morita type. We achieve such a transfer via stable equivalences between Brauer correspondents of a group algebra, with algebras in question being wild blocks of group algebras whose Sylow p-subgroups satisfy the trivial intersection property. In this case, a stable equivalence between the corresponding blocks is given by Green correspondence.

1.1. Notation. Throughout this text k will denote a field. All algebras will be defined over the field k, and all modules will be left modules. The category of finite dimensional A–modules is denoted by A–mod, and the stable category of

2010 *Mathematics Subject Classification.* Primary 16A03; Secondary 16W50, 20C20.
Key words and phrases. Graded algebras, graded stable equivalence, wild blocks.

the category A–mod is denoted by A-$\overline{\text{mod}}$. The space of morphisms in the stable category between A-modules M and N is denoted by $\overline{\text{Hom}}_A(M,N)$. For a given module M, the projective cover of M is denoted by P_M (see [**9**] for more details).

1.2. Graded modules. We say that an algebra A is a graded algebra if A is the direct sum of subspaces $A = \bigoplus_{i\in\mathbb{Z}} A_i$, such that $A_iA_j \subset A_{i+j}$, $i,j \in \mathbb{Z}$. If $A_i = 0$ for $i < 0$, we say that A is positively graded. An A-module M is graded if it is the direct sum of subspaces $M = \bigoplus_{i\in\mathbb{Z}} M_i$, such that $A_iM_j \subset M_{i+j}$, for all $i,j \in \mathbb{Z}$. If M is a graded A–module, then $N = M\langle i\rangle$ denotes the graded module given by $N_j = M_{i+j}$, $j \in \mathbb{Z}$. An A-module homomorphism f between two graded modules M and N is a homomorphism of graded modules if $f(M_i) \subseteq N_i$, for all $i \in \mathbb{Z}$. For a graded algebra A, we denote by A–modgr the category of graded finite dimensional A–modules. We set $\text{Homgr}_A(M,N) := \bigoplus_{i\in\mathbb{Z}} \text{Hom}_{A-gr}(M, N\langle i\rangle)$, where $\text{Hom}_{A-gr}(M, N\langle i\rangle)$ denotes the space of all graded homomorphisms between M and $N\langle i\rangle$ (the space of homogeneous morphisms of degree i). There is an isomorphism of vector spaces $\text{Hom}_A(M,N) \cong \text{Homgr}_A(M,N)$ that gives us a grading on $\text{Hom}_A(M,N)$ (cf. [6], Corollary 2.4.4). By Corollary 2.4.7 in [6], there is an isomorphism of vector spaces $\text{Ext}^1_A(M,N) \cong \text{Extgr}^1_A(M,N)$, where $\text{Extgr}^1_A(M,N) := \bigoplus_{i\in\mathbb{Z}} \text{Ext}^1_{A-gr}(M, N\langle i\rangle)$, which gives us a grading on the space $\text{Ext}^1_A(M,N)$. Here, $\text{Ext}^1_{A-gr}(M, N\langle i\rangle)$ denotes the Ext–space computed in the category of graded A–modules.

We note here that if we have two different gradings on an indecomposable module, then they differ only by a shift (cf. [3], Lemma 2.5.3). Unless otherwise stated, for a graded algebra A given by a quiver and relations, we will assume that the projective indecomposable A-modules are graded in such a way that the top of a projective indecomposable A-module is in degree 0.

2. Transfer of gradings

The main theorem that enables us to transfer gradings via stable equivalences is due to Rouquier.

THEOREM 2.1 ([8], Theorem 6.1). *Let M be an (A,B)-bimodule and let N be a (B,A)-bimodule inducing mutually inverse stable equivalences between self-injective k-algebras A and B, where k is an algebraically closed field of positive characteristic. If B is graded, then A admits a grading and there is a graded structure on the bimodules M and N. Moreover, the graded bimodules M and N induce mutually inverse equivalences between the stable categories of graded A-modules and graded B-modules.*

Let F be an equivalence between the stable categories of A-mod and B-mod. By [4, Section 4.1], for A-modules S and T, there is an isomorphism of vector spaces

$$\text{Ext}^1_A(S,T) \cong \text{Ext}^1_B(F(S), F(T)).$$

Having in mind Theorem 2.1, if S and T are graded modules, then the graded version of the above isomorphism holds as well, i.e. we have a graded isomorphism

