BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org / JOURNALS / BULLETIN
Vol. **4(2014)**, 37-43

doi: 10.7251/BIMVI1401037S

DOUBLE DIFFUSIVE CONVECTION IN A NON-NEWTONIAN FLUID SATURATED POROUS LAYER WITH THROUGHFLOW

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ABSTRACT: *The effects of quadratic drag and vertical throughflow on the onset of double diffusive convection in a non-Newtonian fluid saturated horizontal porous layer are investigated. A modified Forchheimer-extended Darcy model which takes inertia into account and viscoelastic effects is employed to describe the flow in a porous medium. The boundaries are considered to be impermeable but perfect conductors of heat and solute concentration. Conditions for the occurrence of stationary and oscillatory convection are obtained analytically using the Galerkin technique. In contrast to the single component system, it is found that a small amount of throughflow in either of its direction destabilizes the system.*

1. INTRODUCTION

Double diffusive convection in porous media has generated considerable interest in recent years because of its importance in many engineering applications such as biomechanical and chemical engineering, geothermal systems, enhanced recovery of petroleum reservoirs, underground spreading of chemical waste among others [1]. The major available literature on these are mainly concerned with heat and mass transfer and flow of Newtonian fluids in porous media. However, many practical problems cited above involve non-Newtonian fluids saturating a porous media. For example, the performance of an oil reservoir depends, to a large extent, upon the physical nature of crude oil present in the reservoir. The light crude is essentially Newtonian while the heavy crude is non-Newtonian. The study of a heavy crude oil is based on a generalized Darcy equation, which takes into account of non-Newtonian effects as well as nonlinear nature of Darcy- Forchheimer Oldroyd-B fluid. Such an equation is useful in the study of mobility control in oil displacement mechanism, which improves the efficiency of oil recovery. There exist many different types of non-Newtonian fluids. However, some oil sand contains waxy crude oil at shallow depths of the reservoirs which are considered to be viscoelastic fluids. In such situations, a viscoelastic model of a fluid will be more realistic than inelastic non- Newtonian fluids. Also, many geophysical and technological applications involve non-isothermal flow of fluids through porous media, called throughflow.

Copious literature is available on the thermal instability of a viscoelastic fluid saturated porous layer with and without throughflow effects [3, 4]. However, the importance of double diffusive convection in a viscoelastic fluid saturated porous layer with throughflow becomes significant when precise processing is required. To date, flow within the porous layer is invariably modeled using a Newtonian-fluid approximation despite viscoelastic properties with combined thermal and solute concentration gradients which are crucial in a variety of laboratory and geophysical situations. The difficulty in dealing with such instability problems is that one has to solve the time dependent equations with variable coefficients. Such a phenomenon, to our knowledge has not been given cognizance and the study of it is the main objective of this paper.

2. FORMULATION OF THE PROBLEM

We consider an incompressible binary viscoelastic fluid saturated horizontal porous layer of thickness d with constant vertical throughflow of magnitude w_0 which is either gravity aligned or otherwise in its direction. A Cartesian co-ordinate system (x, y, z) is chosen such that the origin is at the bottom of the layer and z-axis is vertically upward. The boundaries of the porous layer are kept at constant but different temperatures and solute concentrations. That is, T_0 and S_0 at the lower boundary $z = 0$, while T_1 ($< T_0$) and S_1 ($< S_0$) at the upper boundary $z = d$. The viscoelastic fluid is approximated by the Oldroyd-B constitutive model. The governing equations, following [1, 2], are :

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$(1+\lambda_1 \frac{\partial}{\partial t})(\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{\rho_0 C_d}{\sqrt{k}}|\vec{q}|\vec{q} + \rho\vec{g} + \nabla p) + \frac{\mu}{k}(1+\lambda_2 \frac{\partial}{\partial t})\vec{q} = 0 \tag{2}$$

$$A\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \kappa_t \nabla^2 T \tag{3}$$

$$\varepsilon\frac{\partial S}{\partial t} + (\vec{q}\cdot\nabla)S = \kappa_s \nabla^2 S \tag{4}$$

$$\rho = \rho_0\{1-\alpha_t(T-T_0)+\alpha_s(S-S_0)\} \tag{5}$$

where $\vec{q}$ is the velocity vector, T the temperature, S the solute concentration, p the hydrostatic pressure, μ the viscosity of the fluid, λ_1 and λ_2 are constant relaxation and retardation times , κ_t the effective thermal diffusivity, κ_s is the solute analog of κ_t, ε the porosity of the porous medium, k the permeability of the porous medium,

