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Weakly $\mathcal{I}_g$-closed sets

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ABSTRACT. In this paper, the notion of weakly $\mathcal{I}_g$-closed sets in ideal topological spaces is studied. The relationships of weakly $\mathcal{I}_g$-closed sets and various properties of weakly $\mathcal{I}_g$-closed sets are investigated.

1. Introduction

The first step of generalizing closed sets was done by Levine in 1970 [**7**]. He defined a subset A of a topological space (X, τ) to be g-closed if its closure belongs to every open superset of A. As the weak form of g-closed sets, the notion of weakly g-closed sets was introduced and studied by Sundaram and Nagaveni [**11**]. Sundaram and Pushpalatha [**12**] introduced and studied the notion of strongly g-closed sets, which is implied by that of closed sets and implies that of g-closed sets. Park and Park [**9**] introduced and studied mildly g-closed sets, which is properly placed between the classes of strongly g-closed and weakly g-closed sets. Moreover, the relations with other notions directly or indirectly connected with g-closed were investigated by them. In 1999, Dontchev et al. studied the notion of generalized closed sets in ideal topological spaces called $\mathcal{I}_g$-closed sets [**2**]. In 2008, Navaneethakrishnan and Paulraj Joseph have studied some characterizations of normal spaces via $\mathcal{I}_g$-open sets [**8**]. In 2013, Ekici and Ozen [**4**] introduced a generalized class of τ^*. Ravi et. al [**10**] introduced another generalized class of τ^* called weakly $\mathcal{I}_g$-closed sets.

The main aim of this paper is to study the notion of weakly $\mathcal{I}_g$-closed sets in ideal topological spaces. The relationships of weakly $\mathcal{I}_g$-closed sets and various properties of weakly $\mathcal{I}_g$-closed sets are discussed.

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2. Preliminaries

In this paper, (X, τ) represents topological space on which no separation axioms are assumed unless explicitly stated. The closure and the interior of a subset G of a space X will be denoted by cl(G) and int(G), respectively.

DEFINITION 2.1. A subset G of a topological space (X, τ) is said to be

(1) g-closed [**7**] if cl(G) $\subseteq$ H whenever G $\subseteq$ H and H is open in X;
(2) g-open [**7**] if X\G is g-closed.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

(1) A $\in \mathcal{I}$ and B $\subseteq$ A imply B $\in \mathcal{I}$ and
(2) A $\in \mathcal{I}$ and B $\in \mathcal{I}$ imply A$\cup$B $\in \mathcal{I}$ [**6**].

Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X if $\mathcal{P}$(X) is the set of all subsets of X, a set operator $(\bullet)^\star : \mathcal{P}(X) \to \mathcal{P}(X)$, called a local function [**6**] of A with respect to τ and $\mathcal{I}$ is defined as follows: for A $\subseteq$ X, $A^\star(\mathcal{I}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $cl^\star(\bullet)$ for a topology $\tau^\star(\mathcal{I}, \tau)$, called the $\star$-topology and finer than τ, is defined by $cl^\star(A) = A \cup A^\star(\mathcal{I}, \tau)$ [**13**]. We will simply write $A^\star$ for $A^\star(\mathcal{I}, \tau)$ and $\tau^\star$ for $\tau^\star(\mathcal{I}, \tau)$. If $\mathcal{I}$ is an ideal on X, then (X, τ, $\mathcal{I}$) is called an ideal topological space. On the other hand, (A, τ_A, $\mathcal{I}_A$) where τ_A is the relative topology on A and $\mathcal{I}_A = \{A \cap J : J \in \mathcal{I}\}$ is an ideal topological space for an ideal topological space (X, τ, $\mathcal{I}$) and A $\subseteq$ X [**5**]. For a subset A $\subseteq$ X, cl*(A) and int*(A) will, respectively, denote the closure and the interior of A in (X, τ^*).

