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# Approximate analytical solutions of MHD flow of a viscous fluid on a nonlinear porous shrinking sheet

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Abstract. The paper presents the semi-numerical solution for the magnetohydrodynamic (MHD) flow due to nonlinear porous shrinking sheet caused by boundary layer of an incompressible viscous flow. The governing partial differential equations of momentum equations are reduced into ordinary differential equation by using a classical similarity transformation along with appropriate boundary conditions. Both nonlinearity and infinite interval demand novel mathematical tools for their analysis. We use fast converging Dirichlet series and Method of stretching of variables for the solution of these nonlinear differential equations. These methods have the advantages over pure numerical methods for obtaining the derived quantities accurately for various values of the parameters involved at a stretch and also they are valid in much larger parameter domain as compared with HAM, HPM, ADM and the classical numerical schemes.

## 1. Introduction

The boundary layer flow induced by stretching surface moving with a certain velocities in an otherwise quiescent fluid medium often occurs in several engineering processes. Such flows have many important applications in industries, for example in the extrusion of a polymer sheet from a die or in the drawing of plastic films. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The mechanical properties of the final product strictly depend on the stretching and cooling rates in the process. The phenomena of velocities on the boundary towards a fixed point are known as shrinking phenomena, which often occur in the situations such as rising shrinking

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balloon. Only limited attention has been focussed on the study of shrinking phenomena [1-9]. In certain situations, the shrinking sheet solutions do not exist, since the velocity cannot be confined in a boundary layer. These solutions may exist if either the magnetic field or the stagnation flow is taken into account. Crane [10] found a closed form solution for steady two-dimensional stretching where the velocity on the boundary is away and proportional to the distance from the fixed point. The more basic stretching solutions differ from Cranes are as follows. Gupta and Gupta [11] added suction or injection on the surface. Brady and Acrivos [12] considered the flow inside a stretching channel or tube and the flow outside a stretching tube by Wang [13]. The three-dimensional and axisymmetric stretching surface was studied by Wang [14]. The unsteady stretching film was studied by Wang [15] and Usha and Sridharan [16].

From continuity of Cranes stretching sheet solution would induce a far field suction towards the sheet, while the shrinking sheet would cause a velocity away from the sheet. Thus from the physical grounds vorticity of the shrinking sheet is not confined within the boundary layer and the flow unlikely to exist unless adequate suction on the boundary is imposed. The purpose of this paper is to study the properties of the flow due to a shrinking sheet with suction. Recently, Ali et al. [17] discussed boundary layer flow and heat transfer due to permeable shrinking sheet with prescribed surface heat flux by Keller-box method. Noor et al [18] examined the simple non-perturbative solution for MHD viscous flow due to a shrinking sheet by series solution using Adomain decomposition method (ADM). Raftari and Yildirim [19], examined the MHD viscous flow due to a shrinking sheet by employing the homotopy perturbation method (HPM) and Pade approximants. Bhattacharyya [20] analysed the effects of heat (source/sink) on the steady two dimensional MHD boundary layer flow and heat transfer over a shrinking sheet with wall mass suction using finite difference method.

The present paper is to discuss the semi-numerical solution of two-dimensional MHD flow of a viscous fluid on a nonlinear porous shrinking sheet. The solution of the resulting third order nonlinear boundary value problem with an infinite interval is obtained using Dirichlet series method and method of stretching of variables. We seek solution of the general equation of the type