α_t is the volumetric thermal expansion coefficient, α_s is the solute analog of α_t, ρ the fluid density, ρ_0 is the reference density, C_d the dimensionless Forchheimer coefficient, $\vec{g}$ the acceleration due to gravity $A = (\rho_0 c)_m / (\rho_0 c_p)_f = [(1-\varepsilon)(\rho_0 c)_s + \varepsilon (\rho_0 c_p)_f] / (\rho_0 c_p)_f$ the ratio of heat capacities of the fluid saturated porous medium to that of the fluid, and c_p is the specific heat. The subscripts m, s and f refer respectively to the porous medium, solid and fluid.

The basic state is not quiescent (i.e., $\vec{q}_b = w_0 \hat{k}$) and as a result the basic temperature and solute concentration distributions vary from linear to nonlinear with porous layer height z. Following the standard linear stability analysis procedure as outlined in [4], the governing stability equations in dimensionless form can then be written as

$$\left[\frac{\sigma}{\mathrm{Pr}} + \mathrm{G}|\mathrm{Q}| + \mathrm{Da}^{-1}\frac{(1+\Gamma\Lambda\sigma)}{(1+\Gamma\sigma)}\right](\mathrm{D}^2 - a^2)W = -R_t a^2 \theta + R_s a^2 \mathrm{C} \quad (6)$$

$$\left[\mathrm{D}^2 - a^2 - \mathrm{M}\sigma - \mathrm{QD}\right]\Theta = f(z)W \quad (7)$$

$$\left[\tau(\mathrm{D}^2 - a^2) - \sigma - \mathrm{QD}\right]\mathrm{C} = \mathrm{g}(z)W \quad (8)$$

where, W, Θ and C are respectively the perturbed amplitudes of z-component of velocity, temperature and solute concentration, $D = d/dz$ and a is the horizontal wavenumber. Here $R_t = \alpha_t\, g\, \Delta T\, d^3 / \nu\, \kappa_t$ is the thermal Rayleigh number, $R_s = \alpha_s\, g\, \Delta S\, d^3 / \nu\, \kappa_t$ is the solute Rayleigh number, $Q = w_0 d / \kappa_t$ is the Peclet number, $\mathrm{Pr} = \nu\, \varepsilon^2 / \kappa_t$ is the modified Prandtl number, $Da = k / d^2$ is the Darcy number, $\tau = \kappa_s / \kappa_t$ is the ratio of diffusivities, $f(z) = -Q e^{Qz} / (e^Q - 1)$ and $g(z) = -\tilde{Q} e^{\tilde{Q} z} / (e^{\tilde{Q}} - 1)$ are the dimensionless steady state non-linear basic temperature and solute concentration gradients respectively, $\tilde{Q} = Q / \tau$ and σ is the growth rate which is complex in general.

The boundary conditions are

$$W = \Theta = C = 0 \ \text{at} \ \ z = 0,1 \quad (9)$$

2. SOLUTION TO THE EIGENVALUE PROBLEM

Equations (6) – (8) together with the boundary conditions given by Eq.(9) constitute a double eigenvalue problem and a single term Galerkin expansion technique, which yields sufficiently accurate and useful results, is used to solve it. Accordingly, the variables in Eqs.(6) –(8) are written in terms of trial functions as $W = A_1 W_1(z), \Theta = A_2 \Theta_1(z)$ and $C = A_3 C_1(z)$, where $W_1(z)$, $\Theta_1(z)$ and $C_1(z)$ will be generally chosen in such a way that they satisfy the respective boundary conditions, and A_1 - A_3 are constants. Multiplying Eq.(6) by W_1, Eq. (7) by Θ_1 and Eq.(8) by C_1;

integrating the equations with respect to z from 0 to 1, eliminating the constants A_1 - A_3 from the resulting equations, taking $\sigma = i\omega$ and clearing the complex quantities from the denominator of the expression obtained for R_t ,we get

$$R_t = R_s \frac{(\tau\delta^4 + M\omega^2)}{(\tau^2\delta^4 + \omega^2)}\delta_1 + (\frac{Da^{-1}\delta^2(1+\Lambda\Gamma^2\omega^2)}{(1+\Gamma^2\omega^2)} + G|Q|\delta^2 - \frac{M\omega^2 Da^{-1}\Gamma(\Lambda-1)}{(1+\Gamma^2\omega^2)} + \frac{M\omega^2}{Pr})\frac{\delta^2}{a^2\delta_2} + i\omega N \quad (10)$$

where

$$N = R_s[\frac{(M\tau\delta^2 - \delta^2)}{(\tau^2\delta^4 + \omega^2)}]\delta_1 + [\frac{MDa^{-1}(1+\Lambda\Gamma^2\omega^2)}{(1+\Gamma^2\omega^2)} + MG|Q| + \frac{\delta^2 Da^{-1}\Gamma(\Lambda-1)}{(1+\Gamma^2\omega^2)} + \frac{\delta^2}{Pr})\frac{\delta^2}{a^2\delta_2}. \quad (11)$$