DEFINITION 2.2. A subset G of an ideal topological space (X, τ, $\mathcal{I}$) is said to be

(1) $\mathcal{I}_g$-closed [**2**] if $G^* \subseteq$ H whenever G $\subseteq$ H and H is open in (X, τ, $\mathcal{I}$).
(2) $pre^*_{\mathcal{I}}$-open [**3**] if G $\subseteq$ int*(cl(G)).
(3) $pre^*_{\mathcal{I}}$-closed [**3**] if X\G is $pre^*_{\mathcal{I}}$-open.
(4) $\mathcal{I}$-R closed [**1**] if G = cl*(int(G)).
(5) $*$-closed [**5**] if G = cl*(G) or $G^* \subseteq$ G.

REMARK 2.1. [**4**] In any ideal topological spaces, every $\mathcal{I}$-R closed set is $*$-closed but not conversely.

DEFINITION 2.3. [**4**] Let (X, τ, $\mathcal{I}$) be an ideal topological space. A subset G of X is said to be a weakly $\mathcal{I}_{rg}$-closed set if $(int(G))^* \subseteq$ H whenever G $\subseteq$ H and H is a regular open set in X.

DEFINITION 2.4. [**4**] Let (X, τ, $\mathcal{I}$) be an ideal topological space. A subset G of X is said to be a weakly $\mathcal{I}_{rg}$-open set if X\G is a weakly $\mathcal{I}_{rg}$-closed set.

REMARK 2.2. [**4**] Let (X, τ, $\mathcal{I}$) be an ideal topological space. The following diagram holds for a subset G $\subseteq$ X:

$$\begin{array}{ccccc}
\mathcal{I}_g\text{-closed} & \longrightarrow & \mathcal{I}_{rg}\text{-closed} & \longrightarrow & \text{weakly } \mathcal{I}_{rg}\text{-closed} \\
\uparrow & & & & \uparrow \\
*\text{-closed} & & & & \text{pre}^*_{\mathcal{I}}\text{-closed} \\
\uparrow & & & & \\
\mathcal{I}\text{-R-closed} & & & &
\end{array}$$

These implications are not reversible as shown in [**4**].

Definition 2.5. [**10**] Let (X, τ, $\mathcal{I}$) be an ideal topological space. A subset G of X is said to be

(1) a mildly $\mathcal{I}_g$-closed set if (int(G))* $\subseteq$ H whenever G $\subseteq$ H and H is a g-open set in X;
(2) a weakly $\mathcal{I}_g$-closed set if (int(G))* $\subseteq$ H whenever G $\subseteq$ H and H is an open set in X;
(3) a strongly $\mathcal{I}_g$-closed set if G* $\subseteq$ H whenever G $\subseteq$ H and H is a g-open set in X.

Remark 2.3. [**10**] Let (X, τ, $\mathcal{I}$) be an ideal topological space. The following diagram holds for a subset G $\subseteq$ X:

$$\begin{array}{ccc}
\text{strongly } \mathcal{I}_g\text{-closed set} & \longrightarrow & \mathcal{I}_g\text{-closed set} \\
\downarrow & & \downarrow \\
\text{mildly } \mathcal{I}_g\text{-closed set} & \longrightarrow & \text{weakly } \mathcal{I}_g\text{-closed set}
\end{array}$$

These implications are not reversible.

3. Properties of weakly $\mathcal{I}_g$-closed sets

Theorem 3.1. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. The following properties are equivalent:*

(1) *G is a weakly $\mathcal{I}_g$-closed set,*
(2) *cl*(int(G)) $\subseteq$ H whenever G $\subseteq$ H and H is an open set in X.*

Proof. It is similar to that of Theorem 3.5 of [4].

