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Quadruple Coincidence Point Results in Partially Ordered Metric Spaces

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ABSTRACT. In this paper, we prove quadruple coincidence point theorems for mixed g-monotone mappings satisfying the compatibility property in partially ordered metric space.

1. Introduction

The fixed point theorems in metric spaces are playing a major role to construct methods in mathematics and to solve problems in applied mathematics and sciences. The existence of a fixed point in partially ordered metric and G-metric spaces has been considered in ([**1**]-[**4**]) and ([**5**]-[**10**]). The notion of coupled fixed points have been introduced by Guo and Laksmikantham [**3**] in connection with monotone operators, which is further generalized by Choudhury [**1**], Bessem Samet [**2**] and many more. Berinde and Borcut [**7**] introduced the concept of triple fixed point and proved some related theorems. The concept of quadruple fixed point is considered by Erdal Karapinar [**4**], Mustafa [**10**]. Here, our aim is to prove a unique quadruple coincidence point theorem for g-monotone mappings satisfying the compatibility property in partially ordered metric space.

2. Preliminaries

DEFINITION 2.1. [**10**] Let $(X, \leqslant)$ be partially ordered set and $F : X^4 \to X$. We say that F has the mixed g-monotone property, if for any $x, y, z, w \in X$,
$x_1, x_2 \in X, gx_1 \leqslant gx_2 \Rightarrow F(x_1, y, z, w) \leqslant F(x_2, y, z, w)$,
$y_1, y_2 \in X, gy_1 \leqslant gy_2 \Rightarrow F(x, y_2, z, w) \leqslant F(x, y_1, z, w)$,
$z_1, z_2 \in X, gz_1 \leqslant gz_2 \Rightarrow F(x, y, z_1, w) \leqslant F(x, y, z_2, w)$,

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$w_1, w_2 \in X, gw_1 \leqslant gw_2 \Rightarrow F(x,y,z,w_2) \leqslant F(x,y,z,w_1)$.

Definition 2.2. [**10**] An element $(x,y,z,w) \in X^4$ is called a quadruple coincidence point of $F : X^4 \to X$ and $g : X \to X$, if the following conditions are satisfied, $F(x,y,z,w) = g(x)$, $F(y,z,w,x) = g(y)$, $F(z,w,x,y) = g(z)$, $F(w,x,y,z) = g(w)$.

Definition 2.3. [**5**] Let (X,d) be a metric space and $\{x_n\} \subseteq X$. The mappings $f, g : X \to X$ are said to be compatible if,

$$\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that for some $x \in X$, such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x.$$

Now, we define a mapping $\overline{d} : X^4 \times X^4 \to X$ on (X,d) by:

$$\overline{d}((x,y,z,w),(u,v,h,l)) = d(x,u) + d(y,v) + d(z,h) + d(w,l),$$

which will be denoted for convenience by d. Also, let ψ denotes all functions $\phi : [0,\infty) \to [0,\infty)$, which satisfy:

(1) ϕ is non-decreasing,
(2) $\phi(t) < t$ for all $t > 0$,
(3) $lim_{r\to t^+} \phi(r) < t$ for all $t > 0$.

3. Main Result

Theorem 3.1. *Let $(X,\leqslant)$ be a partially ordered set and (X,d) be a complete metric space. Let $F : X^4 \to X$ be a mapping having the mixed g-monotone property on X, such that there exist four elements $x_0, y_0, z_0, w_0 \in X$, with*

$$gx_0 \leqslant F(x_0,y_0,z_0,w_0), gy_0 \geqslant F(y_0,z_0,w_0,x_0), \quad gz_0 \leqslant F(z_0,w_0,x_0,y_0) \text{ and } gw_0 \geqslant F(w_0,x_0,y_0,z_0). \tag{3.1}$$

Suppose there exist $\phi \in \psi$, $M \geqslant 0$ such that

$$d(F(x,y,z,w), F(u,v,h,l)) \leqslant \phi\left(\frac{d(gx,gu) + d(gy,gv) + d(gz,gh) + d(gw,gl)}{4}\right) \tag{3.2}$$

$\forall x,y,z,u,v,h,l \in X$ with $gx \geqslant gu, gy \leqslant gv, gz \geqslant gh$ and $gw \leqslant gl$. Also, let $F(X^4) \subseteq g(X)$ and F, g being continuous, monotone increasing and compatible mappings. Then F and g have quadruple coincidence point in X.

