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Extension of Some Fixed Point Theorems of $\{a, b, c\}$-Type Generalized Mappings in Weakly Cauchy Normed Spaces

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Abstract. Let C be a closed convex weakly Cauchy subset of a normed space X. Then we define new $\{a, b, c\}$-type generalized nonexpansive mapping and $\{a, b, c\}$-type generalized contraction mapping T from C into C. These type of mappings will be denoted respectively by $\{a, b, c\}$-gntype and $\{a, b, c\}$-gctype. The aim of this paper is to establish some strong convergence results for such type of mappings. Our results extends and generalizes some of the results given in [**2**].

1. Introduction

Let C be a closed convex subset of a normed space X and T be a mapping from C into C such that

$$\|T(x) - T(y)\| \leqslant a\|x - y\| + b\|y - T(y)\|\| + c\|x - T(x)\|$$

for all $x, y \in C$ and for some real numbers $a, b, c \in [0, 1]$.

When $0 < a < 1, b = c = 0$, T is said to be a *contraction mapping* , if X is complete, S. Banach gave his famous Banach contraction principle, namely, T has a unique fixed point.

When $a = 1, b = c = 0$, T is said to be a *nonexpansive mapping*, if C is a bounded closed convex subset of a Banach space X, W.A. Kirk proved fixed point theorems concerning this type of mappings [**9**].

Recently, the existence of fixed points of T when the domain of T is unbounded discussed in [**7**]. If $a = 0$, T is said to be Kannan type mapping [**8**]. If $a+b+c < 1$

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a unique fixed point of T defined on a closed convex subset of a weakly Cauchy normed space is proved [**4**].

THEOREM 1.1. ([**4**]) *Let X be a normed space, C be a closed convex and weakly Cauchy subset of X and T be a mapping from C into C which satisfies*

$$\|T(x) - T(y)\| \leqslant a\|x - y\| + b\|y - T(y)\|\| + c\|x - T(x)\|$$

for all $x, y \in C$ and for some real numbers $a, b, c \in [0, 1]$ with $a + b + c < 1$. Then T has a unique fixed point $y \in C$.

If $0 < a < 1, b, c \geqslant 0$ & $a + b + c = 1$, T becomes Gregus type mapping, M. Gregus [**6**] proved the existence of a unique fixed point of such a mapping provided that C is closed convex subset of a Banach space X .

THEOREM 1.2. ([**6**]) *Let C be a closed convex subset of a Banach space X and T be a mapping from C into C which satisfies*

$$\|T(x) - T(y)\| \leqslant a\|x - y\| + b\|y - T(y)\|\| + c\|x - T(x)\|$$

for all $x, y \in C$ and for some real numbers $a, b, c \in [0, 1]$ with $0 < a < 1$ & $a + b + c = 1$. Then T has a unique fixed point $y \in C$.

More general contraction type mapping was given in [**5**], [**10**], [**11**]. It is proved that

THEOREM 1.3. ([**5**]) *Let (X, d) be a complete metric space and T be a mapping from X into X which satisfies*

$$d(T(x), T(y)) \leqslant ad(x, y) + bd(y, Ty) + cd(x, Tx) + ed(x, Ty) + fd(y, Tx)$$

for all $x, y \in C$ and for some real numbers $a, b, c, e, f \in [0, 1]$ with $a+b+c+e+f < 1$. Then T has a unique fixed point.

If $a + b + c = 1$, T becomes $\{a, b, c\}$-Generalized nonexpansive type mapping, Sahar Ali proved the existence of a unique fixed point of such a mapping when C is containing contraction point, closed, convex and weakly Cauchy subset of a normed space X [**1**].

The purpose of this paper is to introduce a new $\{a, b, c\}$-gntype and $\{a, b, c\}$-gctype mappings defined on a closed convex weakly Cauchy subset of a normed space not necessarily Banach in general and prove some strong convergence result for such mappings. Our result extends and generalizes some of the results given in [**2**].

