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AN ITERATION FOR FINDING COMMON RANDOM FIXED POINTS FOR QUASI-NONEXPANSIVE TYPE MAPPINGS IN HILBERT SPACES

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Abstract. We define a random iteration scheme and consider its convergence to a common random fixed point of two random operators defined on a convex subset of a separable Hilbert space.

1. Introduction

The study of random fixed points has been an active area of contemporary research in mathematics. Some of the works in this field are noted in [**1, 2, 3, 4, 8**]. In particular, random iteration schemes leading to random fixed points were introduced in [**2**]. After that, random iterations for finding solutions of random operator equations and fixed points of random operators have been discussed, as, for example, in [**3, 4**]. The aim of this paper is to define an iteration scheme for two random operators on a nonempty closed convex subset of a separable Hilbert space and consider its convergence to a common random fixed point of two random operators. The two random operators satisfy some contractive inequality. Contractive mappings have often been subjects of fixed point studies. For a review of the subject matter, we refer to [**7**].

2. Preliminaries

First we review the following concepts which are essential for our study in this paper. These concepts are obtainable in [**2**], [**3**].

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Throughout this paper, (Ω, Σ) denotes a measurable space and H is a separable Hilbert space. C is a nonempty subset of H.

DEFINITION 2.1. A function $f : \Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \Sigma$ for every Borel subset B of H.

DEFINITION 2.2. A function $F : \Omega \times C \to C$ is said to be random operator if $F(\cdot, x) : \Omega \to C$ is measurable for every $x \in C$.

DEFINITION 2.3. A measurable function $g : \Omega \to C$ is said to be a random fixed point of the random operator $F : \Omega \times C \to C$ if , for all $t \in \Omega$, $F(t, g(t)) = g(t)$.

LEMMA 2.1. ([**6**]) *Let H be a Hilbert space. Then, for any $x, y, z \in H$ and any real λ, the following equality holds:*

$$\|(1-\lambda)x + \lambda y - z\|^2 = (1-\lambda)\|x-z\|^2 + \lambda\|y-z\|^2 - \lambda(1-\lambda)\|x-y\|^2 \tag{2.1}$$

We define a random iteration scheme in the following definition.

DEFINITION 2.4. **(Random iteration scheme):** Let $S, T : \Omega \times C \to C$, where C is a nonempty convex subset of a separable Hilbert space H, be two random operators. Let

$$g_0 : \Omega \to C \tag{2.2}$$

be any measurable function. The random iteration scheme is defined as the following sequence of functions:

$$g_{n+1}(t) = (1-\alpha_n)g_n(t) + \alpha_n h_n(t) \tag{2.3}$$

$$h_n(t) = \beta_n S(t, g_n(t)) + (1-\beta_n)T(t, g_n(t)) \text{ , } \forall n \geqslant 0 \tag{2.4}$$

$$0 < a < \alpha_n < b < 1, \text{ a, b are constants } \forall n = 0, 1, 2 \cdots \tag{2.5}$$

$$0 < \beta_n < 1, \quad \forall n = 0, 1, 2 \cdots \tag{2.6}$$

$$\overrightarrow{\lim_{n\to\infty}} \beta_n < 1 \tag{2.7}$$

LEMMA 2.2. ([**9**]) *Let $\{x_n\}$ and $\{y_n\}$ be two sequences in a Hilbert space H such that*

$$\|x_n\| \leqslant 1, \quad \|y_n\| \leqslant 1, \ \forall n = 0, 1, 2, \cdots \tag{2.8}$$

Let $\{c_n\}$, with $0 < a < c_n < b < 1$, where a and b are fixed constants, be a sequence of real numbers. Then

$$\lim_{n\to\infty} \|(1-c_n)x_n + c_n y_n\| = 1. \tag{2.9}$$

Implies

$$\lim_{n\to\infty} \|x_n - y_n\| = 0. \tag{2.10}$$

In [**9**], the result was proved for a uniformly convex Banach space. Since Hilbert spaces are particular cases of uniformly convex Banach spaces, the result is also true in Hilbert spaces. We have stated the result in the form of the above lemma only in the context of Hilbert spaces.

