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HEAT TRANSFER ON MHD VISCOUS FLOW OVER A STRETCHING SHEET WITH PRESCRIBED HEAT FLUX

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Abstract. A steady three-dimensional Magnetohydrodynamic (MHD) boundary layer viscous flow and heat transfer due to a permeable stretching sheet with prescribed surface heat flux is studied in presence of a uniform applied magnetic field transverse to the flow. Using the implicit finite-difference scheme, known as the Keller-box method, the nonlinear ordinary differential equations are solved. The velocity and temperature profiles, skin friction coefficient and wall temperature are discussed for various parameters.

1. Introduction

The problem of boundary layer flow over a fixed flat plate was discussed by Blasius [1]. Sakiadis [2] first considered the boundary layer flow over a stretching sheet which was extended by Crane [3] for the two-dimensional case when the velocity was proportional to the distance from the plate. Gupta et al. [4] and Magyari et al. [5] studied the heat and mass transfer over a stretching sheet subject to suction or injection. Using integral methods, the problem of mixed convection along a vertical surface in the presence of a uniform transverse magnetic field in a porous medium was investigated by Cheng [6]. Two-dimensional stagnation flows adjacent to a vertical heated surface with both prescribed wall temperature and prescribed wall heat flux was considered by Ramachandran et al.[7].
The exact solution for viscous flow induced by a shrinking sheet was investigated by Miklavcic et al.[8] and established non-unique solutions for certain range of suction

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parameter for both two-dimensional and axisymmetric cases. Using the homotopy analysis method (HAM), Sajid et al. [9] extended the above problem for MHD viscous flow. Lok et al.[10] studied MHD stagnation point flow towards a shrinking sheet in micropolar fluid.

In this paper, the steady three-dimensional MHD boundary layer viscous flow and heat transfer due to a permeable stretching sheet with prescribed surface heat flux is studied in presence of a transverse uniform applied magnetic field.

2. Basic Equations

Consider a steady, three-dimensional, laminar, viscous flow of an incompressible electrically conducting fluid bounded by a stretching sheet. Here we assume that the magnetic Reynolds number is small and the electric field is zero so that the induced magnetic field can be neglected. The magnetic field B_0 is applied in z-direction. The governing equations are ([20] , [21]) then

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}v, \tag{3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right), \tag{5}$$

The boundary Conditions are:

$$at \quad z=0: \quad u=ax, \quad v=a(m-1)y, \quad w=-W, \quad \frac{\partial T}{\partial z}=-\frac{q_w}{\kappa}, \tag{6}$$

$$at \quad z\to\infty: \quad u=0, \quad T=T_\infty. \tag{7}$$

where (u,v,w) the fluid velocity, $\nu=\frac{\mu}{\rho}$ the kinematic viscosity, μ the dynamic viscosity, σ the electrical conductivity, α the thermal diffusivity, κ the thermal conductivity, ρ the fluid density, p the fluid pressure, a>0 the stretching constant, q_w the surface heat flux, W the suction velocity, m=1 when the sheet stretches in x-direction (two-dimensional) and m=2 when the sheet stretches axisymmetrically, T_w the sheet temperature and T_∞ is the free stream temperature.

Applying the following similarity transformations

$$u=axf^{/}(\eta), \quad v=a(m-1)yf^{/}(\eta), \quad w=-\sqrt{a\nu}.mf(\eta),$$

$$\eta = \sqrt{\frac{a}{\nu}} z, \quad \theta(\eta) = \frac{\kappa(T - T_\infty)}{q_w}\sqrt{\frac{a}{\nu}}, \tag{8}$$

into the equations (1)-(5), it is seen that equation (1) is satisfied while equation (4) can be integrated to give

$$\frac{p}{\rho} = \nu\frac{\partial w}{\partial z} - \frac{w^2}{2} + constant, \tag{9}$$

Equations (2), (3) and (5) transform to

$$f^{///} - M^2 f^{/} - (f^{/})^2 + m f f^{//} = 0, \tag{10}$$

$$\frac{\theta^{//}}{P_r} + m f \theta^{/} = 0, \tag{11}$$

where η the independent dimensionless similarity variable and θ is the dimensionless temperature. The primes denotes the differentiation w.r.t. η. The $f^{/}(\eta)$ and $\theta(\eta)$ give the velocity and temperature respectively.
The boundary conditions (6) and (7) respectively reduce to:

$$f(0) = s, \quad f^{/}(0) = 1, \quad \theta^{/}(0) = -1, \tag{12}$$

$$f^{/}(\infty) = 0, \quad \theta(\infty) = 0, \tag{13}$$

where $P_r = \frac{\nu}{\alpha}$ is the Prandtl number, $s = \frac{W}{m\sqrt{a\nu}}$ is the suction parameter, $M = B_0\sqrt{\frac{\sigma}{\rho a}}$ is the Hartmann number.