2. Preliminaries

DEFINITION 2.1. Let X be normed space. Then a subset C of X is said to be weakly Cauchy if and only if every Cauchy sequence in C has a subsequence converging weakly to some point in X

DEFINITION 2.2. Let C be a subset of a normed space X and T be a mapping from C into C satisfying

$$\begin{aligned}\|T(x)-T(y)\| &\leqslant a\|x-y\| + b\max\{\|x-T(x)\|, \|y-T(y)\|\}\\ &\quad + c[\|x-T(y)\| + \|y-T(x)\|]\end{aligned}$$

for all $x, y \in C$ and some real numbers $a, b, c \in [0,1]$. Then
(a) T is said to be $\{a,b,c\}$-gntype mapping, if $0 < a < 1$, $0 < b$, $0 \leqslant c < 1$ and $a + b + 2c = 1$
(b) T is said to be $\{a,b,c\}$-gctype mapping, if $0 \leqslant c < \frac{1}{3}$ and $a + b + 2c < 1$

LEMMA 2.1. ([**2**]) *Let X be a normed space, C be a closed convex subset of X, $\{x_n\}_{n\in\mathcal{N}}$ be a Cauchy sequence in C that has a subsequence converging weakly to some point $y \in X$. Then $\{x_n\}_{n\in\mathcal{N}}$ has a subsequence converging strongly to y and $y \in C$.*

LEMMA 2.2. ([**2**]) *Let X be a normed space and T be a mapping from X into X, if there is a real number t, $t < 1$ which satisfies that for every $x \in X$ there exists $y \in X$ such that*

$$\|T(y)-y\| \leqslant t\|T(x)-x\|$$

Then $\inf\{\|T(x)-x\| : x \in X\} = 0$

3. Main Results

LEMMA 3.1. *Let X be a normed space, T be a mapping from X into X satisfying*

$$\begin{aligned}\|T(x)-T(y)\| &\leqslant a\|x-y\| + b\max\{\|x-T(x)\|, \|y-T(y)\|\}\\ &\quad + c[\|x-T(y)\| + \|y-T(x)\|]\end{aligned}$$

for all $x, y \in C$ and some positive real numbers $a, b, c \in [0,1]$. Then for any $x \in X$, the sequence of iterates $\{T_n(x)\}_{n\in\mathcal{N}}$ satisfy

$$\|T^{n+1}(x)-T^n(x)\| \leqslant k^n\|T(x)-x\| \tag{3.1}$$

PROOF. Let $x \in X$, we have

$$\begin{aligned}\|T^2(x)-T(x)\| &\leqslant a\|T(x)-x\| + b\max\{\|T(x)-T(T(x))\|, \|x-T(x)\|\}\\ &\quad + c[\|T(x)-T(x)\| + \|x-T(T(x))\|]\\ &\leqslant a\|T(x)-x\| + b\max\{\|T(x)-T(T(x))\|, \|x-T(x)\|\}\\ &\quad + c[\|x-T(x)\| + \|T(x)-T(T(x))\|]\end{aligned}$$

Now there are two cases
i.e., If $\max\{\|T(x)-T(T(x))\|, \|x-T(x)\|\} = \|T(x)-T(T(x))\|$, then

$$\begin{aligned} \|T^2(x)-T(x)\| &\leqslant (a+c)\|T(x)-x\| + (b+c)\|T(x)-T(T(x))\| \\ &\leqslant \Big(\frac{a+c}{1-(b+c)}\Big)\|T(x)-x\| \end{aligned} \tag{3.2}$$

Again, if $\max\{\|T(x)-T(T(x))\|, \|x-T(x)\|\} = \|T(x)-x\|$, then

$$\begin{aligned} \|T^2(x)-T(x)\| &\leqslant (a+b+c)\|T(x)-x\| + c\|T(x)-T(T(x))\| \\ &\leqslant \Big(\frac{a+b+c}{1-c}\Big)\|T(x)-x\| \end{aligned} \tag{3.3}$$

Again since

$$\begin{aligned} \|T^3(x)-T^2(x)\| = \|T(T^2(x))-T(T(x))\| \\ &\leqslant a\|T^2(x)-T(x)\| \\ &\quad + b\max\{\|T^2(x)-T(T^2(x))\|, \|T(x)-T(T(x))\|\} \\ &\quad + c[\|T^2(x)-T(T(x))\| + \|T(x)-T(T^2(x))\|] \\ &\leqslant a\|T^2(x)-T(x)\| + b\max\{\|T^2(x)-T^3(x)\|, \|T(x)-T^2(x)\|\} \\ &\quad + c[\|T(x)-T^2(x)\| + \|T^2(x)-T^3(x)\|] \end{aligned}$$