$$f''' + Aff'' + Bf'^2 + Cf' = 0 \tag{1.1}$$

The boundary conditions for the problem under consideration are

$$f(0) = \alpha_1 = f_w, \quad f'(0) = \beta_1, \quad f'(\infty) = 0 \tag{1.2}$$

where $A$, $B$ and $C$ are constants and prime denotes derivative with respect to the independent variable $\eta$. This equation admits a Dirichlet series solution; necessary conditions for the existence and uniqueness of these solutions may also be found in [21, 22]. For a specific type of boundary conditions i.e. $f'(\infty) = 0$, the Dirichlet series solution is particularly useful for obtaining the derived quantities exactly. A general discussion of the convergence of the Dirichlet series may also be found in Riesz [23]. The accuracy as well as uniqueness of the solution can be confirmed using other powerful semi-numerical schemes. Sachdev et al. [24] have analysed various problems from fluid dynamics of stretching sheet using this approach and found more accurate solution compared with earlier numerical findings. Recently, Vishwanath et al. [25, 26] and Ramesh et al. [27] have analysed the problems from MHD boundary layer flow with nonlinear stretching sheet using the above methods and found more accurate results compared with the classical numerical methods. In this article, we also present Dirichlet series solution and an approximate analytical method called method of stretching of variables. This method is quite easy to use especially for nonlinear ordinary differential equations and requires less computer time compared with pure numerical methods and easy to solve compared with other approximate methods (for example, Homotopy analysis method (HAM)).

## 2. Mathematical Formulation of the problem

Consider the MHD flow of an incompressible viscous fluid over a nonlinear porous shrinking sheet at $y = 0$. The fluid is electrically conducting under the influence of an applied magnetic filed $B(x)$ normal to the porous and shrinking sheet. By neglecting induced magnetic field, the resulting steady two-dimensional boundary layer equations are of the form (see Nadeem and Hussain [28])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma {B_0}^2}{\rho}u \tag{2.2}$$

where $u$ and $v$ are the velocity components in the $x$ and $y$ directions, $\nu$ is the kinematic viscosity, $\rho$ is the fluid density and $\sigma$ is the electrical conductivity of the fluid. To obtain similarity solutions, we assume that the external electrical and polarization effects are negligible in Eq. (2.2) and the magnetic field $B(x)$ is considered in the form (see Chaim [29])

$$B(x) = B_0 x^{\frac{n-1}{2}} \tag{2.3}$$

The relevant boundary conditions for the present flow are

$$u(x,y) = -cx^n, \quad v(x,y) = -V_0 x^{\frac{n-1}{2}}, \quad u(x,y) \to 0 \ \text{ as } \ y \to 0 \tag{2.4}$$

where $V_0$ is the porosity of the plate. Eqs. (2.1) to (2.3) along with the boundary conditions (2.4) admit similarity solution. We use following similarity variables and non-dimensional variables

$$\eta = \sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}y, \quad u = cx^n f'(\eta) \tag{2.5}$$

$$v = -\sqrt{\frac{c\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right] \tag{2.6}$$

and substituting them into Eqs. (2.1)-(2.4) to obtain the following nonlinear ordinary differential equation

$$f''' + ff'' - \beta f'^2 - M^2 f' = 0, \quad ' = \frac{d}{d\eta}, \tag{2.7}$$

and the boundary conditions are

$$f = f_w, \quad f' = -1, \quad \text{at} \quad \eta = 0 \tag{2.8}$$

$$f' \to 0 \quad \eta \to \infty \tag{2.9}$$

where $f_w = \frac{V_0}{\sqrt{\frac{c\nu(n+1)}{2}}}$ is the wall mass transfer parameter, $M = \frac{2\sigma {B_0}^2}{\rho c(1+n)}$ is the magnetic parameter and $\beta = \frac{2n}{n+1}$ is the non-dimensional parameter.

## 3. Dirichlet Series Solution

We use Dirichlet series which is an elegant semi-numerical scheme to solve the problem exactly. We seek Dirichlet series solution of Eq. (1.1) satisfying last boundary condition $f'(\infty) = 0$ automatically in the form (Kravchenko & Yablonskii [21 ,22])

$$f = \gamma_1 + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i e^{-i\gamma\eta} \tag{3.1}$$

where $\gamma$ and $a$ are parameters which are to be determined. Substituting Eq.(3.1) into Eq.(1.1), we get

(3.2)
$$\sum_{i=1}^{\infty}\{-\gamma i^3 + A\gamma\gamma_1 i^2 - Ci\} b_i a^i e^{-i\gamma\eta} + \frac{6\gamma}{A}\sum_{i=2}^{\infty}\sum_{k=1}^{i-1}\{Ak^2 + Bk(i-k)\} b_k b_{i-k} a^i e^{-i\gamma\eta} = 0$$

$$\text{For } \ i = 1, \text{ we have } \ \gamma_1 = \frac{\gamma + C}{A} \tag{3.3}$$