3. Numerical Method

The equations (10) and (11) subject to the boundary conditions (12) and (13) are solved numerically using an implicit finite-difference scheme scheme known as the Keller-box method [12, 13, 14]. The method has following four basic steps:

- I Reduce Equations (10) and (11) to a first order equation;
- II Write the difference equations using central differences;
- III Linearise the resulting algebraic equations by Newton's method and write them in Matrix-vector form;
- IV Use the Block-tridiagonal elimination technique to solve the linear system.

3.1. The Finite Difference Scheme. In this secion, steps (I) and (II) are combined. First we inroduce new dependent variables $u(x,\eta)$, $v(x,\eta)$, $t(x,\eta)$ and $q(x,\eta) = \theta(x,\eta)$ such that

$$f^{/} = u, \quad u^{/} = v, \quad q^{/} = t, \tag{14}$$

so that equations (10) and (11) reduce to

$$v^{/} - M^2u - u^2 + mfv = 0, \tag{15}$$

$$t^{/} + mP_rft = 0. \tag{16}$$

We now consider the net rectangle in the $x - \eta$ plane as shown in figure 1 and the net points defined as follows:

$$x^0 = 0, \quad x^n = x^{n-1} + k_n, \quad n = 1, 2, \ldots N, \tag{17}$$

$$\eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_j, \quad j = 1, 2, \ldots J, \quad \eta_j = \eta_\infty, \tag{18}$$

where k_n is the $\triangle x$ - spacing and h_j is the $\triangle \eta$ - spacing. Here n and j are the sequence of numbers that indicate the coordinate location, not tensor indices or exponents.

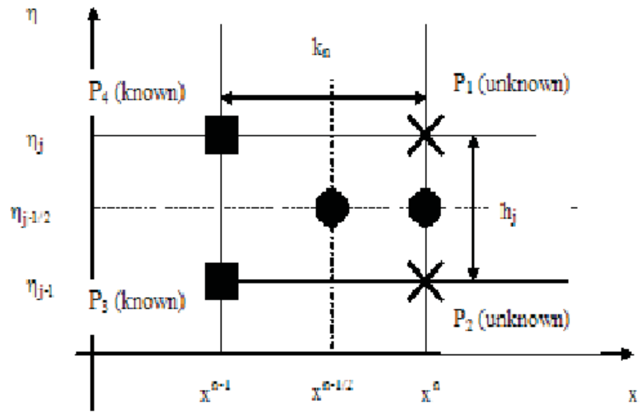

FIGURE 1. Net Rectangle for difference approximation.

Here we use the following finite-differences:

$$()^n_{j-0.5} = 0.5[()^n_j + ()^n_{j-1}], \tag{19}$$

$$()^{n-0.5}_j = 0.5[()^n_j + ()^{n-1}_j], \tag{20}$$

$$\left(\frac{\partial u}{\partial x}\right)_{j-0.5}^{n-0.5} = \frac{[(u)_{j-0.5}^{n} - (u)_{j-0.5}^{n-1}]}{k_n}, \tag{21}$$

$$\left(\frac{\partial u}{\partial \eta}\right)_{j-0.5}^{n-0.5} = \frac{[(u)_{j}^{n-0.5} - (u)_{j-1}^{n-0.5}]}{h_j}, \tag{22}$$