Here

$$\delta_1 = \left[\frac{6\tilde{Q}\coth(\tilde{Q}/2) - \tilde{Q}^2 - 12}{6Q\coth(Q/2) - Q^2 - 12}\right]\tau^4, \qquad \delta_2 = \frac{60}{Q^4}[Q^2 - 6Q\coth(Q/2) + 12], \qquad \delta^2 = a^2 + 10.$$

Since R_t is a physical quantity, it must be real and it implies either $\omega = 0$ (stationary convection) or $N = 0$ (oscillatory convection) in Eq.(10). For oscillatory onset $N = 0$ $(\omega \neq 0)$ then Eq.(11) yields a dispersion relation of the form.

$$a_1(\omega^2)^2 + a_2(\omega^2) + a_3 = 0 \quad (12)$$

where a_1, a_2 and a_3 are functions of known parameters.

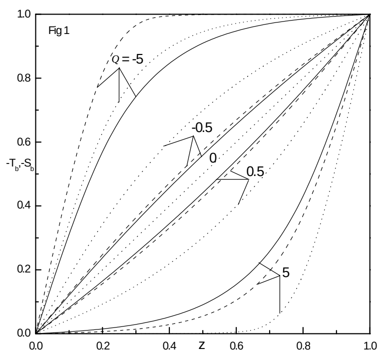
4. RESULTS AND DISCUSSION

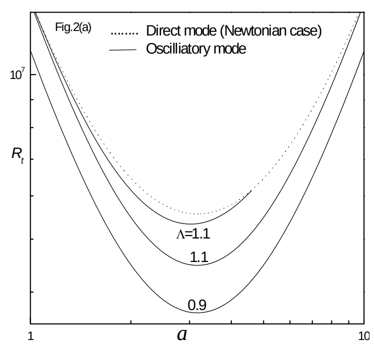
It is observed that the steady case $(\omega = 0)$ results correspond to those of Newtonian fluid. This is because the basic state remains the same for both Newtonian and viscoelastic fluids saturated porous layer as it corresponds to pure conduction. The basic temperature and solute concentration distributions are obtained for representative values of Q and τ, and are presented graphically in Fig.1, in order to understand their influence on the stability of the system. As can be seen from the figure, the basic state distributions are linear in the absence of throughflow (Q = 0). However, when throughflow exists, the distributions become nonlinear, and deviate from each other with an increase in Q. In fact, the nonlinearity in base-state solute concentration stratification (**dotted lines**) becomes more pronounced as compared to temperature stratification (**solid lines**) with decrease in τ. The change in base-state stratification between the solute concentration and temperature affect the stability of the system significantly.

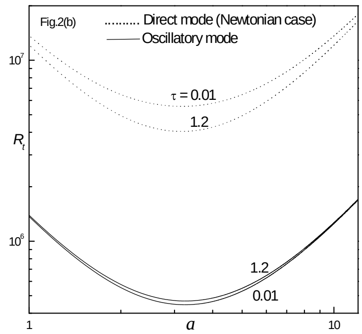
The marginal stability curves for steady and oscillatory modes in the R_t - a plane are shown in Figs.2(a,b) for values of $Q = 0.1$, $Da = 10^{-5}$, Γ =10, $Pr = 10$,