Substituting Eq. (3.3) into Eq. (3.2) the recurrence relation for obtaining coefficients is given by

$$b_i = \frac{6\gamma^2}{Ai(i-1)(\gamma^2 i - C)}\sum_{k=1}^{i-1}\{Ak^2 + Bk(i-k)\} b_k b_{i-k} \tag{3.4}$$

For $i = 2, 3, 4, \ldots$. If the Eq. (3.1) converges absolutely when $\gamma > 0$ for some $\eta_0$, this series converges absolutely and uniformly in the half plane $\mathrm{Re}\eta \geqslant \mathrm{Re}\eta_0$ and represents an analytic $\frac{2\pi i}{\gamma}$ periodic function $f = f(\eta_0)$ such that $f'(\infty) = 0$ (Kravchenko & Yablonskii [21]).

The Eq. (3.1) contains two free parameters namely $a$ and $\gamma$. These unknown parameters are determined from the remaining boundary conditions of Eq. (1.2) at $\eta = 0$.

$$f(0) = \frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i = \alpha_1 \tag{3.5}$$

and

$$f'(0) = \frac{6\gamma^2}{A}\sum_{i=1}^{\infty}(-i)b_i a^i = \beta_1 \tag{3.6}$$

The solution of the above transcendental Eq. (3.5) and Eq. (3.6) yield constants $a$ and $\gamma$. The solution of the above transcendental equations is equivalent to the unconstrained minimization of the functional

$$\left[\frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i - \alpha_1\right]^2 + \left[\frac{6\gamma^2}{A}\sum_{i=1}^{\infty}(-i)b_i a^i - \beta_1\right]^2 \tag{3.7}$$

We use Powells method of conjugate directions (Press et al. [29]) which is one of the most efficient techniques for solving unconstrained optimization problems. This helps in finding the unknown parameters $a$ and $\gamma$ uniquely for different values of the parameters $A$, $B$, $C$, $\alpha_1$ and $\beta_1$. Alternatively, Newtons method is also used to determine the unknown parameters $a$ and $\gamma$ accurately.
The shear stress at the surface of the problem is given by

$$f''(0) = \frac{6\gamma^2}{A}\sum_{i=1}^{\infty} b_i a^i (i\gamma)^2 \tag{3.8}$$

The velocity profiles of the problem is given by

$$f''(0) = \frac{6\gamma^2}{A}\sum_{i=1}^{\infty} -ib_i a^i e^{i\gamma\eta} \tag{3.9}$$

## 4. Method of Stretching of Variables

Many nonlinear ODE arising in MHD problems are not amenable for obtaining analytical solutions. In such situations, attempts have been made to develop approximate methods for the solution of these problems. The numerical approach is always based on the idea of stretching of variables of the flow problems. Method of stretching of variables is used here for the solution of such problems. In this method, we have to choose suitable derivative function $H'$ such that the derivative boundary conditions are satisfied automatically and integration of $H'$ will satisfy the remaining boundary condition. Substitution of this resulting function into the given equation gives the residual of the form $R(\zeta, \alpha)$ which is called defect function. Using Least squares method, the residual of the defect function can be minimized (for details see (Ariel, [30]).
Using the transformation $f = f_w + F$ into Eq. (1.1), we get

$$F''' + A(f_w + F)F'' + BF'^2 + CF' = 0, \;\; ' = \frac{d}{d\eta} \tag{4.1}$$

and the boundary conditions (1.2) become

$$F(0) = 0, \;\; F'(0) = -1, \;\; F'(\infty) = 0 \tag{4.2}$$

We introduce two variables $\zeta$ and $G$ in the form

$$G(\zeta) = \alpha F\eta \;\text{ and }\; \zeta = \alpha\eta \tag{4.3}$$

where $\alpha > 0$, is an amplification factor. In view of Eq. (4.3), the system (4.1-4.2) are transformed to the form

$$\alpha^2 G''' + A(f_w\alpha + G)G'' + BG'^2 + CG' = 0, \ ' = \frac{d}{d\zeta} \tag{4.4}$$

and the boundary conditions in Eq. (4.2) become

$$G(0) = 0, \ G'(0) = -1, \ G'(\infty) = 0 \tag{4.5}$$

We choose a trail velocity profile

$$G' = -\exp(-\zeta) \tag{4.6}$$

which satisfies the derivative conditions in Eq. (4.5). Integrating Eq. (4.6) with respect to $\zeta$ from 0 to $\zeta$ using conditions (4.5), we get

$$G = \exp(-\zeta) - 1 \tag{4.7}$$

Substituting Eq. (4.7) into Eq. (4.4), we get the residual of defect function

$$R(\zeta, \alpha) = (-\alpha^2 + Af_w\alpha - A - C)\exp(-\zeta) + (A + B)\exp(-2\zeta) \tag{4.8}$$