Now we write the finite-difference for the midpoint $(x^n, \eta_{j-0.5})$ of the segment P_1P_2 using $(19) - (22)$. This process is called "centering about $(x^n, \eta_{j-0.5})$". We get by ommitting upper indices n:

$$f_j - f_{j-1} - \frac{h_j}{2}(u_j + u_{j-1}) = 0, \tag{23}$$

$$u_j - u_{j-1} - \frac{h_j}{2}(v_j + v_{j-1}) = 0, \tag{24}$$

$$q_j - q_{j-1} - \frac{h_j}{2}(t_j + t_{j-1}) = 0, \tag{25}$$

$$v_j - v_{j-1} - \frac{h_j}{2}M^2(u_j + u_{j-1}) - \frac{h_j}{4}(u_j + u_{j-1})^2 + \frac{h_j}{4}m(f_j + f_{j-1})(v_j + v_{j-1}) = 0, \tag{26}$$

$$t_j - t_{j-1} - \frac{h_j}{4}P_r m(t_j + t_{j-1})(f_j + f_{j-1}) = 0. \tag{27}$$

The boundary condiions at $x = x^N$ are

$$f_0^N = s, \quad u_0^N = 1, \quad t_0^N = -1, \quad u_J^N = 0, \quad v_J^N = 0, \quad q_J^N = 0. \tag{28}$$

3.2. Newton's method for linearisation. To linearise the nonlinear system (23)-(27), we introduce the following i-th iterate at $x = x^n$:

$$\begin{aligned} f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)}, \quad u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)}, \quad v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)}, \\ q_j^{(i+1)} = q_j^{(i)} + \delta q_j^{(i)}, \quad t_j^{(i+1)} = t_j^{(i)} + \delta t_j^{(i)}, \end{aligned} \tag{29}$$

Substituting these in (23)-(27) and then retaining only the linear terms in $\delta f_j^{(i)}$, $\delta u_j^{(i)}$, $\delta v_j^{(i)}$, $\delta q_j^{(i)}$ and $\delta t_j^{(i)}$, we get the following linear tridiagonal system:

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}(\delta u_j + \delta u_{j-1}) = (r_1)_j, \tag{30}$$

$$\delta u_j - \delta u_{j-1} - \frac{h_j}{2}(\delta v_j + \delta v_{j-1}) = (r_2)_j, \tag{31}$$

$$\delta q_j - \delta q_{j-1} - \frac{h_j}{2}(\delta t_j + \delta t_{j-1}) = (r_3)_j, \tag{32}$$

$$(a_1)_j \delta v_j + (a_2)_j \delta v_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta u_j + (a_6)_j \delta u_{j-1} = (r_4)_j, \tag{33}$$

$$(b_1)_j \delta t_j + (b_2)_j \delta t_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} = (r_5)_j, \tag{34}$$

where $(a_1)_j = 1+\frac{1}{2}mh_j f_{j-0.5}$, $(a_2)_j = (a_1)_j - 2$, $(a_3)_j = \frac{1}{2}mh_j v_{j-0.5}$, $(a_4)_j = (a_3)_j$, $(a_5)_j = -\frac{1}{2}M^2 h_j - h_j u_{j-0.5}$, $(a_6)_j = (a_5)_j$, $(b_1)_j = 1+\frac{1}{2}mP_r h_j f_{j-0.5}$, $(b_2)_j = (b_1)_j - 2$, $(b_3)_j = \frac{1}{2}P_r h_j t_{j-0.5}$, $(b_4)_j = (b_3)_j$, $(r_1)_j = f_{j-1} - f_j + h_j u_{j-0.5}$, $(r_2)_j = u_{j-1} - u_j + h_j v_{j-0.5}$, $(r_3)_j = q_{j-1} - q_j + h_j t_{j-0.5}$, $(r_4)_j = v_{j-1} - v_j + M^2 h_j u_{j-0.5} + h_j (u_{j-0.5})^2 - mh_j f_{j-0.5} v_{j-0.5}$, $(r_5)_j = t_{j-1} - t_j - P_r m h_j f_{j-0.5} t_{j-0.5}$.
For all iterates, we take

$$\delta f_0 = 0, \quad \delta u_0 = 0, \quad \delta t_0 = 0, \quad \delta u_J = 0, \quad \delta q_J = 0. \tag{35}$$