Proof. Suppose $x_0, y_0, z_0, w_0 \in X$ be given by (3.1) As $F(X^4) \subseteq g(X)$, therefore we can choose $x_1, y_1, z_1, w_1 \in X$ such that $gx_1 = F(x_0,y_0,z_0,w_0)$, $gy_1 = F(y_0,z_0,w_0,x_0)$,
$gz_1 = F(z_0,w_0,x_0,y_0)$, $gw_1 = F(w_0,x_0,y_0,z_0)$. Then we have,
$gx_0 \leqslant gx_1$, $gy_0 \geqslant gy_1$, $gz_0 \leqslant gz_1$ and $gw_0 \geqslant gw_1$. In the same way, we have
$gx_2 = F(x_1,y_1,z_1,w_1)$, $gy_2 = F(y_1,z_1,w_1,x_1)$,
$gz_2 = F(z_1,w_1,x_1,y_1)$, $gw_2 = F(w_1,x_1,y_1,z_1)$.

Since F has mixed g-monotone property, therefore we have
$gx_0 \leqslant gx_1 \leqslant gx_2$, $gy_2 \leqslant gy_1 \leqslant gy_0$, $gz_0 \leqslant gz_1 \leqslant gz_2$ and $gw_2 \leqslant gw_1 \leqslant gw_0$.
Continuing this process, we can construct four sequences $\{gx_n\}, \{gy_n\}, \{gz_n\}$ and $\{gw_n\}$ such that
$gx_n = F(x_{n-1}, y_{n-1}, z_{n-1}, w_{n-1}) \leqslant gx_{n+1} = F(x_n, y_n, z_n, w_n)$,
$gy_{n+1} = F(y_n, z_n, w_n, x_n) \leqslant gy_n = F(y_{n-1}, z_{n-1}, w_{n-1}, x_{n-1})$,
$gz_n = F(z_{n-1}, w_{n-1}, x_{n-1}, y_{n-1}) \leqslant gz_{n+1} = F(z_n, w_n, x_n, y_n)$,
$gw_{n+1} = F(w_n, x_n, y_n, z_n) \leqslant gw_n = F(w_{n-1}, x_{n-1}, y_{n-1}, z_{n-1})$.
Now, for any $n \in N$, we have

$$\begin{aligned} d(gx_{n+1}, gx_n) &= d(F(x_n, y_n, z_n, w_n), F(x_{n-1}, y_{n-1}, z_{n-1}, w_{n-1})) \\ &\leqslant \phi\left[\frac{d(gx_n, gx_{n-1}) + d(gy_n, gy_{n-1}) + d(gz_n, gz_{n-1}) + d(gw_n, gw_{n-1})}{4}\right], \end{aligned} \tag{3.3}$$

$$\begin{aligned} d(gy_n, gy_{n+1}) &= d(F(y_{n-1}, z_{n-1}, w_{n-1}, x_{n-1}), F(y_n, z_n, w_n, x_n)) \\ &\leqslant \phi\left[\frac{d(gy_{n-1}, gy_n) + d(gz_{n-1}, gz_n) + d(gw_{n-1}, gw_n) + d(gx_{n-1}, gx_n)}{4}\right], \end{aligned} \tag{3.4}$$

$$\begin{aligned} d(gz_{n+1}, gz_n) &= d(F(z_n, w_n, x_n, y_n), F(z_{n-1}, w_{n-1}, x_{n-1}, y_{n-1})) \\ &\leqslant \phi\left[\frac{d(gz_n, gz_{n-1}) + d(gw_n, gw_{n-1}) + d(gx_n, gx_{n-1}) + d(gy_n, gy_{n-1})}{4}\right], \end{aligned} \tag{3.5}$$

$$\begin{aligned} d(gw_n, gw_{n+1}) &= d(F(w_{n-1}, x_{n-1}, y_{n-1}, z_{n-1}), F(w_n, x_n, y_n, z_n)) \\ &\leqslant \phi\left[\frac{d(gw_{n-1}, gw_n) + d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n) + d(gz_{n-1}, gz_n)}{4}\right]. \end{aligned} \tag{3.6}$$