$\mathrm{Ext}^1_{A-gr}(S,T) \cong \mathrm{Ext}^1_{B-gr}(F(S),F(T))$. It follows that

$$\mathrm{Ext}^1_A(S,T) \cong \bigoplus_{i\in\mathbb{Z}} \mathrm{Ext}^1_{A-gr}(S,T\langle i\rangle) \cong \bigoplus_{i\in\mathbb{Z}} \mathrm{Ext}^1_{B-gr}(F(S),F(T)\langle i\rangle).$$

From this isomorphism it follows that if there is a stable equivalence between two symmetric algebras A and B, then we can determine the quiver of A by calculating Ext-spaces in B. This is true because the dimension of $\mathrm{Ext}^1_A(S,T)$ is equal to the number of arrows in the quiver of A from the vertex corresponding to the simple module T to the vertex corresponding to the simple module S. Moreover, if B is graded then we can put a grading on A as well by Theorem 2.1. By this theorem, the corresponding stable categories of graded A-modules and graded B-modules are equivalent and the last graded isomorphism above gives us $\mathrm{Ext}^1_A(S,T)$ as a graded space, hence giving us a grading on the quiver of A, i.e. giving us a grading on A.

It follows that all we have to do to transfer gradings via stable equivalences from B to A is to compute the graded space

$$\bigoplus_{i\in\mathbb{Z}} \mathrm{Ext}^1_{B-gr}(F(S),F(T)\langle i\rangle)$$

for all simple A-modules S and T. This graded space is the graded space spanned by all arrows from T to S in the path algebra of the quiver of A. The resulting grading on A is unique up to shifts of graded correspondents of simple A-modules. We will assume that these modules are graded in such a way that their tops are in degree 0. It is very difficult to conduct these computations in concrete situations, but in some cases it is possible to do so, as we present it in the following section.

2.1. Green correspondence. If a stable equivalence between algebras A and B is of Morita type, then the functor F inducing the stable equivalence between A and B is given by a bimodule ${}_AM_B$, i.e. $F = {}_B M^*_A \otimes -$, where M^* is the dual of M, and its quasi-inverse is given by ${}_AM_B \otimes -$. We note here that we disregard the projective summands, because they do not contribute to the extension spaces. The idea of transfer is to put a grading on $\mathrm{Ext}^1_B(M^* \otimes_A S, M^* \otimes_A T)$ and use the same grading to grade $\mathrm{Ext}^1_A(S,T)$, i.e. to grade the corresponding arrows of the quiver of A. In order to transfer gradings via stable equivalences of Morita type between A and B we need to have: a bimodule ${}_AM_B$ that induces a stable equivalence of Morita type, the images of simple A-modules under this equivalence, and a structure of graded modules on these images.

The trivial intersection case is the typical situation where we have stable equivalences of Morita type. For a given group G and a prime number p, we say that the Sylow p-subgroups of G have the trivial intersection property if $g \notin N_G(P)$ implies that $P \cap P^g = 1$, for every Sylow p-subgroup P of G.

In the case of the trivial intersection Sylow p-subgroups, by Theorem 3.10.1 and Theorem 3.10.3 in [1] there is a stable equivalence of Morita type between kG and kL induced by Green correspondence (which is given either by restriction or induction (i.e. with a bimodule kG), and taking non-projective summands), where $L = N_G(P)$ and P is a Sylow p-subgroup of G. Here, $\mathrm{char}\, k = p$.

In the next section we give an example of such a correspondence where the transfer of gradings via stable equivalences can be done explicitly. The blocks in question are blocks of wild representation type.

3. The principal block of A_6 and its Brauer correspondent

Throughout this section k will be a field of characteristic 3. For the alternating group A_6, a Sylow 3-subgroup P is isomorphic to $C_3 \times C_3$ and the normalizer $N_{A_6}(P)$ of P is isomorphic to $P \rtimes C_4$, where the action of C_4 on P is given as follows (cf. [7], Appendix). If $C_3 \times C_3 = \langle x \rangle \times \langle y \rangle$, then the action of a generator f of C_4 on a set of generators $\{x, y\}$ of P is given by $x^f = xy$ and $y^f = xy^2$.