3.3. The Block tridiagonal matrix. The linearised difference system (30)−(34) has a block tridiagonal structure as follows:

$$\begin{bmatrix} [A_1] & [C_1] & & & & \\ [B_2] & [A_2] & [C_2] & & & \\ & & \dots & \dots & & \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] & \\ & & & [B_J] & [C_J] & \end{bmatrix} \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \dots \\ [\delta_{J-1}] \\ [\delta_J] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ \dots \\ [r_{J-1}] \\ [r_J] \end{bmatrix}$$

or,

$$\mathbf{A}\delta = \mathbf{r}, \tag{36}$$

where

$$[A_1] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ d & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ (a_2)_j & 0 & (a_3)_j & (a_1)_j & 0 \\ 0 & 0 & (b_3)_j & 0 & (b_1)_j \end{bmatrix},$$

$$[A_j] = \begin{bmatrix} d & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ (a_6)_j & 0 & (a_3)_j & (a_1)_j & 0 \\ 0 & 0 & (b_3)_j & 0 & (b_1)_j \end{bmatrix}, \quad 2 \leqslant j \leqslant J;$$

$$[B_j] = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & d \\ 0 & 0 & (a_4)_j & (a_2)_j & 0 \\ 0 & 0 & (b_4)_j & 0 & (b_2)_j \end{bmatrix}, \quad 2 \leqslant j \leqslant J;$$

$$[C_j] = \begin{bmatrix} d & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ (a_5)_j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad 1 \leqslant j \leqslant J-1.$$

Here $d = -\frac{h_j}{2}$,

$$[\delta_1] = \begin{bmatrix} \delta v_0 \\ \delta q_0 \\ \delta f_1 \\ \delta v_1 \\ \delta t_1 \end{bmatrix}, \quad [\delta_j] = \begin{bmatrix} \delta u_{j-1} \\ \delta q_{j-1} \\ \delta f_j \\ \delta v_j \\ \delta t_j \end{bmatrix}, 2 \leqslant j \leqslant J; \quad [r_j] = \begin{bmatrix} (r_1)_j \\ (r_2)_j \\ (r_3)_j \\ (r_4)_j \\ (r_5)_j \end{bmatrix}, 1 \leqslant j \leqslant J.$$

Forward sweep:
To solve equation (36) , assume the matrix $\mathbf{A}$ to be nonsingular and it can be factored as

$$\mathbf{A} = \mathbf{LU}, \tag{37}$$

where

$$L = \begin{bmatrix} [\alpha_1] & & & & \\ [B_2] & [\alpha_2] & & & \\ & & \dots & \dots & \\ & & & [\alpha_{J-1}] & \\ & & & [B_J] & [\alpha_J] \end{bmatrix}, \quad U = \begin{bmatrix} I & [\Gamma_1] & & & \\ & I & [\Gamma_2] & & \\ & & \dots & \dots & \\ & & & I & [\Gamma_{J-1}] \\ & & & & I \end{bmatrix},$$

[I] is the identity matrix of order 5, and $[\alpha_j], [\Gamma_j]$ are 5×5 matrices whose elements are determined by the following equations:

$$[\alpha_1] = [A_1], \tag{38}$$

$$[A_1][\Gamma_1] = [C_1], \tag{39}$$

$$[\alpha_j] = [A_j] - [B_j][\Gamma_{j-1}], j = 2, 3, \cdots J, \tag{40}$$

$$[\alpha_j][\Gamma_j] = [C_j], j = 2, 3, \cdots J-1. \tag{41}$$

Backward sweep:

$$\mathbf{LU}\delta = \mathbf{r}, \tag{42}$$

Let

$$\mathbf{U}\delta = \mathbf{w}, \tag{43}$$

so that

$$\mathbf{Lw} = \mathbf{r}, \tag{44}$$

where

$$\mathbf{w} = \begin{bmatrix} [w_1] \\ [w_2] \\ \cdots \\ [w_{J-1}] \\ [w_J] \end{bmatrix},$$

and the $[w_j]$ are 5×1 column matrices. The elements $\mathbf{w}$ can be solved from the equation (44) by

$$[\alpha_1][w_1] = [r_1], \tag{45}$$

$$[\alpha_j][w_j] = [r_j] - [B_j][w_{j-1}], 2 \leqslant j \leqslant J. \tag{46}$$

With these $[w_j]$ and from equation (43) we get $[\delta_j]$:

$$[\delta_J] = [w_J], \tag{47}$$

$$[\delta_j] = [w_j] - [\Gamma_j][\delta_{j+1}], 1 \leqslant j \leqslant J-1. \tag{48}$$

These iterations will be stopped when

$$|\delta v_0^{(i)}| < \varepsilon, \tag{49}$$

where ε is the desired level of accuracy.