By using the least squares method as discussed in Ariel [30], the Eq. (4.8) can be minimized for which

$$\frac{\partial}{\partial\alpha}\int_0^\infty R^2(\zeta, \alpha)d\zeta = 0 \tag{4.9}$$

Substituting (4.8) into Eq. (4.9) and solving cubic equation in $\alpha$ for a positive root, we get

$$\alpha = \frac{1}{6}(3Af_w \pm \sqrt{3(-4A + 8B - 12C + 3A^2 f_w^2)}) \tag{4.10}$$

Once the amplification factor is calculated, then using Eq. (4.1), original function $f$ can be written as

$$f = f_w + \frac{1}{\alpha}(\exp(-\alpha\eta) - 1) \tag{4.11}$$

with $\alpha$ defined in Eq. (4.10). Thus Eq. (4.11) gives the solution of Eq. (1.1) for all $A$, $B$ & $C$.

## 5. Results and Discussion

In the present paper we discuss the semi-numerical solution of two-dimensional electrically conducting viscous fluid past a porous nonlinear porous shrinking sheet. The governing equations are simplified by suitable similarity transformation and the reduced third order nonlinear boundary value problems with infinite domain are solved semi-numerically using an elegant powerful technique which are Dirichlet series method and an approximate analytical method- method of stretching of variables. We have given exact analytical solution of the nonlinear boundary value problem in more general form. In this method it is important that the edge boundary layer $\eta \to \infty$ automatically satisfied. Numerical computations are performed for various values of the physical parameters involved in the equation viz. the magnetic parameter $M$, non-dimensional parameter $\beta$ and the wall mass transfer parameter $s = f_w$. The present solution is also validated by comparing it with the previously published work of Nadeem and Hussain [28] (see Tables 1-3). Table 4 presents shear stress at the wall i.e $f''(0)$ for different parameters viz. $M, \beta$ and $s = f_w$ is obtained by using Dirichlet series and method of stretching of variables and the results are compared.

The graphs for the function $f'(\eta)$ i.e. velocity profiles which corresponds to velocity component $u$ and $v$ are drawn against $\eta$ for different values of the parameters $s = f_w$ at $M = 4$, $\beta = 1$ and $M = 1$, $\beta = 1$ are shown in Fig.1 and 2 respectively. It is observed that in Fig. 1 when $s = f_w$ increases, the velocity profiles are far away from the wall for mass injection, and the boundary layer thickness is more and more thicker. From Fig. 2 it is evident that the boundary layer is near the wall for large values of $s = f_w$. It is concluded that the results obtained are comparable with those in [4] by taking $\beta = 1$. The velocity profiles $f'(\eta)$ against $\eta$ for different values of $M$ with $s = f_w = 0.1$ and $\beta = 0.1$ are shown in Fig.3. It is evident that in Fig.3 when $M$ increases, the velocity profile is more and more far away from the wall, and the boundary layer thickness is more and more thicker.

The above computations work is very well by using Dirichlet series and method of stretching of variables. It is also susceptible to the computers memory limitations and takes very less computer memory. In this work we utilize Mathematica and FORTRAN compiler running on a personal computer with Pentium processor.