DEFINITION 2.5. ([**3**]) **(Modified Tricomi's condition):** Two functions $S, T : C \to C$, where C is any nonempty subset of a Hilbert space H, are said to satisfy modified Tricomi's condition if

$$Sp = Tp = p. \tag{2.11}$$

Implies

$$\|Sx - p\| \leqslant \|x - p\|, \quad \|Tx - p\| \leqslant \|x - p\|, \quad \forall x \in C. \tag{2.12}$$

3. Main Results

In this section we discuss the convergence of the random iteration scheme (Definition (2.4)) to a common random fixed point of two random operators which satisfy certain conditions.

First we prove the following lemma which is essential for the main result.

LEMMA 3.1. *Let $S, T : C \to C$, where C is any nonempty subset of a Hilbert space H, satisfy*

$$\begin{aligned} \|Sx - Ty\| \leqslant \max\{&\frac{\|x - Sx\| + \|y - Ty\|}{2}, \frac{\|x - Ty\| + \|y - Sx\|}{3} \\ &, \frac{\|x - y\| + \|x - Sx\| + \|y - Ty\|}{3}\}, \end{aligned} \tag{3.1}$$

for all $x, y \in C$. Then S and T satisfy modified Tricomi's condition.

PROOF. Let $Tp = Sp = p$, putting $y = p$ in (3.1)

$$\begin{aligned} \|Sx - p\| &\leqslant \max\{\frac{\|x - Sx\| + \|p - Tp\|}{2}, \frac{\|x - Tp\| + \|p - Sx\|}{3} \\ &\quad , \frac{\|x - p\| + \|x - Sx\| + \|p - Tp\|}{3}\} \\ &\leqslant \max\{\frac{\|x - Sx\|}{2}, \frac{\|x - p\| + \|p - Sx\|}{3}, \frac{\|x - p\| + \|x - Sx\|}{3}\} \\ &\leqslant \max\{\frac{\|x - p\| + \|p - Sx\|}{2}, \frac{\|x - p\| + \|p - Sx\|}{3}, \frac{2\|x - p\| + \|p - Sx\|}{3}\} \\ &\leqslant \frac{2\|x - p\| + \|p - Sx\|}{3} \\ &\leqslant \|x - p\|. \end{aligned}$$

Similarly , $\|Tx - p\| \leqslant \|x - p\|$.

This completes the proof of the lemma. □

REMARK 3.1. The condition (3.1) is also a quasi-non-expansive type mapping.

THEOREM 3.1. *Let $S, T : \Omega \times C \to C$, where $C$is a nonempty closed convex subset of a separable Hilbert space H , be two continuous random operators which satisfy the following inequality; for all $x, y \in C$ and $t \in \Omega$,*

$$\|S(t,x) - T(t,y\| \leqslant \max\{\tfrac{\|x-S(t,x)\|+\|y-T(t,y)\|}{2}, \tfrac{\|x-T(t,y)\|+\|y-S(t,x)\|}{3}, \tfrac{\|x-y\|+\|x-S(t,x)\|+\|y-T(t,y)\|}{3}\} \tag{3.2}$$

Further, it is assumed that there exists a common random fixed point of $S\&T$. Then the random iteration scheme (Definition (2.4)), if convergent, converges to a common random fixed point of $S\&T$.

PROOF. Let $p : \Omega \to C$ be a common random fixed point of $S\&T$. It is observe that (3.2) implies that for fixed $t \in \Omega$, $S(t, \cdot)$ and $T(t, \cdot)$ satisfy (3.1). Therefore by lemma (3.1),

$$\|S(t,x) - p(t)\| \leqslant \|x - p(t)\| \tag{3.3}$$

$$\|T(t,x) - p(t)\| \leqslant \|x - p(t)\| \tag{3.4}$$

for all $x \in C\&t \in \Omega$.