Let $B := kP \rtimes C_4$. The quiver of B is given by

$$\begin{tikzcd}
 & \overset{0}{\bullet} \arrow[dl, bend left=10, "\delta_{0,1}"] \arrow[dr, bend left=20, "\delta_{0,3}"] & \\
\overset{1}{\bullet} \arrow[ur, bend left=20, "\delta_{1,0}"] \arrow[dr, bend left=10, "\delta_{1,2}"] & & \overset{3}{\bullet} \arrow[ul, bend left=10, "\delta_{3,0}"] \arrow[dl, bend left=20, "\delta_{3,2}"] \\
 & \underset{2}{\bullet} \arrow[ul, bend left=20, "\delta_{2,1}"] \arrow[ur, bend left=10, "\delta_{2,3}"] &
\end{tikzcd}$$

and the relations are $\delta_{i,i+1}\delta_{i+1,i} = \delta_{i,i-1}\delta_{i-1,i}$, for $i = 0, 1, 2, 3$, and $\delta_{i,j}\delta_{j,i+2} = 0$, for $i = 0, 1, 2, 3$ and $j = i+1, i-1$ (cf. [7], Section 4). Here, the addition in indices is modulo 4. We recommend [**2**] as a good introduction to path algebras of quivers.

We will assume that B is graded in such a way that the vertices and the arrows of the quiver of Q are homogeneous. Furthermore, we assume that $\deg(\delta_{i,j}) = d_{i,j}$. From the relations of B we see that $d_{i,j} + d_{j,i}$ must be constant, independent of i and j. In particular, if $d_{i,j} = 1$, then we get a tight grading on B, i.e. we have that (see [**5**]):

$$B \cong \bigoplus_{i \geqslant 0} \mathrm{rad}^i(B)/\mathrm{rad}^{i+1}(B).$$

The simple B-module corresponding to the vertex i of the quiver of B will be denoted by T_i. The radical layers of the corresponding projective indecomposable B-module P_i are given by

$$\begin{array}{ccccc}
 & & T_i & & \\
 & T_{i-1} & & T_{i+1} & \\
T_{i+2} & & T_i & & T_{i+2} \\
 & T_{i-1} & & T_{i+1} & \\
 & & T_i & &
\end{array},$$

where $i \in \{0, 1, 2, 3\}$ and the addition in indices is modulo 4.

3.1. The principal block of kA_6. The Sylow 3-subgroups of A_6 have the trivial intersection property. There is a stable equivalence between B_0, the principal block of kA_6, and its Brauer correspondent B, the principal block of $kP \rtimes C_4$, given by Green correspondence, i.e. by induction and restriction, and taking non-projective summands (see [**1**]). Let S_i, $i = 1, 2, 3, 4$, be the simple B_0-modules, where we set a convention that S_i is the simple B_0-module such that the top of its Green correspondent is isomorphic to T_i. Then the Green correspondents of these simple modules are given by (cf. [7], Section 4)

$$X_0 := T_0, \quad X_1 := \begin{matrix} T_1 \\ T_0 \\ T_3 \end{matrix}, \quad X_2 := \begin{matrix} & T_2 & \\ T_1 & & T_3 \\ & T_2 & \end{matrix}, \quad X_3 := \begin{matrix} T_3 \\ T_0 \\ T_1 \end{matrix}.$$

The module X_i is a graded module because it is a quotient of a projective indecomposable B-module by a homogeneous submodule. As a graded module, X_0 is just the simple module T_0 concentrated in degree 0, and for the other Green correspondents we have

$$\begin{matrix} T_1 & 0 \\ T_0 & d_{0,1} \\ T_3 & d_{3,0}+d_{0,1} \end{matrix}, \quad \begin{matrix} & & T_2 & & 0 \\ d_{1,2} & T_1 & & T_3 & d_{3,2} \\ & & T_2 & & d_{3,2}+d_{2,3} \end{matrix}, \quad \begin{matrix} T_3 & 0 \\ T_0 & d_{0,3} \\ T_1 & d_{1,3}+d_{0,3} \end{matrix},$$

where the numbers to the left or to the right of a composition factor denote the degree of that composition factor.