## 6. Conclusions

In this article, we describe the analysis of boundary value problem for third order nonlinear ordinary differential equation over an infinite interval arising in MHD boundary layer flow of viscous fluid of a nonlinear porous shrinking sheet. The semi-numerical schemes described here offer advantages over solutions obtained by HAM and numerical methods etc. The convergence of the Dirichlet series method is given. The results are presented in Tables and graphically, the effects of the emerging parameters are discussed semi-numerically.

| $C = -M$ | Dirichlet Series Method | | | HAM[28] | Method of stretching of variable |
|---|---|---|---|---|---|
| | $a$ | $\gamma$ | $f''(0)$ | | |
| -2.0 | 0.054731 | 1.643822 | 1.849758 | 1.61804 | 1.86015 |
| -2.1 | 0.052450 | 1.685285 | 1.883383 | 1.66190 | 1.89642 |
| -2.2 | 0.050357 | 1.725497 | 1.916484 | 1.70416 | 1.93178 |
| -2.3 | 0.048301 | 1.764556 | 1.942798 | 1.74499 | 1.96629 |
| -2.4 | 0.046382 | 1.803412 | 1.990647 | 1.78452 | 2.00000 |
| -2.5 | 0.044769 | 1.840313 | 2.021417 | 1.82288 | 2.03297 |
| -2.6 | 0.043268 | 1.876307 | 2.051775 | 1.86015 | 2.06525 |
| -2.7 | 0.041867 | 1.911455 | 2.081734 | 1.89642 | 2.09687 |
| -2.8 | 0.040386 | 1.946396 | 2.118864 | 1.93178 | 2.12788 |
| -2.9 | 0.039178 | 1.979943 | 2.147477 | 1.96629 | 2.15831 |
| -3.0 | 0.038042 | 2.012796 | 2.175738 | 2.00000 | 2.18819 |

TABLE 1. Comparison of the values of $f''(0)$ for different values of $M$ with $A = 1$, $B = -\beta = 0$ and $\alpha_1 = f_w = 1.0$ obtained by Dirichlet series method, Method of stretching of variables and other applied methods

| $\alpha_1 = f_w$ | Dirichlet Series Method | | | HAM[28] | Method of stretching of variable |
|---|---|---|---|---|---|
| | $a$ | $\gamma$ | $f''(0)$ | | |
| 0.0 | 0.124039 | 1.047661 | 1.281245 | 1.27094 | 1.26491 |
| 0.1 | 0.113082 | 1.098286 | 1.338243 | 1.31874 | 1.31590 |
| 0.2 | 0.105488 | 1.147827 | 1.375143 | 1.36855 | 1.36886 |
| 0.3 | 0.096194 | 1.203188 | 1.436081 | 1.42041 | 1.42377 |
| 0.4 | 0.089326 | 1.258175 | 1.481007 | 1.47430 | 1.48062 |
| 0.5 | 0.081631 | 1.322876 | 1.571061 | 1.53024 | 1.53938 |
| 0.6 | 0.075527 | 1.378300 | 1.598066 | 1.58819 | 1.60000 |
| 0.7 | 0.069887 | 1.440873 | 1.653000 | 1.64812 | 1.66244 |
| 0.8 | 0.064606 | 1.505872 | 1.711642 | 1.71001 | 1.72665 |
| 0.9 | 0.059143 | 1.574649 | 1.786470 | 1.77379 | 1.79257 |
| 1.0 | 0.054731 | 1.643822 | 1.849758 | 1.83942 | 1.86015 |

TABLE 2. Comparison of the values of $f''(0)$ for different values of $\alpha_1 = f_w$ with $A = 1$, $B = -\beta = -1$, and $C = -M = -2.0$ obtained by Dirichlet series method, Method of stretching of variables and other applied methods

| $B=-\beta$ | Dirichlet Series Method | | | HAM[28] | Method of stretching of variable |
|---|---|---|---|---|---|
| | $a$ | $\gamma$ | $f''(0)$ | | |
| 0.0 | 0.053918 | 1.646281 | 1.873659 | 1.86201 | 1.88444 |
| 0.1 | 0.054731 | 1.643822 | 1.849758 | 1.83942 | 1.86015 |
| 0.2 | 0.055574 | 1.641291 | 1.825494 | 1.81648 | 1.83542 |
| 0.3 | 0.056449 | 1.638687 | 1.800918 | 1.79318 | 1.81022 |
| 0.4 | 0.057358 | 1.636006 | 1.775968 | 1.76949 | 1.78452 |
| 0.5 | 0.058303 | 1.633244 | 1.750655 | 1.74539 | 1.75831 |
| 0.6 | 0.059282 | 1.630411 | 1.725166 | 1.72087 | 1.73153 |
| 0.7 | 0.060308 | 1.627467 | 1.699012 | 1.69591 | 1.70416 |
| 0.8 | 0.061377 | 1.624427 | 1.672453 | 1.67046 | 1.67615 |
| 0.9 | 0.062494 | 1.621285 | 1.645467 | 1.64452 | 1.64746 |
| 1.0 | 0.063661 | 1.618034 | 1.618034 | 1.61804 | 1.61803 |