Due to equations (3.3)-(3.6), we obtain

$$\begin{aligned} & d(gx_{n+1}, gx_n) + d(gy_n, gy_{n+1}) + d(gz_{n+1}, gz_n) + d(gw_n, gw_{n+1}) \\ & \leqslant 4\phi\left[\frac{d(gx_n, gx_{n+1}) + d(gy_n, gy_{n-1}) + d(gz_n, gz_{n-1}) + d(gw_n, gw_{n-1})}{4}\right]. \end{aligned} \tag{3.7}$$

Let $d_n = d(gx_n, gx_{n+1}) + d(gy_n, gy_{n+1}) + d(gz_n, gz_{n+1}) + d(gw_n, gw_{n+1})$
Then equation (3.7) implies $d_n \leqslant 4\phi\left(\frac{d_{n-1}}{4}\right) \Rightarrow d_n < d_{n-1}$.
Thus (d_n) is decreasing sequence. Therefore there is some $d > 0$, such that

$$\lim_{n\to\infty} d_n = d \tag{3.8}$$

Now, we claim that $d = 0$. If not, then taking $n \to \infty$ of both sides of equation (3.6), we get
$d \leqslant \lim_{n\to\infty} 4\phi\left(\frac{d_n}{4}\right) < d$,
which is a contradiction. Hence $d = 0$, that is,

$$\lim_{n\to\infty} [d(gx_n, gx_{n+1}) + d(gy_n, gy_{n+1}) + d(gz_n, gz_{n+1}) + d(gw_n, gw_{n+1})] = 0. \tag{3.9}$$

Now, we will prove that $\{gx_n\}, \{gy_n\}, \{gz_n\}$ and $\{gw_n\}$ are Cauchy sequences. Suppose to contrary that at least one of these sequences is not a Cauchy sequence.

Then there exist an $\epsilon > 0$ for which we can find subsequences of integers (m_k) and (n_k) ,with $n(k) > m(k) > k$ such that

$$\begin{aligned}[t] &\left[d\left(gx_{n(k)}, gx_{m(k)}\right) + d\left(gy_{n(k)}, gy_{m(k)}\right) + \right. \\ &\left. d\left(gz_{n(k)}, gz_{m(k)}\right) + d\left(gw_{n(k)}, gw_{m(k)}\right)\right] \geqslant \epsilon. \end{aligned} \tag{3.10}$$

Further corresponding to $m(k)$, we can choose $n(k)$ in such a way, that it is the smallest integer with $n(k) > m(k)$ and satisfying equation (3.10), then

$$\begin{aligned}[t] &\left[d\left(gx_{n(k)-1}, gx_{m(k)}\right) + d\left(gy_{n(k)-1}, gy_{m(k)}\right) + \right. \\ &\left. d\left(gz_{n(k)-1}, gz_{m(k)}\right) + d\left(gw_{n(k)-1}, gw_{m(k)}\right)\right] < \epsilon. \end{aligned} \tag{3.11}$$

From equation (3.10), (3.11) and applying triangle inequality, we have

$$\begin{aligned} \epsilon \leqslant r_k &= d\left(gx_{n(k)}, gx_{m(k)}\right) + d\left(gy_{n(k)}, gy_{m(k)}\right) + d\left(gz_{n(k)}, gz_{m(k)}\right) \\ &+ d\left(gw_{n(k)}, gw_{m(k)}\right) \\ &\leqslant d\left(gx_{n(k)}, gx_{n(k)-1}\right) + d\left(gy_{n(k)}, gy_{n(k)-1}\right) \\ &+ d\left(gz_{n(k)}, gz_{n(k)-1}\right) + d\left(gw_{n(k)}, gw_{n(k)-1}\right). \end{aligned}$$