4. Results and Discussions

With the help of the implicit finite-difference scheme known as the Keller-box method [12, 13, 14], the equation (10) and (11) subject to the boundary conditions (12) and (13) are solved numerically. Taking the step size $\Delta\eta = 0.01$ in η and within the interval $[0, \eta_\infty]$, where η_∞ is the boundary layer thickness, we run the programme in MATLAB up to the desired level of accuracy which the difference between the input and output values of $v(x, 0)$ (or $f^{//}(0)$) i.e.,equal to 0.00001.
For the validation of the numerical method used in this study,the case of $M^2 = 4$ and $s = 1$ are compared with those of Ali et al. [15], where the skin friction coefficient are $f^{//}(0) = 2.3028$ for $m = 1$, and $f^{//}(0) = 2.8916$ for $m = 2$ by changing one of the boundary conditions ($f^{/}(0) = -1$) in (12). The present results are found to be in good agreement with those of [15].

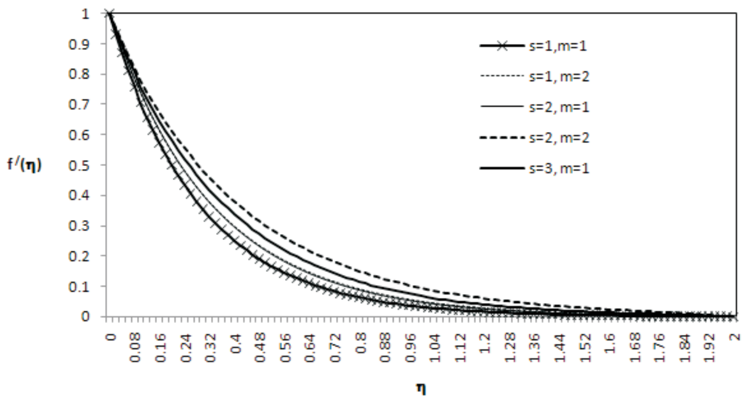

FIGURE 2. Velocity profiles for different s.

The effects of suction parameter s on velocity and temperature profiles are shown in figures 2 and 3. When the value of s increases the velocity profile also increases (Fig.2). But the temperature profile decreases with the increase of s (Fig.3).

The effects of Hartmann number M on velocity and temperature profiles can be observed from the figures 4 and 5 respectively. It is shown that the velocity profiles decreases with the increase of M (Fig.4). When $m = 1$ (the sheet stretches in x-direction two-dimensionally)the temperature profiles increase with the increase of M; but the reverse effect has been observed when $m = 2$ (the sheet stretches axisymmetrically)(Fig.5).

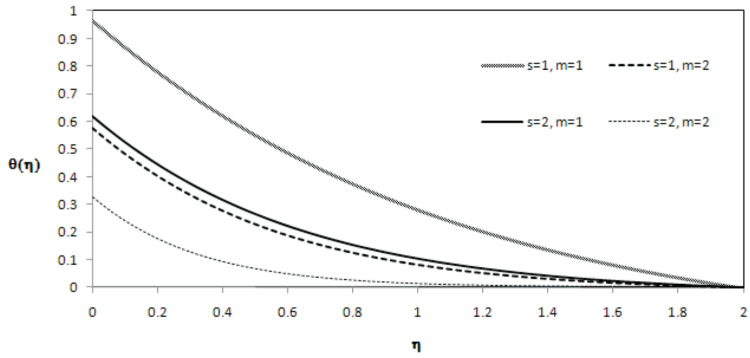

FIGURE 3. Temperature profiles for different s.