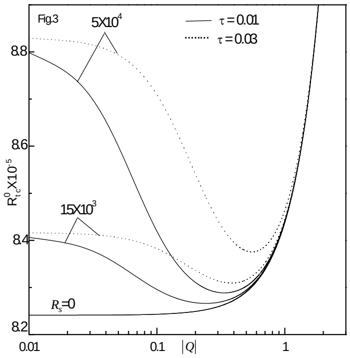
$R_s = 5\times10^4$, $C_d = 0.5$, M=1.25 , $\varepsilon = 0.4$. Figure 2(a) shows the neutral curves for Λ = 0.9 and 1.1 when τ =0.01, while Fig.2 (b) illustrates the neutral curves for two values of τ =0.01 and 1.2 when Λ =0.1. From these figures it is important to note that oscillatory convection is possible even if Λ >1 and $M\,\tau > 1$; a contrast result when compared to single component viscoelastic fluid saturated porous layer (i.e., R_s =0) and double diffusive Newtonian fluid saturated porous layer (i.e., $\Gamma = 0$) respectively. Besides, there exist two different onset frequencies at the same wave number when Λ >1 and increase in Λ is to delay the onset of oscillatory convection (see Fig. 2a). Also, the critical wave number for the direct mode (i.e., Newtonian case) is found to be smaller than that of oscillatory mode.

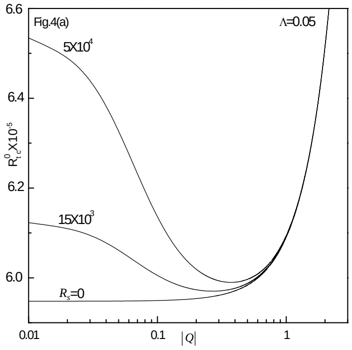
The critical oscillatory Rayleigh number R^{0}_{tc}, is computed numerically and the results are shown in Figs 3-5 for different values of R_s ($= 0$, 15×10^3, 5×10^4), Λ (=0.05,0.1) and Γ (=0.5,1.0) respectively, as a function of $|Q|$ with fixed value of $Da = 10^{-5}$, M=1.25, $Pr = 10$, $C_d = 0.5$ and $\varepsilon = 0.4$. From these figures, it is interesting to note that in the absence of an additional diffusing component (i.e., R_s =0) the effect of throughflow is always stabilizing by a degree which is independent of the flow direction. Nonetheless, when a viscoelastic fluid saturated porous medium is stratified with $R_s = 5\times10^3$ and 5×10^4 (i.e., in the presence of an additional diffusing component) then the effect of throughflow is destabilizing up to a certain value of $|Q|$, and the destabilization manifests itself as a minimum in the R^{0}_{tc} - $|Q|$ plot. The
destabilization may be due to the distortion of steady state basic temperature and solute concentration distribution by the throughflow. Further, decrease in the value of τ (see Fig.3) as well as Λ (see Fig.4 a,b) and increase in the value of Γ (see Fig. 5 a,b) is to decrease the critical oscillatory Rayleigh number and hence their effect is to hasten the onset of convection up to a certain value of $|Q|$ and exceeding which the curves for different R_s coalesce. Whereas, increase in R_s is to delay the onset of convection.

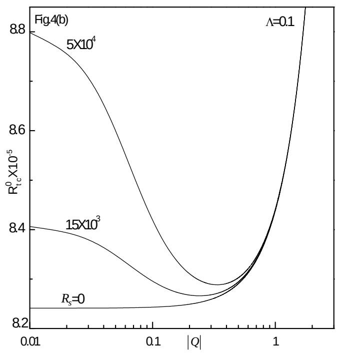


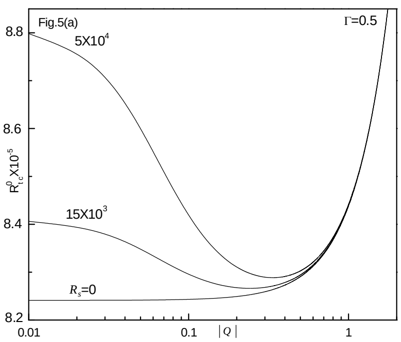


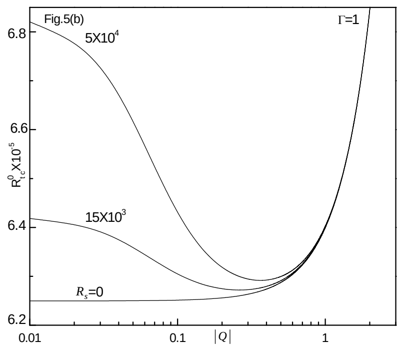












ACKNOWLEDGEMENT*:*

The authors are thankful to the referee for his valuable suggestions and comments for the improvement of the paper. Also, the first author is grateful to **Dr. M. N. Channabasappa**, Director and **Dr. Shivakumaraiah**, Principal, Siddaganga Institute of Technology, Tumkur, for their constant support and encouragement.

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Received by editors 27.11.2013. revised version 24.02.2014;
available online 03.03.2014.