Theorem 3.2. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is open and weakly $\mathcal{I}_g$-closed, then G is $*$-closed.*

Proof. Let G be an open and weakly $\mathcal{I}_g$-closed set in (X, τ, $\mathcal{I}$). Since G is open and weakly $\mathcal{I}_g$-closed, cl*(G) = cl*(int(G)) $\subseteq$ G. Thus, G is a $*$-closed set in (X, τ, $\mathcal{I}$).

Theorem 3.3. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_g$-closed set, then (int(G))* $\setminus$G contains no any nonempty closed set.*

Proof. Let G be a weakly $\mathcal{I}_g$-closed set in (X, τ, $\mathcal{I}$). Suppose that H is a closed set such that H $\subseteq$ (int(G))*$\setminus$G. Since G is a weakly $\mathcal{I}_g$-closed set, X$\setminus$H is open and G $\subseteq$ X$\setminus$H, then (int(G))* $\subseteq$ X$\setminus$H. We have H $\subseteq$ X$\setminus$(int(G))*. Hence, H

$\subseteq$ (Int(G))* $\cap$ (X\(int(G))*) = $\emptyset$. Thus, (int(G))*\G contains no any nonempty closed set.

THEOREM 3.4. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_g$-closed set, then cl*(int(G))\G contains no any nonempty closed set.*

PROOF. Suppose that H is a closed set such that H $\subseteq$ cl*(int(G))\G. By Theorem 3.3, it follows from the fact that cl*(int(G))\G = ((int(G))* $\cup$ int(G))\G.

THEOREM 3.5. *Let (X, τ, $\mathcal{I}$) be an ideal topological space. The following properties are equivalent:*

(1) *G is pre$^*_{\mathcal{I}}$-closed for each weakly $\mathcal{I}_g$-closed set G in (X, τ, $\mathcal{I}$),*
(2) *Each singleton $\{x\}$ of X is a closed set or $\{x\}$ is pre$^*_{\mathcal{I}}$-open.*

PROOF. (1) $\Rightarrow$ (2) : It follows from Theorem 3.11 of [4].
(2) $\Rightarrow$ (1) : It is similar to that of Theorem 3.11 of [4].

THEOREM 3.6. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If cl*(int(G))\G contains no any nonempty $*$-closed set, then G is a weakly $\mathcal{I}_g$-closed set.*

PROOF. Suppose that cl*(int(G))\G contains no any nonempty $*$-closed set in (X, τ, $\mathcal{I}$). Let G $\subseteq$ H and H be an open set. Assume that cl*(int(G)) is not contained in H. It follows that cl*(int(G))$\cap$(X\H) is a nonempty $*$-closed subset of cl*(int(G))\G. This is a contradiction. Hence G is a weakly $\mathcal{I}_g$-closed set.

THEOREM 3.7. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_g$-closed set, then int(G) = H\K where H is $\mathcal{I}$-R closed and K contains no any nonempty closed set.*

PROOF. Let G be a weakly $\mathcal{I}_g$-closed set in (X, τ, $\mathcal{I}$). Take K = (int(G))*\G. Then, by Theorem 3.3, K contains no any nonempty closed set.

Take H = cl*(int(G)). Then H = cl*(int(H)). Moreover, we have H\K = ((int(G))* $\cup$ int(G))\((int(G))*\G) = ((int(G))* $\cup$ int(G)) $\cap$ (X\(int(G))* $\cup$ G) = int(G).

THEOREM 3.8. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. Assume that G is a weakly $\mathcal{I}_g$-closed set. The following properties are equivalent:*

(1) *G is pre$^*_{\mathcal{I}}$-closed,*
(2) *cl*(int(G))\G is a closed set,*
(3) *(int(G))* \G is a closed set.*

PROOF. (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (1) : It follows from Theorem 3.14 of [4].
(2) $\Leftrightarrow$ (3) : It is similar to that of Theorem 3.14 of [4].