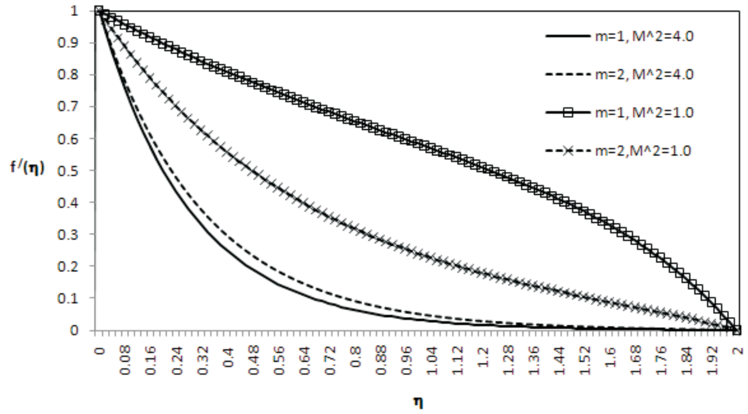

FIGURE 4. Velocity profiles for different M.

The temperature profiles decreases with the increase of P_r for both $m = 1$ and $m = 2$ (fig.6).This occurs because when Pr increases, the thermal diffusivity decreases, and it leads to the decrease of the energy transfer ability that decreases the thermal boundary layer.

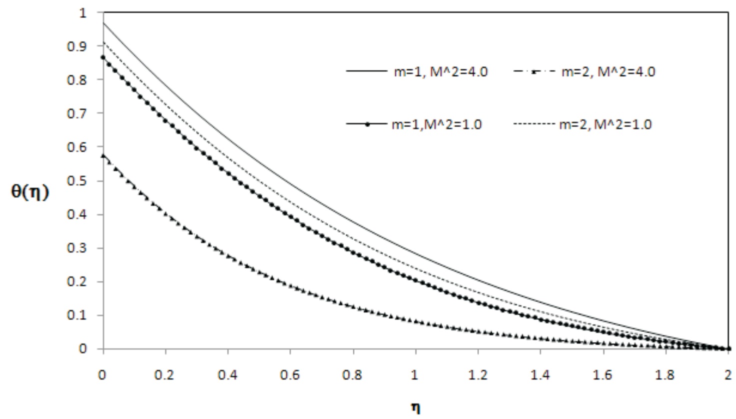

FIGURE 5. Temperature profiles for diffrent M.

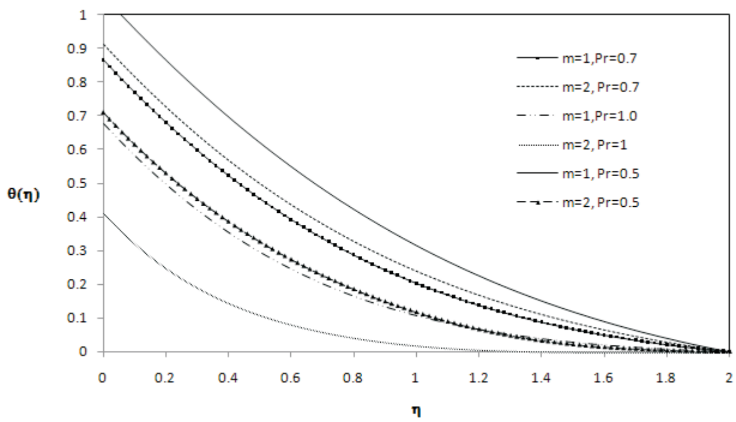

FIGURE 6. Temperature profiles for different Pr.

The variation of skin friction coefficient $f^{//}(0)$ and Nusselt number $\frac{1}{\theta(0)}$ with s are respectively plotted in the figures 7 and 8. It is seen that $f^{//}(0)$ and $\frac{1}{\theta(0)}$ decrease with M^2 for both the cases $m = 1$ and $m = 2$.

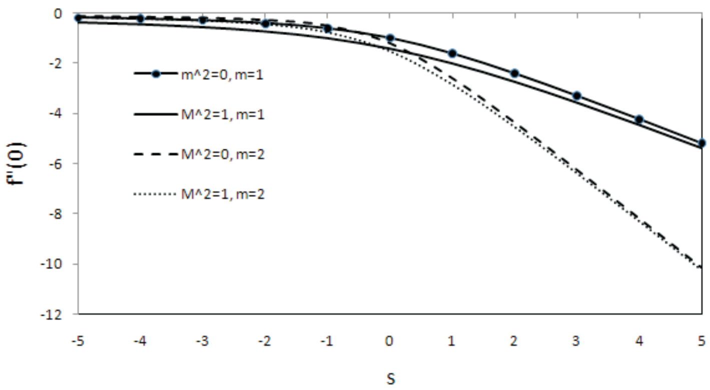

FIGURE 7. Skin friction coefficient with s.