Letting $k \to \infty$ in above inequality with keeping in mind equation (3.8), we conclude that

$$\lim_{k\to\infty} r_k = \epsilon \tag{3.12}$$

Again employing triangle inequality, we obtain

$$\begin{aligned} r_k &= d\left(gx_{n(k)}, gx_{m(k)}\right) + d\left(gy_{n(k)}, gy_{m(k)}\right) + d\left(gz_{n(k)}, gz_{m(k)}\right) \qquad (3.13)\\ &+ d\left(gw_{n(k)}, gw_{m(k)}\right) \\ &\leqslant d_{n(k)} + d_{m(k)} + d\left(gx_{n(k)+1}, gx_{m(k)+1}\right) + d\left(gy_{n(k)+1}, gy_{m(k)+1}\right) \qquad (3.14)\\ &+ d\left(gz_{n(k)+1}, gz_{m(k)+1}\right) + d\left(gw_{n(k)+1}, gw_{m(k)+1}\right). \end{aligned}$$

As $n(k) > m(k)$, we have
$gx_{n(k)} \geqslant gx_{m(k)}$, $gy_{n(k)} \leqslant gy_{m(k)}$, $gz_{n(k)} \geqslant gz_{m(k)}$ and $gw_{n(k)} \leqslant gw_{m(k)}$.

Using equation (3.2), we obtain

$$d\left(gx_{n(k)+1}, gx_{m(k)+1}\right) \leqslant \phi\left(\frac{r_k}{4}\right). \tag{3.15}$$

Similarly,

$$d\left(gy_{m(k)+1}, gy_{n(k)+1}\right) = \phi\left(\frac{r_k}{4}\right), \tag{3.16}$$

$$d\left(gz_{n(k)+1}, gz_{m(k)+1}\right) = \phi\left(\frac{r_k}{4}\right), \tag{3.17}$$

$$d\left(gw_{m(k)+1}, gw_{n(k)+1}\right) = \phi\left(\frac{r_k}{4}\right). \tag{3.18}$$

Due to equation (3.14)-(3.17) and keeping in view the property of function ϕ, we get

$$d\left(gx_{n(k)+1}, gx_{m(k)+1}\right) + d\left(gy_{m(k)+1}, gy_{n(k)+1}\right) + d\left(gz_{n(k)+1}, gz_{m(k)+1}\right) + d\left(gw_{m(k)+1}, gw_{n(k)+1}\right) < r_k. \tag{3.19}$$

Hence, from equation (3.14) and (3.18), we get $r_k < d_{n(k)} + d_{m(k)} + r_k$.

Taking $k \to \infty$ and using equation (3.9), we conclude $r_k < r_k$. It is a contradiction.

Thus $\{gx_n\}, \{gy_n\}, \{gz_n\}$ and $\{gw_n\}$ are Cauchy sequences in X and since X is a complete metric space, therefore there exist $x, y, z, w \in X$ such that

$$\lim_{n\to\infty} F(x_n, y_n, z_n, w_n) = \lim_{n\to\infty} gx_n = x, \tag{3.20}$$

$$\lim_{n\to\infty} F(y_n, z_n, w_n, x_n) = \lim_{n\to\infty} gy_n = y, \tag{3.21}$$

$$\lim_{n\to\infty} F(z_n, w_n, x_n, y_n) = \lim_{n\to\infty} gz_n = z, \tag{3.22}$$

$$\lim_{n\to\infty} F(w_n, x_n, y_n, z_n) = \lim_{n\to\infty} gw_n = w. \tag{3.23}$$

Now, as F and g are compatible mappings, we have

$$\lim_{n\to\infty} d\left(g\left(F(x_n, y_n, z_n, w_n)\right), F(gx_n, gy_n, gz_n, gw_n)\right) = 0, \tag{3.24}$$

$$\lim_{n\to\infty} d\left(g\left(F(y_n, z_n, w_n, x_n)\right), F(gy_n, gz_n, gw_n, gx_n)\right) = 0, \tag{3.25}$$

$$\lim_{n\to\infty} d\left(g\left(F(z_n, w_n, x_n, y_n)\right), F(gz_n, gw_n, gx_n, gy_n)\right) = 0, \tag{3.26}$$

$$\lim_{n\to\infty} d\left(g\left(F(w_n, x_n, y_n, z_n)\right), F(gw_n, gx_n, gy_n, gz_n)\right) = 0. \tag{3.27}$$