THEOREM 3.9. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X be a weakly $\mathcal{I}_g$-closed set. Then G $\cup$ (X\(int(G))*) is a weakly $\mathcal{I}_g$-closed set in (X, τ, $\mathcal{I}$).*

PROOF. Let G be a weakly $\mathcal{I}_g$-closed set in (X, τ, $\mathcal{I}$). Suppose that H is an open set such that G $\cup$ (X\(int(G))*) $\subseteq$ H. We have X\H $\subseteq$ X\(G $\cup$ (X\(int(G))*))

$= (X \backslash G) \cap (\text{int}(G))^* = (\text{int}(G))^* \backslash G$. Since $X \backslash H$ is a closed set and G is a weakly $\mathcal{I}_g$-closed set, it follows from Theorem 3.3 that $X \backslash H = \emptyset$. Hence, X = H. Thus, X is the only open set containing $G \cup (X \backslash (\text{int}(G))^*)$. Consequently, $G \cup (X \backslash (\text{int}(G))^*)$ is a weakly $\mathcal{I}_g$-closed set in $(X, \tau, \mathcal{I})$.

Corollary 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $G \subseteq X$ be a weakly $\mathcal{I}_g$-closed set. Then $(\text{int}(G))^* \backslash G$ is a weakly $\mathcal{I}_g$-open set in $(X, \tau, \mathcal{I})$.*

Proof. Since $X \backslash ((\text{int}(G))^* \backslash G) = G \cup (X \backslash (\text{int}(G))^*)$, it follows from Theorem 3.9 that $(\text{int}(G))^* \backslash G$ is a weakly $\mathcal{I}_g$-open set in $(X, \tau, \mathcal{I})$.

Theorem 3.10. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $G \subseteq X$. The following properties are equivalent:*

(1) *G is a $*$-closed and open set,*
(2) *G is $\mathcal{I}$-R closed and open set,*
(3) *G is a weakly $\mathcal{I}_g$-closed and open set.*

Proof. It is similar to that of Theorem 3.17 of [4].

Proposition 3.1. *Every $\text{pre}^*_{\mathcal{I}}$-closed set is weakly $\mathcal{I}_g$-closed but not conversely.*

Proof. Let $H \subseteq G$ and G an open set in X. Since H is $\text{pre}^*_{\mathcal{I}}$-closed, $\text{cl}^*(\text{int}(H)) \subseteq H \subseteq G$. Hence H is weakly $\mathcal{I}_g$-closed set.

Example 3.1. Let $(X, \tau, \mathcal{I})$ be an ideal topological space such that X = {a, b, c, d}, $\tau = \{\emptyset$, {a}, {c}, {a, c}, {c, d}, {a, c, d}, {b, c, d}, X} and $\mathcal{I} = \{\emptyset\}$. Then {b, c} is weakly $\mathcal{I}_g$-closed set but not $\text{pre}^*_{\mathcal{I}}$-closed.

Remark 3.1. Let $(X, \tau, \mathcal{I})$ be an ideal topological space. The following diagram holds for a subset $G \subseteq X$:

strongly $\mathcal{I}_g$-closed set	$\longrightarrow$	$\mathcal{I}_g$-closed set		
$\downarrow$		$\downarrow$		
mildly $\mathcal{I}_g$-closed set	$\longrightarrow$	weakly $\mathcal{I}_g$-closed set	$\longrightarrow$	weakly $\mathcal{I}_{rg}$-closed set

The reverse implications in this diagram are true by Remark 2.8 and the following Example.

Example 3.2. Let X = {a, b, c}, $\tau = \{\emptyset$, {a}, {a, b}, {a, c}, X} and $\mathcal{I} = \{\emptyset$, {c}}. Then {a} is weakly $\mathcal{I}_{rg}$-closed set but not weakly $\mathcal{I}_g$-closed.