For all $x \in C\&t \in \Omega$, we have

$$\begin{aligned}
\|g_{n+1}(t) - p(t)\|^2 &= \|(1-\alpha_n)g_n(t) + \alpha_n h_n(t) - p(t)\|^2 (by\ 2.3)\\
&= (1-\alpha_n)\|g_n(t) - p(t)\|^2 + \alpha_n\|h_n(t) - p(t)\|^2\\
&- \alpha_n(1-\alpha_n)\|g_n(t) - h_n(t)\|^2 (by\ 2.1)\\
&\leqslant (1-\alpha_n)\|g_n(t) - p(t)\|^2 + \alpha_n\|h_n(t) - p(t)\|^2 (by\ 2.5)\\
&\leqslant (1-\alpha_n)\|g_n(t) - p(t)\|^2\\
&+ \alpha_n\|\beta_n S(t, g_n(t)) + (1-\beta_n)T(t, g_n(t)) - p(t)\|^2\ (by\ 2.4)\\
&\leqslant (1-\alpha_n)\|g_n(t) - p(t)\|^2\\
&+ \alpha_n\{\beta_n\|g_n(t) - p(t)\|^2 + (1-\beta_n)\|g_n(t) - p(t)\|^2\}\\
&\leqslant (1-\alpha_n + \alpha_n\beta_n + \alpha_n - \alpha_n\beta_n)\|g_n(t) - p(t)\|^2\\
&\leqslant \|g_n(t) - p(t)\|^2
\end{aligned}$$

Implies,

$$\|g_{n+1}(t) - p(t)\| \leqslant \|g_n(t) - p(t)\| \tag{3.5}$$

for all $t \in \Omega$ and $n = 0, 1, 2, \cdots$

Since H is a separable Hilbert space and S and T are continuous random operators, it follows from [**5**] that $S(t, x(t))$ and $T(t, x(t))$ are both measurable functions whenever $x(t)$ is measurable.

From the construction of the random iteration scheme (Definition (2.4)) and from the fact that C is convex, it is immediately follows that $\{g_n(t)\}$ is a sequence of measurable functions from Ω to C.

For $t \in \Omega$, let

$$\{g_n(t)\} \to g(t) \quad \text{as } n \to \infty \tag{3.6}$$

Then $g(t)$, being limit of a sequence of measurable functions, is also measurable. Further, C is closed, which implies that g is a function from Ω to C. If $g(t) = p(t)$ for all $t \in \Omega$, then the result is true. Otherwise, let $\Omega^{'}$ be the subset of Ω such that for $t \in \Omega$, $g(t) \neq p(t)$.
Then for $t \in \Omega'$, there exists a positive integer $N = N(t)$ such that for all $n > N(t)$, $g(t) \neq p(t)$. We construct, for $t \in \Omega'$ and $n > N(t)$

$$x_n(t) = \frac{(g_n(t) - p(t))}{\|g_n(t) - p(t)\|} \tag{3.7}$$

$$y_n(t) = \frac{(h_n(t) - p(t))}{\|g_n(t) - p(t)\|}. \tag{3.8}$$

Then clearly for $t \in \Omega'$, $n > N(t)$

$$\|x_n(t)\| = 1. \tag{3.9}$$

Now for $t \in \Omega'$ and $n > N(t)$

$$\begin{aligned}
\|h_n(t) - p(t)\| &= \|\beta_n S(t, g_n(t)) + (1 - \beta_n) T(t, g_n(t)) - p(t)\| \\
&\leqslant \beta_n \|S(t, g_n(t)) - p(t)\| + (1 - \beta_n)\|T(t, g_n(t)) - p(t)\| \\
&\leqslant \beta_n \|g_n(t) - p(t)\| + (1 - \beta_n)\|g_n(t) - p(t)\| \text{ (by 3.3 \& 3.4)} \\
&\leqslant \|g_n(t) - p(t)\|.
\end{aligned}$$