TABLE 3. Comparison of the values of $f''(0)$ for different values of $B=-\beta$ with $A=1$, $C=-M=-2.0$ and $\alpha_1=f_w=1.0$ obtained by Dirichlet series method, Method of stretching of variables and other applied methods

| $B=-\beta$ | $C=-M$ | $\alpha_1=f_w$ | Dirichlet Series Method | | | Method of stretching of variable |
|---|---|---|---|---|---|---|
| | | | $a$ | $\gamma$ | $f''(0)$ | |
| -0.1 | -2.1 | 0.1 | 0.106838 | 1.140902 | 1.365373 | 1.354798 |
| | -2.3 | | 0.094124 | 1.225686 | 1.442787 | 1.429310 |
| | -2.5 | | 0.084191 | 1.305187 | 1.514538 | 1.500000 |
| | -2.7 | | 0.077012 | 1.378848 | 1.569146 | 1.567399 |
| -0.1 | -1.0 | 0.1 | 0.323703 | 0.556776 | 0.833753 | 0.826209 |
| | | 0.3 | 0.271518 | 0.623609 | 0.987067 | 0.938987 |
| | | 0.5 | 0.218699 | 0.707107 | 1.125948 | 1.063940 |
| | | 0.7 | 0.228344 | 0.707106 | 1.230019 | 1.200000 |
| -0.1 | -1.0 | 1.0 | 0.105447 | 1.107895 | 1.413125 | 1.421950 |
| -0.3 | | | 0.114006 | 1.090687 | 1.335266 | 1.34656 |
| -0.5 | | | 0.147065 | 1.000000 | 1.223760 | 1.26376 |
| -0.7 | | | 0.137965 | 1.046524 | 1.157616 | 1.17082 |

TABLE 4. Comparison of the values of $f''(0)$ for different values of $\alpha_1=f_w$ $B=-\beta$, and $C=-M=$ with $A=1$, obtained by Dirichlet series method, Method of stretching of variables and other applied methods

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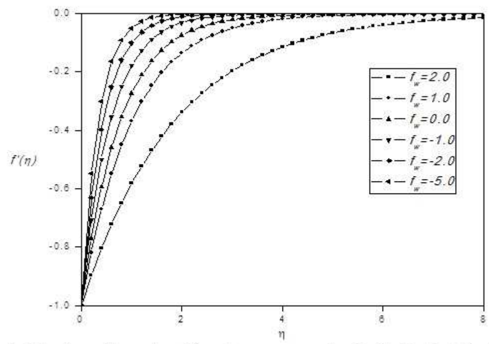


Fig.1 Velocity profiles under different mass parameter $f_w$ with C=-M=-4.0and B=-β=-1.0

FIGURE 1

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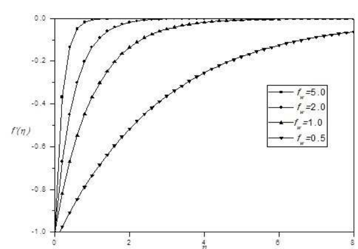


Fig.2 Velocity profiles under different mass parameters $f_w$ with C=-M=-1.0 and B=-β

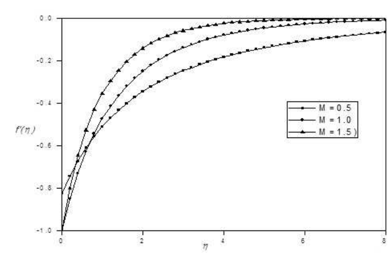


Fig 3 Velocity profiles for different values of C=-M with $f_w$=0.1, B=-β=-0.1

FIGURE 2 FIGURE 3

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