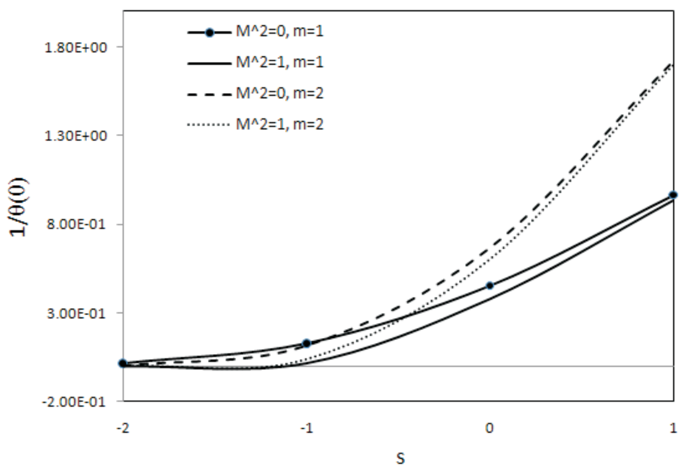

FIGURE 8. Nusselt number with s.

References

[1] H. Blasius, *Grenzschichten in Flussigkeiten mit kleiner Reibung*, Zeitschrift fur Mathematik Physik, **56**(1908), 1-37.

[2] B.C. Sakiadis, *Boundary layer behavior on continuous solid surfaces, II. Boundary layer on a flat surfaces* , AIChEJ, **7**(1961), 221-225.

[3] L.J. Crane, *Flow past a stretching plate*, Jr. of Appl. Maths. And Physics (ZAMP), **21**(1970), 645-647.

[4] P.S. Gupta & A.S. Gupta, *Heat and mass transfer on a stretching sheet with suction and blowing*, Canadian Jr. of Chemical Engineering, **55**(1977), 744-746.

[5] E. Magyari, B. Keller, *Exact solutions for self-similar boundary-layer flows induced by permeable stretching walls*, European J. Mechanics B-fluids, **19**(2000), 109-122.

[6] C.Y. Cheng, *Magnetic field effects on coupled heat and mass transfer embedded in a porous medium by integral methods*, Proceedings of the 4^{th} WSEAS International Conference on Heat and Mass Transfer, Gold Coast, Queensland, Australia, (Jan.2007), 86-91.

[7] N. Ramachandran, T.S. Chen, & B.F. Armaly, *Mixed convection in stagnation flows adjacent to vertical surfaces*, Jr. of Heat Transfer, **110**(1988), 373-377.

[8] M. Miklavcic, C.Y. Wang, *Viscous flow due to a shrinking sheet*, Quarterly of Appl. Maths., **64**(2006), 283-290.

[9] M. Sajid, T. Hayat, *The application of homotopy analysis method for MHD viscous flow due to a shrinking sheet*, Chaos, Solitons and Fractals, **39**(2009), 1317-1323.

[10] Y.Y.Lok, A. Ishak, & I. Pop, *MHD stagnation point flow towards a shrinking sheet*, International Jr. for Numerical Methods in Heat Fluid Flow, 2009, online.

[11] H.B. Keller, T. Cebeci, *Accurate numerical methods for boundary layer flows.II: Two-dimensional turbulent flows*, AIAA Journal, **10**(1972), 1193-1199.

[12] T.Y. Na, *Computational Methods in Engineering Boundary Value Problem*, Academic Press, 1979.

[13] T. Cebeci, P. Bradshaw, *Physical and Computational Aspects of Convective Heat Transfer*, Springer, 1984.

[14] T. Cebeci, J. Cousteix, *Modeling and Computing of Boundary-Layer Flows: Laminar, Turbulent and Transitional Boundary Layers in Incompressible and Compressible Flows*, Springer, 2005.

[15] F.M. Ali, R. Nazar and N.M.Arifin, *MHD Viscous Flow and Heat Transfer Induced by a permeable Shrinking Sheet with Prescribed Surface Heat Flux*, WSEAS Transactions on Mathematics, **9** (5) (2010), 365-375.

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