This shows that,

$$\|y_n(t)\| \leqslant 1. \tag{3.10}$$

Also for $t \in \Omega'$ and $n > N(t)$,

$$\begin{aligned}
(1 - \alpha_n)x_n(t) + \alpha_n y_n(t) &= \frac{(1 - \alpha_n)g_n(t) + \alpha_n h_n(t) - p(t)}{\|g_n(t) - p(t)\|} \\
&= \frac{g_{n+1}(t) - p(t)}{\|g_n(t) - p(t)\|},
\end{aligned}$$

implies that

$$\|(1 - \alpha_n)x_n(t) + \alpha_n y_n(t)\| = \frac{\|g_{n+1}(t) - p(t)\|}{\|g_n(t) - p(t)\|} \to 1, \tag{3.11}$$

as $n \to \infty$ (since $g_n(t) \to g(t) \neq p(t)$, as $n \to \infty$)
From (2.5), (3.9), (3.10), and (3.11), by using lemma (2.2), we get $\lim_{n\to\infty} \|x_n(t) - y_n(t)\| = 0$ for $t \in \Omega'$ and $n > N(t)$.
Now ,

$$\lim_{n\to\infty} \frac{\|g_n(t) - T(t, h_n(t))\|}{\|g_n(t) - p(t)\|} \leqslant \lim_{n\to\infty} \frac{\|g_n(t) - h_n(t)\|}{\|g_n(t) - p(t)\|} \to 0 \text{ as } n \to \infty$$

i.e.

$$\lim_{n\to\infty} \|g_n(t) - T(t, h_n(t))\| = 0, \tag{3.12}$$

for $t \in \Omega'$ and $n > N(t)$.
or

$$\lim_{n\to\infty} T(t, h_n(t)) = \lim_{n\to\infty} g_n(t) = g(t). \tag{3.13}$$

Again, for $t \in \Omega'$,

$$\|g_n(t) - S(t, g_n(t))\| = \|g_n t(t) - T(t, h_n(t)) + T(t, h_n(t)) - S(t, g_n(t))\|$$

$$\begin{aligned}
&\leqslant \|g_n(t) - T(t, h_n(t))\| \\
&\quad + \max\{\frac{\|g_n(t) - S(t, g_n(t))\| + \|h_n(t) - T(t, h_n(t))\|}{2}, \\
&\quad \frac{\|g_n(t) - T(t, h_n(t))\|\|h_n(t) - S(t, g_n(t))\|}{3}, \\
&\quad \frac{\|h_n(t) - g_n(t)\| + \|g_n(t) - S(t, g_n(t))\| + \|h_n(t) - T(t, h_n(t))\|}{3}\} \\
&\leqslant \|g_n(t) - T(t, h_n(t))\| \\
&\quad + \max\{\frac{\|g_n(t) - S(t, g_n(t))\| + \|h_n(t) - g_n(t)\| + \|g_n(t) - T(t, h_n(t))\|}{2}, \\
&\quad \frac{\|g_n(t) - T(t, h_n(t))\| + \|h_n(t) - g_n(t)\| + \|g_n(t) - S(t, g_n(t))\|}{3}, \\
&\quad \frac{2\|h_n(t) - g_n(t)\| + \|g_n(t) - S(t, g_n(t))\| + \|g_n(t) - T(t, h_n(t))\|}{3}\} \\
&\leqslant \|g_n(t) - T(t, h_n(t))\| \\
&\quad + \max\{\frac{\|g_n(t) - S(t, g_n(t))\| + \beta_n\|S(t, g_n(t)) - g_n(t)\|}{2} \\
&\quad + \frac{(1-\beta_n)\|T(t, h_n(t)) - g_n(t)\| + \|g_n(t) - T(t, h_n(t))\|}{2}, \\
&\quad \frac{\|g_n(t) - T(t, h_n(t))\| + \beta_n\|S(t, g_n(t)) - g_n(t)\|}{3} \\
&\quad + \frac{(1-\beta_n)\|T(t, h_n(t)) - g_n(t)\| + \|g_n(t) - S(t, g_n(t))\|}{3}, \\
&\quad \frac{2\beta_n\|S(t, g_n(t)) - g_n(t)\| + 2(1-\beta_n)\|T(t, h_n(t)) - g_n(t)\|}{3} \\
&\quad + \frac{\|g_n(t) - S(t, g_n(t))\| + \|g_n(t) - T(t, h_n(t))\|}{3}\} \\
&\leqslant \|g_n(t) - T(t, h_n(t))\| \\
&+ \frac{(2\beta_n + 1)\|g_n(t) - S(t, g_n(t))\| + (3 - 2\beta_n)\|g_n(t) - T(t, h_n(t))\|}{3}
\end{aligned}$$