We will use a stable equivalence between B and B_0 to recover the quiver of B_0 and to transfer gradings from B to B_0. In order to do that we need to compute

$$\operatorname{Ext}^1_B(S_i\downarrow_B, S_j\downarrow_B)$$

for $i, j \in \{0, 1, 2, 3\}$, in the category of graded B-modules. For a simple B_0-module S_i we have

$$S_i\downarrow_B \cong X_i \oplus Q_i,$$

where Q_i is a projective B-module. Since the bifunctor $\operatorname{Ext}^1_B(-,-)$ is additive and $\operatorname{Ext}^1_B(M, P) = \operatorname{Ext}^1_B(P, M) = 0$ for any B-module M and any projective B-module P, we have that

$$\operatorname{Ext}^1_B(S_i\downarrow_B, S_j\downarrow_B) \cong \operatorname{Ext}^1_B(X_i, X_j).$$

Therefore, we need to compute $\operatorname{Ext}^1_B(X_i, X_j)$ for $i, j \in \{0, 1, 2, 3\}$, to recover the quiver of B_0. We notice here that these projective summands Q_i are not equal to zero.

We start our computation by constructing a projective resolution of X_0:

$$\cdots \longrightarrow P_1\langle r\rangle \oplus P_0\langle s\rangle \oplus P_3\langle t\rangle \xrightarrow{G} P_1\langle -d_{1,0}\rangle \oplus P_3\langle -d_{3,0}\rangle \xrightarrow{F} P_0 \longrightarrow T_0,$$

where r, s, t are the necessary shifts, and P_i is the projective cover of T_i.

If we apply the functor $\operatorname{Hom}_{B-gr}(-, T_0\langle j\rangle)$ to the above resolution, we get that $\operatorname{Extgr}^1_B(T_0, T_0) = 0$, because $\operatorname{Hom}_B(P_1 \oplus P_3, T_0) = 0$. This tells us that there are no loops starting and ending at S_0 in the quiver of B_0.

To compute $\operatorname{Ext}^1_{B-gr}(T_0, X_2\langle j\rangle)$ we apply the functor $\operatorname{Hom}_{B-gr}(-, X_2\langle j\rangle)$ to the above resolution. It is obvious that $\operatorname{Hom}_B(P_0, X_2) = 0$, because T_0 is not a composition factor of X_2. Since for any homomorphism $f \in \operatorname{Hom}_B(P_1 \oplus P_3, X_2)$ we have that $f \circ G = 0$, it follows that $\operatorname{Ext}^1_{B-gr}(T_0, X_2\langle j\rangle)$ is isomorphic to

$$\operatorname{Hom}_{B-gr}(P_1\langle -d_{1,0}\rangle \oplus P_3\langle -d_{3,0}\rangle, X_2\langle j\rangle).$$

From this we conclude that $\mathrm{Extgr}^1_B(T_0, X_2) \cong k\langle d_{3,0} - d_{3,2}\rangle \oplus k\langle d_{1,0} - d_{1,2}\rangle$. This tells us that there are two arrows in the quiver of B_0 starting at S_2 and ending at S_0. These arrows are in degrees $d_{3,2} - d_{3,0}$ and $d_{1,2} - d_{1,0}$.

To compute $\mathrm{Ext}^1_{B-gr}(T_0, X_1\langle j\rangle)$ we apply the functor $\mathrm{Hom}_{B-gr}(-, X_1\langle j\rangle)$ to the above resolution. The space $\mathrm{Hom}_B(P_1 \oplus P_3, X_1)$ is 2-dimensional. For any non-zero map $g \in \mathrm{Hom}_B(P_1, X_1)$ we have that $g \circ G \neq 0$. Also, we have that $f \circ F \neq 0$ for all non-zero $f \in \mathrm{Hom}_B(P_0, X_1)$. From this we conclude that $\mathrm{Ext}^1_B(X_0, X_1) = 0$. By interchanging the roles of X_1 and X_3, and using the same arguments we get that $\mathrm{Ext}^1_B(X_0, X_3) = 0$. This means that there are no arrows from S_1 to S_0, nor from S_3 to S_0 in the quiver of B_0.

We have recovered all arrows (and computed their degrees) in the quiver of B_0 that have S_0 as their target.