4. Further properties

Theorem 4.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. The following properties are equivalent:*

(1) *Each subset of $(X, \tau, \mathcal{I})$ is a weakly $\mathcal{I}_g$-closed set,*
(2) *G is $\text{pre}^*_{\mathcal{I}}$-closed for each open set G in X.*

Proof. (1) $\Rightarrow$ (2) : It follows from Theorem 4.1 of [4].
(2) $\Rightarrow$ (1) : It is similar to that of Theorem 4.1 of [4].

THEOREM 4.2. *Let (X, τ, $\mathcal{I}$) be an ideal topological space. If G is a weakly $\mathcal{I}_g$-closed set and G $\subseteq$ H $\subseteq$ cl*(int(G)), then H is a weakly $\mathcal{I}_g$-closed set.*

PROOF. Let H $\subseteq$ K and K be an open set in X. Since G $\subseteq$ K and G is a weakly $\mathcal{I}_g$-closed set, then cl*(int(G)) $\subseteq$ K. Since H $\subseteq$ cl*(int(G)), then cl*(int(H)) $\subseteq$ cl*(int(G)) $\subseteq$ K. Thus, cl*(int(H)) $\subseteq$ K and hence, H is a weakly $\mathcal{I}_g$-closed set.

COROLLARY 4.1. *Let (X, τ, $\mathcal{I}$) be an ideal topological space. If G is a weakly $\mathcal{I}_g$-closed and open set, then cl*(G) is a weakly $\mathcal{I}_g$-closed set.*

PROOF. Let G be a weakly $\mathcal{I}_g$-closed and open set in (X, τ, $\mathcal{I}$). We have G $\subseteq$ cl*(G) $\subseteq$ cl*(G) = cl*(int(G)). Hence, by Theorem 4.2, cl*(G) is a weakly $\mathcal{I}_g$-closed set in (X, τ, $\mathcal{I}$).

THEOREM 4.3. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a nowhere dense set, then G is a weakly $\mathcal{I}_g$-closed set.*

PROOF. It is similar to that of Theorem 4.4 of [4].

REMARK 4.1. The reverse of Theorem 4.3 is not true in general as shown in the following example.

EXAMPLE 4.1. Let X, τ and $\mathcal{I}$ be as in Example 3.2. Then {a, b} is a weakly $\mathcal{I}_g$-closed set but not a nowhere dense set.

REMARK 4.2.
(1) The union of two weakly $\mathcal{I}_g$-closed sets in an ideal topological space need not be a weakly $\mathcal{I}_g$-closed set.
(2) The intersection of two weakly $\mathcal{I}_g$-closed sets in an ideal topological space need not be a weakly $\mathcal{I}_g$-closed set.

EXAMPLE 4.2. Let (X, τ, $\mathcal{I}$) be an ideal topological space such that X = {a, b, c, d}, τ = {$\emptyset$, {a}, {b}, {a, b}, {a, c}, {a, b, c}, X} and $\mathcal{I}$ = {$\emptyset$, {a}}. Then {a} and {c} are weakly $\mathcal{I}_g$-closed sets but their union {a, c} is not a weakly $\mathcal{I}_g$-closed set.

EXAMPLE 4.3. Let (X, τ, $\mathcal{I}$) be an ideal topological space such that X = {a, b, c}, τ = {$\emptyset$, {a}, X} and $\mathcal{I}$ = {$\emptyset$}. Then {a, b} and {a, c} are weakly $\mathcal{I}_g$-closed sets but their intersection {a} is not a weakly $\mathcal{I}_g$-closed set.

THEOREM 4.4. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. Then G is a weakly $\mathcal{I}_g$-open set if and only if H $\subseteq$ int*(cl(G)) whenever H $\subseteq$ G and H is a closed set.*

PROOF. It is similar to that of Theorem 4.13 of [4].