Again we have two cases.
i.e. If $\max\{\|T^2(x)-T^3(x)\|, \|T(x)-T^2(x)\|\} = \|T^2(x)-T^3(x)\|$, then

$$\begin{aligned} \|T^2(x)-T^3(x)\| &\leqslant \Big(\frac{a+c}{1-(b+c)}\Big)\|T^2(x)-T(x)\| \\ &\leqslant \Big(\frac{a+c}{1-(b+c)}\Big)^2\|T(x)-x\| \\ &\leqslant K_1^2\|T(x)-x\| \end{aligned} \tag{3.4}$$

where $K_1^2 = \Big(\frac{a+c}{1-(b+c)}\Big)$

Also, if$\max\{\|T^2(x)-T^3(x)\|, \|T(x)-T^2(x)\|\} = \|T(x)-T^2(x)\|$, then we have

$$\begin{aligned} \|T^2(x)-T^3(x)\| &\leqslant \Big(\frac{a+b+c}{1-c}\Big)\|T^2(x)-T(x)\| \\ &\leqslant \Big(\frac{a+b+c}{1-c}\Big)^2\|T(x)-x\| \\ &\leqslant K_2^2\|T(x)-x\| \end{aligned} \tag{3.5}$$

where $K_2^2 = \Big(\frac{a+b+c}{1-c)}\Big)$

Continuing in this way,we get

$$\|T^{n+1}(x)-T^n(x)\| \leqslant k^n\|T(x)-x\|$$

where $k = \max\{K_1^2, K_2^2\}$

This completes the proof of the Lemma. □

THEOREM 3.1. *Let C be a closed convex and weakly Cauchy subset of a normed space X, T be $\{a,b,c\}$-gntype mapping, then* $\inf\{\|T(x)-x\| : x \in X\} = 0$, *accordingly T has a unique fixed point. Moreover, any sequence $\{x_n\}_{n\in\mathcal{N}}$ in C with* $\lim_{n\to\infty} \|T(x_n) - x_n\| = 0$ *has a subsequence strongly convergent to the unique fixed point of T.*

PROOF. Using Lemma (3.1) with the fact that $k = 1$ as $a + b + 2c = 1$, the inequality (3.1) insure that for every $x \in C$ and $n \in \mathcal{N}$, we have

$$\|T^{n+1}(x) - T^n(x)\| \leqslant \|T^n(x) - T^{n-1}(x)\| \leqslant \|T(x) - x\|$$

On the other hand

$$\begin{aligned}
\|T^3(x) - T(x)\| &\leqslant a\|T^2(x) - x\| + b\max\{\|T^2(x) - T^3(x))\|, \|x - T(x)\|\} \\
&\quad + c[\|T^2(x) - T(x)\| + \|x - T^3(x)\|] \\
&\leqslant a\|T^2(x) - x\| + b\|T(x) - x\| \\
&\quad + c[\|T^2(x) - T(x)\| + \|T(x) - x\| \\
&\quad + \|T(x) - T^2(x)\| + \|T^2(x) - T^3(x)\|] \\
&\leqslant (a + b + 4c)\|T(x) - x\| = (1 + 2c)\|T(x) - x\|
\end{aligned}$$

Since C is convex, the element $y = \frac{1}{2}(T^2(x) + T^3(x))$ is in C, one has

$$\begin{aligned}
\|y - T(x)\| &\leqslant \frac{1}{2}[\|T(x) - T^2(x)\| + \|T(x) - T^3(x)\|] \\
&\leqslant \frac{1}{2}[\|T(x) - x\| + (1 + 2c)\|T(x) - x\|] \\
&\leqslant \frac{1}{2}(2 + 2c)\|T(x) - x\| \leqslant (1 + c)\|T(x) - x\|
\end{aligned}$$

$$\begin{aligned}
\|y - T^2(x)\| &= \frac{1}{2}\|T^3(x) - T^2(x)\| \leqslant \frac{1}{2}\|T(x) - x\| \\
\|y - T^3(x)\| &= \frac{1}{2}\|T^3(x) - T^2(x)\| \leqslant \frac{1}{2}\|T(x) - x\|
\end{aligned}$$