Since F is continuous for all $n \geqslant 0$, we get

$$d\left(gx, F(gx_n, gy_n, gz_n, gw_n)\right) \leqslant$$
$$d\left(gx, g\left(F(x_n, y_n, z_n, w_n)\right)\right) + d\left(g\left(F(x_n, y_n, z_n, w_n)\right), F(gx_n, gy_n, gz_n, gw_n)\right).$$

On applying $n \to \infty$ and combining equation (3.18) and (3.22), we obtain

$$F(x, y, z, w) = gx,\ F(y, z, w, x) = gy,\ F(z, w, x, y) = gz \ and \ F(w, x, y, z) = gw.$$

Hence we conclude that F and g have a quadruple coincidence point in X. □

THEOREM 3.2. *In addition to the hypothesis of Theorem 3.1, suppose that for every* $(x, y, z, w), (x_1, y_1, z_1, w_1)$ *in* X^4*, there exists* (u, v, h, l) *that is comparable to* (x, y, z, w) *and* (x_1, y_1, z_1, w_1)*, then F and g have a unique quadruple coincidence point.*

Proof. From Theorem 3.1, the set of quadruple fixed points of F and g is non-empty. Suppose (x, y, z, w) and (x_1, y_1, z_1, w_1) are quadruple coincidence points of F and g, that is

$$gx = F(x, y, z, w), gy = F(y, z, w, x),$$
$$gz = F(z, w, x, y), gw = F(w, x, y, z) \text{ and}$$

And

$$gx_1 = F(x_1, y_1, z_1, w_1), gy_1 = F(y_1, z_1, w_1, x_1),$$
$$gz_1 = F(z_1, w_1, x_1, y_1), gw_1 = F(w_1, x_1, y_1, z_1).$$

We shall show that
$gx = gx_1, gy = gy_1, gz = gz_1$ *and* $gw = gw_1$.

By assumption, there exist $(u, v, h, l) \in X$, that is comparable to (x, y, z, w) and (x_1, y_1, z_1, w_1).

Now, we define sequences $\{gu_n\}, \{gv_n\}, \{gh_n\}$ *and* $\{gl_n\}$as follows:
$u_0 = u, v_0 = v, h_0 = h, l_0 = l, gu_{n+1} = F(u_n, v_n, h_n, l_n), gv_{n+1} = F(v_n, h_n, l_n, u_n)$
$gh_{n+1} = F(h_n, l_n, u_n, v_n)$ *and* $gl_{n+1} = F(l_n, u_n, v_n, h_n)$ for all $n \in N$.
Since (u, v, h, l) being comparable with (x, y, z, w), we may assume that

$$(x, y, z, w) \geqslant (u, v, h, l) = (u_0, v_0, h_0, l_0).$$

Applying mathematical induction, it is easy to prove that

$$(x, y, z, w) \geqslant (u_n, v_n, h_n, l_n) \text{ for all } n \in N.$$

Due to equation (3.2), we obtain

$$\begin{aligned} (3.28)\, d(gx, gu_{n+1}) &= d(F(x, y, z, w), F(u_n, v_n, h_n, l_n)) \\ &\leqslant \phi\left[\frac{d(gx, gu_n) + d(gy, gv_n) + d(gz, gh_n) + d(gw, gl_n)}{4}\right], \end{aligned}$$

Analogously

$$(3.29) \quad d(gv_{n+1}, gy) \leqslant \phi\left[\frac{d(gv_n, gy) + d(gh_n, gz) + d(gl_n, gw) + d(gu_n, gx)}{4}\right],$$

$$(3.30) \quad d(gz, gh_{n+1}) \leqslant \phi\left[\frac{d(gz, gh_n) + d(gw, gl_n) + d(gx, gu_n) + d(gy, gv_n)}{4}\right],$$

$$(3.31) \quad d(gw, gl_{n+1}) \leqslant \phi\left[\frac{d(gw, gl_n) + d(gx, gu_n) + d(gy, gv_n) + d(gz, gh_n)}{4}\right].$$