THEOREM 4.5. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_g$-closed set, then cl*(int(G))$\setminus$G is a weakly $\mathcal{I}_g$-open set in (X, τ, $\mathcal{I}$).*

PROOF. Let G be a weakly $\mathcal{I}_g$-closed set in (X, τ, $\mathcal{I}$). Suppose that H is a closed set such that H $\subseteq$ cl*(int(G))$\setminus$G. Since G is a weakly $\mathcal{I}_g$-closed set, it follows from Theorem 3.4 that H = $\emptyset$. Thus, we have H $\subseteq$ int*(cl(cl*(int(G))$\setminus$G)). It follows from Theorem 4.4 that cl*(int(G))$\setminus$G is a weakly $\mathcal{I}_g$-open set in (X, τ, $\mathcal{I}$).

THEOREM 4.6. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_g$-open set, then H = X whenever H is an open set and int*(cl(G)) $\cup$ (X$\setminus$G) $\subseteq$ H.*

PROOF. Let H be an open set in X and int*(cl(G)) $\cup$ (X$\setminus$G) $\subseteq$ H. We have X$\setminus$H $\subseteq$ (X$\setminus$int*(cl(G))) $\cap$ G = cl*(int(X$\setminus$G))$\setminus$(X$\setminus$G). Since X$\setminus$H is a closed set and X$\setminus$G is a weakly $\mathcal{I}_g$-closed set, it follows from Theorem 3.4 that X$\setminus$H = $\emptyset$. Thus, we have H = X.

THEOREM 4.7. *Let (X, τ, $\mathcal{I}$) be an ideal topological space. If G is a weakly $\mathcal{I}_g$-open set and int*(cl(G)) $\subseteq$ H $\subseteq$ G, then H is a weakly $\mathcal{I}_g$-open set.*

PROOF. Let G be a weakly $\mathcal{I}_g$-open set and int*(cl(G)) $\subseteq$ H $\subseteq$ G. Since int*(cl(G)) $\subseteq$ H $\subseteq$ G, then int*(cl(G)) = int*(cl(H)). Let K be a closed set and K $\subseteq$ H. We have K $\subseteq$ G. Since G is a weakly $\mathcal{I}_g$-open set, it follows from Theorem 4.4 that K $\subseteq$ int*(cl(G)) = int*(cl(H)). Hence, by Theorem 4.4, H is a weakly $\mathcal{I}_g$-open set in (X, τ, $\mathcal{I}$).

COROLLARY 4.2. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and G $\subseteq$ X. If G is a weakly $\mathcal{I}_g$-open and closed set, then int*(G) is a weakly $\mathcal{I}_g$-open set.*

PROOF. Let G be a weakly $\mathcal{I}_g$-open and closed set in (X, τ, $\mathcal{I}$). Then int*(cl(G)) = int*(G) $\subseteq$ int*(G) $\subseteq$ G. Thus, by Theorem 4.7, int*(G) is a weakly $\mathcal{I}_g$-open set in (X, τ, $\mathcal{I}$).

DEFINITION 4.1. A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is called P-set if A = M$\cup$N where M is closed and N is pre$^*_{\mathcal{I}}$-open.

REMARK 4.3. Every pre$^*_{\mathcal{I}}$-open (resp. closed) set is P-set but not conversely.

EXAMPLE 4.4. Let X, τ and $\mathcal{I}$ be as in Example 3.2. Then {b} is a P-set but not pre$^*_{\mathcal{I}}$-open. Also {c} is P-set but not closed.

THEOREM 4.8. *For a subset H of (X, τ, $\mathcal{I}$), the following are equivalent.*

(1) *H is pre$^*_{\mathcal{I}}$-open.*
(2) *H is a P-set and weakly $\mathcal{I}_g$-open.*

PROOF. (1) $\Rightarrow$ (2): By Remark 4.3, H is a P-set. By Proposition 3.1, H is weakly $\mathcal{I}_g$-open.
(2) $\Rightarrow$ (1): Let H be a P-set and weakly $\mathcal{I}_g$-open. Then there exist a closed set M and a pre$^*_{\mathcal{I}}$-open set N such that H =M$\cup$N. Since M $\subseteq$ H and H is weakly $\mathcal{I}_g$-open, by Theorem 4.4, M $\subseteq$ int*(cl(H)). Also, we have N $\subseteq$ int*(cl(N)). Since N $\subseteq$ H, N $\subseteq$ int*(cl(N)) $\subseteq$ int*(cl(H)). Then H = M$\cup$N $\subseteq$ int*(cl(H)). So H is pre$^*_{\mathcal{I}}$-open.