Then

$$\begin{aligned}
2\|T(y)-y\| &\leqslant \|T(y)-T^2(x)\|+\|T(y)-T^3(x)\| \\
&\leqslant a\|y-T(x)\|+b\max\{\|y-T(y)\|,\|T(x)-T^2(x)\|\} \\
&\quad +c[\|y-T^2(x)\|+\|T(x)-T(y)\|] \\
&\quad +a\|y-T^2(x)\|+b\max\{\|y-T(y)\|,\|T^2(x)-T^3(x)\|\} \\
&\quad +c[\|y-T^2(x)\|+\|T^2(x)-T(y)\|] \\
&\leqslant a(1+c)\|T(x)-x\|+b\max\{\|y-T(y)\|,\|x-T(x)\|\} \\
&\quad +c\Big[\Big(\frac{1}{2}\Big)\|T(x)-x\|+(1+c)\|T(x)-x\|+\|y-T(y)\|\Big] \\
&\quad +a\Big(\frac{1}{2}\Big)\|T(x)-x\|+c\Big[\Big(\frac{1}{2}\Big)\|T(x)-x\| \\
&\quad +\Big(\frac{1}{2}\Big)\|T(x)-x\|+\|y-T(y)\|\Big] \\
&\leqslant \Big(\frac{1}{2}\Big)\Big\{2a(a+c)+c+2c(1+c)+a+2c\Big\}\|T(x)-x\| \\
&\quad +2c\|y-T(y)\|+b\max\{\|y-T(y)\|,\|x-T(x)\|\}
\end{aligned}$$

Now there are two cases.
i.e. If $\max\{\|y-T(y)\|,\|x-T(x)\|\}=\|x-T(x)\|$, then

$$\begin{aligned}
2\|T(y)-y\| &\leqslant \Big(\frac{1}{2}\Big)\Big\{2a(a+c)+3c+2c(1+c)a+2b\Big\}\|T(x)-x\| \\
&\quad +2c\|y-T(y)\|
\end{aligned}$$

$$\begin{aligned}
\|T(y)-y\| &\leqslant \Big\{\frac{3a+2b+5c+2ac+2c^2}{4(1-c)}\Big\}\|T(x)-x\| \\
&\leqslant \Big\{\frac{4(1-c)-\{1+b+c(3+2a+2c)\}}{4(1-c)}\Big\}\|T(x)-x\| \\
&\leqslant \Big\{1-\frac{(1+b+c(3+2a+2c)}{4(1-c)}\Big\}\|T(x)-x\| \qquad (3.6)
\end{aligned}$$

Also, if $\max\{\|y-T(y)\|,\|x-T(x)\|\}=\|y-T(y)\|$, then

$$\begin{aligned}
2\|T(y)-y\| &\leqslant \Big(\frac{1}{2}\Big)\Big\{2a(1+c)+c+2c(1+c)+a+2c\Big\}\|T(x)-x\| \\
&\quad +(b+2c)\|T(y)-y\|
\end{aligned}$$

$$\|T(y)-y\| \leqslant \Big(\frac{1}{2}\Big)\Big\{\frac{3a+2ac+5c+2c^2}{2-b-2c}\Big\}\|T(x)-x\|$$
$$\leqslant \Big\{\frac{2a+2ac+3c+2c^2+1-b}{2(1+2a)}\Big\}\|T(x)-x\|$$
$$\leqslant \Big\{\frac{2(1+2a)-1-b-2a+3c+2ac+2c^2}{2(1+2a)}\Big\}\|T(x)-x\|$$
$$\leqslant \Big\{1-\frac{(1+b+2a+c(3+2a+2c)}{2(1+2a)}\Big\}\|T(x)-x\| \tag{3.7}$$