On adding equation (3.27)-(3.30) and using the property of function ϕ, we have

$$(3.32) \quad \begin{aligned} &d(gx, gu_{n+1}) + d(gy, gv_{n+1}) + d(gz, gh_{n+1}) + d(gw, gl_{n+1}) \\ &\leqslant 4\phi\left[\frac{d(gx, gu_{n+1}) + d(gy, gv_{n+1}) + d(gz, gh_{n+1}) + d(gw, gl_{n+1})}{4}\right] \end{aligned}$$

$$(3.33) \quad \begin{aligned} &\text{or } d(gx, gu_{n+1}) + d(gy, gv_{n+1}) + d(gz, gh_{n+1}) + d(gw, gl_{n+1}) \\ &< d(gx, gu_{n+1}) + d(gy, gv_{n+1}) + d(gz, gh_{n+1}) + d(gw, gl_{n+1}) \end{aligned}$$

Thus, the sequence $\{d(gx, gu_n) + d(gy, gv_n) + d(gz, gh_n) + d(gw, gl_n)\}$ is decreasing, therefore there exist $\delta \geqslant 0$, such that

$$\lim_{n\to\infty} [d(gx, gu_n) + d(gy, gv_n) + d(gz, gh_n) + d(gw, gl_n)] = \delta \tag{3.34}$$

Suppose that $\delta > 0$, taking limit as $n \to \infty$ in equation (3.30), we have

$$\delta \leqslant 4\left(\frac{\phi(\delta)}{4}\right) \tag{3.35}$$

It is a contradiction. Hence $\delta = 0$, that is

$$\lim_{n\to\infty} [d(gx, gu_n) + d(gy, gv_n) + d(gz, gh_n) + d(gw, gl_n)] = 0.$$

By this we obtain

$$\lim_{n\to\infty} d(gx, gu_n) = \lim_{n\to\infty} d(gy, gv_n) = \lim_{n\to\infty} d(gz, gh_n) = \lim_{n\to\infty} d(gw, gl_n). \tag{3.36}$$

In the same way, it is easy to show that

$$\lim_{n\to\infty} d(gx_1, gu_n) = \lim_{n\to\infty} d(gy_1, gv_n) = \lim_{n\to\infty} d(gz_1, gh_n) = \lim_{n\to\infty} d(gw_1, gl_n). \tag{3.37}$$

On account of equation (3.35) and (3.36), we have

$$gx = gx_1, gy = gy_1, gz = gz_1 \text{ and } gw = gw_1$$

Hence the result. □

Example 3.1. Let (R, d) be a complete metric space with the usual metric defined on R.

Consider $g : X \to X$ and $F : X^4 \to X$ be defined as

$$g(x) = \frac{7}{9}x \text{ and } F(x, y, z, w) = \frac{x - y + z - w}{8}.$$

Also suppose $\phi : [0, \infty) \to [0, \infty)$ be given by $\phi(t) = \frac{6}{7}t$.

Now for all $x, y, z, u, v, h, l \in X$, satisfying $gx \leqslant gu, gv \leqslant gy, gz \leqslant gh$ and $gl \leqslant gw$, the L.H.S of the condition of equation (3.1) is

$$\begin{aligned} d(F(x, y, z, w), F(u, v, h, l)) &= d\left(\frac{x - y + z - w}{8}, \frac{u - v + h - l}{8}\right) \\ &= \left|\frac{x - y + z - w}{8} - \frac{u - v + h - l}{8}\right| \end{aligned}$$

Now, the R.H.S of equation (3.2) becomes

$$\phi\left(\frac{d(gx,gu)+d(gy,gv)+d(gz,gh)+d(gw,gl)}{4}\right) = \frac{6}{7} \times \frac{7}{9}\left(\frac{|x-u|+|y-v|+|z-h|+|w-l|}{4}\right)$$

we find that the hypothesis of equation (3.2) are satisfied.

Also, $(0, 0, 0, 0)$ is the unique quadruple fixed point of F and g.

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