The following Example shows that the concepts of weakly $\mathcal{I}_g$-open set and P-set are independent.

EXAMPLE 4.5. Let X, τ and $\mathcal{I}$ be as in Example 3.2. Then {d} is weakly $\mathcal{I}_g$-open set but not P-set. Also {b} is P-set but not weakly $\mathcal{I}_g$-open set.

5. Pre$^*_{\mathcal{I}}$-normal spaces

DEFINITION 5.1. An ideal topological space (X, τ, $\mathcal{I}$) is said to be pre$^*_{\mathcal{I}}$-normal if for every pair of disjoint closed subsets A, B of X, there exist disjoint pre$^*_{\mathcal{I}}$-open sets U, V of X such that A $\subseteq$ U and B $\subseteq$ V.

THEOREM 5.1. *The following properties are equivalent for a space (X, τ, $\mathcal{I}$).*

(1) *X is pre$^*_{\mathcal{I}}$-normal;*
(2) *for any disjoint closed sets A and B, there exist disjoint weakly $\mathcal{I}_g$-open sets U, V of X such that A $\subseteq$ U and B $\subseteq$ V;*
(3) *for any closed set A and any open set B containing A, there exists a weakly $\mathcal{I}_g$-open set U such that A $\subseteq$ U $\subseteq$ cl*(int(U)) $\subseteq$ B.*

PROOF. (1) $\Rightarrow$ (2): The proof is obvious.

(2) $\Rightarrow$ (3): Let A be any closed set of X and B any open set of X such that A $\subseteq$ B. Then A and X\B are disjoint closed sets of X. By (2), there exist disjoint weakly $\mathcal{I}_g$-open sets U, V of X such that A $\subseteq$ U and X\B $\subseteq$ V. Since V is weakly $\mathcal{I}_g$-open set, by Theorem 4.4, X\B $\subseteq$ int*(cl(V)) and U$\cap$int*(cl(V)) = $\emptyset$. Therefore we obtain cl*(int(U)) $\subseteq$ cl*(int(X\V)) and hence A $\subseteq$ U $\subseteq$ cl*(int(U)) $\subseteq$ B.

(3) $\Rightarrow$ (1): Let A and B be any disjoint closed sets of X. Then A $\subseteq$ X\B and X\B is open and hence there exists a weakly $\mathcal{I}_g$-open set G of X such that A $\subseteq$ G $\subseteq$ cl*(int(G)) $\subseteq$ X\B. Put U = int*(cl(G)) and V = X\cl*(int(G)). Then U and V are disjoint pre$^*_{\mathcal{I}}$-open sets of X such that A $\subseteq$ U and B $\subseteq$ V. Therefore X is pre$^*_{\mathcal{I}}$-normal.

DEFINITION 5.2. A function f : (X, τ, $\mathcal{I}$) $\rightarrow$ (Y, σ) is said to be weakly $\mathcal{I}_g$-continuous if f^{-1}(V) is weakly $\mathcal{I}_g$-closed in X for every closed set V of Y.

DEFINITION 5.3. A function f : (X, τ, $\mathcal{I}$) $\rightarrow$ (Y, σ, $\mathcal{J}$) is called weakly $\mathcal{I}_g$-irresolute if f^{-1}(V) is weakly $\mathcal{I}_g$-closed in X for every weakly $\mathcal{J}_g$-closed of Y.