In both of the cases (3.6) and (3.7), we can write

$$\|T(y)-y\| \leqslant t\|T(x)-x\| \tag{3.8}$$

where t is a positive real number with $t<1$. Now using the Lemma (2.2), we see that $\inf\{\|T(x)-x\| : x \in X\} = 0$.
Pick any sequence $\{x_n\}_{n\in\mathcal{N}}$ with $\lim_{n\to\infty}\|T(x_n)-x_n\| = 0$. We claim that such a sequence is Cauchy sequence in C.
In fact, we have

$$\begin{aligned}
\|x_m-x_n\| &\leqslant \|T(x_m)-x_m\| + \|x_n-T(x_n)\| + \|T(x_m)-T(x_n)\| \\
&\leqslant \|T(x_m)-x_m\| + \|x_n-T(x_n)\| + a\|x_m-x_n\| \\
&\quad + b\max\{\|x_m-T(x_m)\|, \|x_n-T(x_n)\|\} \\
&\quad + c[\|x_m-T(x_n)\| + \|x_n-T(x_m)\|] \\
&\leqslant \|T(x_m)-x_m\| + \|x_n-T(x_n)\| + a\|x_m-x_n\| \\
&\quad + b\max\{\|x_m-T(x_m)\|, \|x_n-T(x_n)\|\} \\
&\quad + c[\|x_m-x_n\| + \|x_n-T(x_n)\| + \|x_n-x_m\| + \|x_m-T(x_m)\|] \\
&\leqslant \|T(x_m)-x_m\| + \|x_n-T(x_n)\| + (a+2c)\|x_m-x_n\| \\
&\quad + b\max\{\|x_m-T(x_m)\|, \|x_n-T(x_n)\|\} \\
&\quad + c[\|x_n-T(x_n)\| + \|x_m-T(x_m)\|]
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\|x_m-x_n\| \leqslant \frac{1}{1-(a+2c)}\Big\{&\|T(x_m)-x_m\| + \|x_n-T(x_n)\| \\
&+ b\max\{\|x_m-T(x_m)\|, \|x_n-T(x_n)\|\}\Big\} \\
&+ c[\|x_n-T(x_n)\| + \|x_m-T(x_m)\|]
\end{aligned}$$

Taking limit as $n\to\infty$ proves that $\{x_n\}_{n\in\mathcal{N}}$ is a Cauchy sequence in C. Since C is weakly Cauchy subset of X, the sequence $\{x_n\}_{n\in\mathcal{N}}$ has subsequence converging weakly to some point $y_0 \in X$, since C is closed convex, using Lemma (2.1), we see that $\{x_n\}_{n\in\mathcal{N}}$ has subsequence converging strongly to y_0 and $y_0 \in C$.

Now,

$$\begin{aligned}\|T(y_0)-y_0\| &\leqslant \|T(y_0)-T(x_n)\|+\|T(x_n)-x_n\|+\|x_n-y_0\|\\ &\leqslant a\|y_0-x_n\|+b\max\{\|y_0-T(y_0)\|,\|x_n-T(x_n)\|\}\\ &\quad + c[\|y_0-T(x_n)\|+\|x_n-T(y_0)\|+\|T(x_n)-x_n\|\\ &\quad + \|x_n-y_0\|\end{aligned}$$

Now, if $\max\{\|y_0-T(y_0)\|,\|x_n-T(x_n)\|\}=\|x_n-T(x_n)\|$, then we see that

$$\|T(y_0)-y_0\| \leqslant \frac{1}{1-c}\{(a+2c+1)\|y_0-x_n\|+(b+c+1)\|x_n-T(x_n)\|\}$$

Taking limit as $n\to\infty$ yields $T(y_0)=y_0$.
Similarly, if $\max\{\|y_0-T(y_0)\|,\|x_n-T(x_n)\|\}=\|y_0-T(y_0)\|$, then

$$\|T(y_0)-y_0\| \leqslant \frac{1}{1-(b+c)}\{(a+2c+1)\|y_0-x_n\|+(c+1)\|x_n-T(x_n)\|\}$$

Taking limit as $n\to\infty$ yields $T(y_0)=y_0$.