We now proceed by recovering the arrows of the quiver of B_0 that have S_2 as their target. To do this we write a projective resolution of X_2:

$$\cdots \longrightarrow P_1\langle n_1\rangle \oplus P_2\langle n_2\rangle \oplus P_3\langle n_3\rangle \xrightarrow{G} P_0\langle l_2\rangle \oplus P_0\langle l_1\rangle \xrightarrow{F} P_2 \longrightarrow X_2\ ,$$

where $l_1 = -d_{0,3} - d_{3,2}$, $l_2 = -d_{0,1} - d_{1,2}$, and n_1, n_2, n_3 are the necessary shifts.

Since $\mathrm{Hom}_B(P_0, X_2) = 0$, it follows that $\mathrm{Extgr}^1_B(X_2, X_2\langle j\rangle) = 0$ for all j, i.e. there are no loops starting and ending at S_2 in the quiver of B_0.

To compute $\mathrm{Ext}^1_{B-gr}(X_2, X_0\langle j\rangle)$ we apply the functor $\mathrm{Hom}_{B-gr}(-, X_0\langle j\rangle)$ to the above resolution. Since $\mathrm{Hom}_B(P_2, T_0) = 0$ and since for every $f \in \mathrm{Hom}_B(P_0, T_0)$ it holds that $f \circ G = 0$, we have that

$$\mathrm{Ext}^1_{B-gr}(X_2, X_0\langle j\rangle) \cong \mathrm{Hom}_{B-gr}(P_0\langle l_2\rangle \oplus P_0\langle l_1\rangle, T_0\langle j\rangle).$$

It follows that $\mathrm{Extgr}^1_B(X_2, X_0) \cong k\langle d_{0,1} + d_{1,2}\rangle \oplus k\langle d_{0,3} + d_{3,2}\rangle$. This tells us that there are two arrows from S_0 to S_2 in the quiver of B_0. Their degrees are $-(d_{0,1} + d_{1,2})$ and $-(d_{0,3} + d_{3,2})$.

We now apply the functor $\mathrm{Hom}_{B-gr}(-, X_1\langle j\rangle)$ to the above resolution. From $\mathrm{Hom}_B(P_2, X_1) = 0$ it follows that $\mathrm{Ext}^1_{B-gr}(X_2, X_1\langle j\rangle)$ is isomorphic to the subspace of $\mathrm{Hom}_{B-gr}(P_0\langle l_2\rangle \oplus P_0\langle l_1\rangle, X_1\langle j\rangle)$ consisting of the maps f such that $f \circ G = 0$. The latter space is isomorphic to $\mathrm{Hom}_{B-gr}(P_0\langle l_1\rangle, X_1\langle j\rangle)$. When composed with G, the non-zero maps from $\mathrm{Hom}_{B-gr}(P_0\langle l_2\rangle, X_2\langle j\rangle)$ give non-zero maps. We conclude that $\mathrm{Extgr}^1_B(X_2, X_1) \cong k\langle d_{0,3} + d_{3,2} + d_{0,1}\rangle$, i.e. in the quiver of B_0, the arrow starting at S_1 and ending at S_2 is in degree $-d_{0,3} - d_{3,2} - d_{0,1}$.

By interchanging the roles of X_1 and X_3, and using the same arguments we get that $\mathrm{Extgr}^1_B(X_2, X_3) \cong k\langle d_{0,1} + d_{1,2} + d_{0,3}\rangle$. We conclude that the arrow starting at S_3 and ending at S_2 is in degree $-d_{0,1} - d_{1,2} - d_{0,3}$.

We have recovered all arrows (and computed their degrees) in the quiver of B_0 that have S_2 as their target.

We are left to recover the arrows of the quiver of B_0 that have S_1 or S_3 as their target. Due to the symmetry of the arguments used for X_1 and X_3, we need to do our computation only for one of them, say X_1.

We start by writing a projective resolution of X_1:

$$\cdots \longrightarrow P_0\langle n_3\rangle \xrightarrow{G} P_2\langle -d_{2,1}\rangle \xrightarrow{F} P_1 \longrightarrow X_1 ,$$

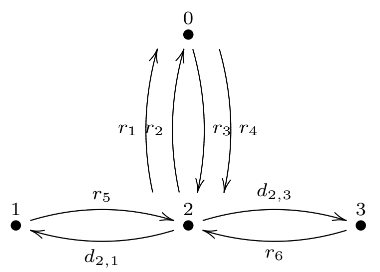
where $n_3 = -(d_{2,1} + d_{3,2} + d_{0,3})$.