THEOREM 5.2. *Let f : X $\rightarrow$ Y be a weakly $\mathcal{I}_g$-continuous closed injection. If Y is normal, then X is pre$^*_{\mathcal{I}}$-normal.*

PROOF. Let A and B be disjoint closed sets of X. Since f is closed injection, f(A) and f(B) are disjoint closed sets of Y. By the normality of Y, there exist disjoint open sets U and V such that f(A) $\subseteq$ U and f(B) $\subseteq$ V. Since f is weakly $\mathcal{I}_g$-continuous, then f^{-1}(U) and f^{-1}(V) are weakly $\mathcal{I}_g$-open sets such that A $\subseteq$ f^{-1}(U) and B $\subseteq$ f^{-1}(V). Therefore X is pre$^*_{\mathcal{I}}$-normal by Theorem 5.1.

THEOREM 5.3. *Let f : X $\rightarrow$ Y be a weakly $\mathcal{I}_g$-irresolute closed injection. If Y is pre$^*_{\mathcal{I}}$-normal, then X is pre$^*_{\mathcal{I}}$-normal.*

PROOF. Let A and B be disjoint closed sets of X. Since f is closed injection, f(A) and f(B) are disjoint closed sets of Y. Since Y is pre$^*_{\mathcal{I}}$-normal, by Theorem 5.1, there exist disjoint weakly $\mathcal{J}_g$-open sets U and V such that f(A) $\subseteq$ U and f(B) $\subseteq$ V. Since f is weakly $\mathcal{I}_g$-irresolute, then f^{-1}(U) and f^{-1}(V) are disjoint weakly $\mathcal{I}_g$-open sets of X such that A $\subseteq$ f^{-1}(U) and B $\subseteq$ f^{-1}(V). Therefore X is pre$^*_{\mathcal{I}}$-normal.

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References

[1] A. Acikgoz and S. Yuksel, *Some new sets and decompositions of $A_{\mathcal{I}-R}$-continuity, α-$\mathcal{I}$-continuity, continuity via idealization,* Acta Math. Hungar., 114(1-2)(2007), 79-89.

[2] J. Dontchev, M. Ganster and T. Noiri, *Unified operation approach of generalized closed sets via topological ideals*, Math. Japonica, 49(1999), 395-401.

[3] E. Ekici, *On $\mathcal{AC}_{\mathcal{I}}$-sets, $\mathcal{BC}_{\mathcal{I}}$-sets, $\beta^*_{\mathcal{I}}$-open sets and decompositions of continuity in ideal topological spaces*, Creat. Math. Inform, 20(1)(2011), 47-54.

[4] E. Ekici and S. Ozen, *A generalized class of τ^* in ideal spaces*, Filomat, 27(4)(2013), 529-535.

[5] D. Jankovic and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(4)(1990), 295-310.

[6] K. Kuratowski, *Topology*, Vol. I, Academic Press, New York, 1966.

[7] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, 19(2)(1970), 89-96.

[8] M. Navaneethakrishnan and J. Paulraj Joseph, *g-closed sets in ideal topological spaces*, Acta Math. Hungar., 119(4)(2008), 365-371.

[9] J. K. Park and J. H. Park, *Mildly generalized closed sets, almost normal and mildly normal spaces*, Chaos, Solitons and Fractals, 20(2004), 1103-1111.

[10] O. Ravi, R. Senthil Kumar and K. Indirani, *Mildly $\mathcal{I}_g$-closed sets*, submitted.

[11] P. Sundaram and N. Nagaveni, *On weakly generalized continuous maps, weakly generalized closed maps and weakly generalized irresolute maps in topological spaces*, Far East J. Math. Sci., 6(6)(1998), 903-1012.

[12] P. Sundaram and A. Pushpalatha, *Strongly generalized closed sets in topological spaces*, Far East J. Math. Sci., 3(4)(2001), 563-575.

[13] R. Vaidyanathaswamy, *Set Topology*, Chelsea Publishing Company, 1946.

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