Uniqueness: Let y and z be two distinct fixed point of T, then

$$\begin{aligned}\|y-z\| &= \|T(y)-T(z)\|\\ &\leqslant a\|y-z\|+b\max\{\|y-T(y)\|,\|z-T(z)\|\}\\ &\quad + c[\|y-T(z)\|+\|z-T(y)\|]\\ &= (a+2c)\|y-z\| < \|y-z\|\end{aligned}$$

This completes the proof of the Theorem. □

THEOREM 3.2. *Let C be a closed convex and weakly Cauchy subset of a normed space X, T be $\{a,b,c\}$-gctype mapping from C into C, then T has a unique fixed point. Moreover, for any $x\in C$ the sequence of iterates $\{T^n(x)\}_{n\in\mathcal{N}}$ has a subsequence strongly convergent to the unique fixed point of T.*

PROOF. Using Lemma (3.1) with the fact that $k<1$, the inequality (3.1) insure that for every $m,n\in\mathcal{N}$ and $n\leqslant m$, we have

$$\|T^m(x)-T^n(x)\| \leqslant \Big[\frac{k^n}{1-k}\Big]\|T(x)-x\|$$

Taking limit as $n\to\infty$ proves that the sequence of iterates $\{T^n(x)\}_{n\in\mathcal{N}}$ is a Cauchy sequence in C, since C is weakly Cauchy, the sequence $\{T^n(x)\}_{n\in\mathcal{N}}$ has subsequence $\{T^{i_n}(x)\}_{n\in\mathcal{N}}$ converging weakly to some point $y\in X$, since C is closed convex, the sequence $\{T^{i_n}(x)\}_{n\in\mathcal{N}}$ is strongly convergent to y and $y\in C$. Taking the limit of each side of the inequality (3.1) as $n\to\infty$ and using the fact that $k<1$, we prove that $\lim_{n\to\infty}\|T^{i_{n+1}}(x)-T^{i_n}(x)\|=0$, hence

$$\lim_{n\to\infty}\|T^{i_{n+1}}(x)-T^{i_n}(x)\|=0$$

On the other hand

$$\begin{aligned}\|T(y)-T^{i_{n+1}}(x)\| &\leqslant a\|y-T^{i_n}(x)\| + b\max\{\|y-T(y)\|, \|T^{i_n}(x)-T^{i_{n+1}}(x)\|\}\\ &\quad + c[\|y-T^{i_{n+1}}(x)\| + \|T^{i_n}(x)-T(y)\|]\\ &\leqslant a\|y-T^{i_n}(x)\| + b\max\{\|y-T(y)\|, \|T^{i_n}(x)-T^{i_{n+1}}(x)\|\}\\ &\quad + c[\|y-T^{i_{n+1}}(x)\| + 2\|T(y)-T^{i_n}(x)\|\\ &\quad + \|T^{i_n}(x)-T(y)\|]\end{aligned}$$

Accordingly, we have

$$\begin{aligned}\|T(y)-y\| &\leqslant \|T(y)-T^{i_{n+1}}(x)\| + \|T^{i_{n+1}}(x)-T^{i_n}(x)\| + \|T^{i_n}(x)-y\|\\ &\leqslant \left[\frac{1}{1-2c}\right]\Big\{a\|y-T^{i_n}(x)\| + b\max\{\|y-T(y)\|, \|T^{i_n}(x)-T^{i_{n+1}}(x)\|\}\\ &\quad + c[\|y-T(y)\| + \|T^{i_n}(x)-T(y)\|]\Big\}\\ &\leqslant \left[\frac{1}{1-2c}\right]\Big\{a\|y-T^{i_n}(x)\| + b\max\{\|y-T(y)\|, \|T^{i_n}(x)-T^{i_{n+1}}(x)\|\}\\ &\quad + c\|T^{i_n}(x)-T(y)\|\Big\} + \left[\frac{c}{1-2c}\right]\|y-T(y)\|\end{aligned}$$

Thus,

$$\begin{aligned}\|T(y)-y\| &\leqslant \left[\frac{1}{1-3c}\right]\Big\{a\|y-T^{i_n}(x)\| + b\max\{\|y-T(y)\|, \|T^{i_n}(x)-T^{i_{n+1}}(x)\|\}\\ &\quad + c\|T^{i_n}(x)-T(y)\|\Big\}\end{aligned}$$

Taking the limit as $n \to \infty$, we get

$$\begin{aligned}\|T(y)-y\| &\leqslant \left[\frac{1}{1-3c}\right][b\|y-T(y)\|]\\ &\leqslant \left[\frac{b}{1-3c}\right]\|y-T(y)\|\end{aligned}$$

which proves that $T(y) = y$. The uniqueness of fixed point follows from the last part of the Theorem (3.1).
This completes the proof of the theorem. □

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