Since $\mathrm{Hom}_B(P_2, X_0) = 0$, we have that $\mathrm{Ext}^1_B(X_1, X_0) = 0$. In other words, there are no arrows from S_0 to S_1 in the quiver of B_0. Similarly, $\mathrm{Ext}^1_B(X_3, X_0) = 0$ and there are no arrows from S_0 to S_3 in the quiver of B_0.

From $\mathrm{Hom}_B(P_2, X_1) = 0$ it follows that $\mathrm{Ext}^1_B(X_1, X_1) = 0$, and similarly from $\mathrm{Hom}_B(P_2, X_3) = 0$ it follows that $\mathrm{Ext}^1_B(X_1, X_3) = 0$. This means that there are no loops starting and ending at S_1, and that there are no arrows starting at S_3 and ending at S_1. The same holds if we interchange X_1 and X_3, i.e. there are no loops starting and ending at S_3, and there are no arrows starting at S_1 and ending at S_3.

We are left to compute $\mathrm{Ext}^1_{B-gr}(X_1, X_2\langle j\rangle)$, for all j. After applying the functor $\mathrm{Hom}_{B-gr}(-, X_2\langle j\rangle)$ to the above projective resolution of X_1, we notice that $\mathrm{Hom}_{B-gr}(P_0, X_2\langle j\rangle) = 0$. The dimension of the vector space $\mathrm{Hom}_B(P_2, X_2)$ is 2, and the dimension of $\mathrm{Hom}_B(P_1, X_2)$ is 1. Every non-surjective map $f \in \mathrm{Hom}_B(P_2, X_2)$ can be written in the form $f = g \circ F$, for some $g \in \mathrm{Hom}_B(P_1, X_2)$. The surjective maps from $\mathrm{Hom}_B(P_2, X_2)$ cannot be written in such a way. It follows that $\mathrm{Extgr}^1_B(X_1, X_2) \cong k\langle d_{2,1}\rangle$. The degree of the corresponding arrow from S_2 to S_1 in the quiver of B_0 is $-d_{2,1}$. By interchanging the roles of X_1 and X_3 we get that $\mathrm{Extgr}^1_B(X_3, X_2) \cong k\langle d_{2,3}\rangle$. The degree of the corresponding arrow from S_2 to S_3 in the quiver of B_0 is $-d_{2,3}$.

From the above computation, we see that we have managed to compute the quiver of B_0, and to put a grading on it. Before we draw the graded quiver of B_0, we will rescale this grading by multiplying all degrees by -1 in order to get rid of the minus signs. The resulting graded quiver of B_0 is given by



where $r_6 = d_{0,1} + d_{1,2} + d_{0,3}$, $r_5 = d_{0,3} + d_{3,2} + d_{0,1}$, $r_4 = d_{0,3} + d_{3,2}$, $r_3 = d_{0,1} + d_{1,2}$, $r_2 = d_{3,0} - d_{3,2}$ and $r_1 = d_{1,0} - d_{1,2}$.

Theorem 3.1. *Let B and B_0 be as above. Let us assume that B is graded, and that we transferred this grading to B_0 via stable equivalence. There exists a grading on B such that the resulting grading on B_0 is positive. Moreover, the resulting grading on B_0 can be tight. In particular, the resulting grading can be such that the homogeneous elements from the radical are in strictly positive degrees.*

Proof. For a suitably chosen strictly positive grading on B, e.g. if we set $d_{1,0} > d_{1,2}$ and $d_{3,0} > d_{3,2}$, we get a strictly positive grading on B_0, i.e. such a grading where all arrows are in positive degrees. Hence, all homogeneous elements from the radical of B_0 are in positive degrees. For example, if we set $d_{3,0} = d_{1,0} = d_{2,1} = d_{2,3} = 2$ and $d_{0,3} = d_{0,1} = d_{1,2} = d_{3,2} = 1$ we get a strictly positive grading on B_0. If we set $d_{1,0} = d_{3,0} = 2$, $d_{1,2} = d_{2,1} = d_{2,3} = d_{3,2} = 1$ and $d_{0,1} = d_{1,0} = 0$, we get a tight grading on B_0. ∎

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Received by editors 20.05.2015; Available online 20.07.2015.

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