implies that

$$3\|g_n(t) - S(t, g_n(t))\| \leqslant$$
$$3\|g_n(t) - T(t, h_n(t))\| + (2\beta_n + 1)\|g_n(t) - S(t, g_n(t))\|$$
$$+(3 - 2\beta_n)\|g_n(t) - T(t, h_n(t))\|$$
$$(2 - 2\beta_n)\|g_n(t) - S(t, g_n(t))\| \leqslant$$
$$(6 - 2\beta_n)\|g_n(t) - T(t, g_n(t))\|$$

$$\|g_n(t) - S(t, g_n(t))\| \leqslant \frac{(6 - 2\beta_n)}{(2 - 2\beta_n)}\|g_n(t) - T(t, g_n(t))\|. \tag{3.14}$$

Making $n \to \infty$ and using (2.7) & (3.12), we get
for $t \in \Omega'$,

$$\lim_{n\to\infty} \|g_n(t) - S(t, g_n(t))\| = 0. \tag{3.15}$$

Or for all $t \in \Omega'$,

$$\lim_{n\to\infty} S(t, g_n(t)) = \lim_{n\to\infty} g_n(t) = g(t). \tag{3.16}$$

Now, for $t \in \Omega'$,

$$\|h_n(t) - g(t)\| = \|\beta_n S(t, g_n(t)) + (1 - \beta_n)T(t, g_n(t)) - g(t)\|$$
$$\leqslant \beta_n\|S(t, g_n(t)) - g(t)\| + (1 - \beta_n)\|T(t, g_n(t)) - g(t)\|$$

Using (3.6) and (3.16) we have by taking limit $n \to \infty$ in the above inequality , for $t \in \Omega'$,

$$\lim_{n\to\infty} h_n(t) = g(t). \tag{3.17}$$

For all $t \in \Omega'$,

$$\|T(t, g(t)) - g(t)\| \leqslant \|T(t, g(t)) - S(t, g_n(t))\| + \|S(t, g_n(t)) - g(t)\|$$
$$\leqslant \max\{\frac{\|g_n(t) - S(t, g_n(t))\| + \|g(t) - T(t, g(t))\|}{2},$$
$$\frac{\|g_n(t) - T(t, g(t))\| + \|g(t) - S(t, g_n(t))\|}{3},$$
$$\frac{\|g_n(t) - g(t)\| + \|g_n(t) - S(t, g_n(t))\| + \|g(t) - T(t, g(t))\|}{3}\}$$

Making $n \to \infty$, we have by (3.6) and (3.15),

$$\|T(t, g(t)) - g(t)\| \leqslant \max\{\frac{\|g(t) - T(t, g(t))\|}{2}, \frac{\|g(t) - T(t, g(t))\|}{3}, \frac{\|g(t) - T(t, g(t))\|}{3}\}$$
$$\leqslant \frac{\|g(t) - T(t, g(t))\|}{2}$$

implies that

$$\|T(t, g(t)) - g(t)\| = 0, \tag{3.18}$$

gives that, for all $t \in \Omega'$,

$$T(t, g(t)) = g(t). \tag{3.19}$$

In the same manner one can find

$$S(t, g(t)) = g(t) \tag{3.20}$$

If $t \in \Omega - \Omega'$, then $g(t) = p(t)$, where $p(t)$ is a common random fixed point of $S\&T$. Thus (3.19) and (3.20) are also satisfied for $t \in \Omega - \Omega'$ and hence for all $t \in \Omega$. Since g is a measurable function (being the limit of a sequence of measurable functions), it follows that $g : \Omega \to C$ is a common random fixed point of $S\&T$. □

THEOREM 3.2. *If, in Theorem* (3.1)*, the subset C of H is further compact, than the random iteration scheme (Definition* (2.4)*) converges to a random fixed point of $S\&T$.*

PROOF. Following the same procedure as in Theorem (3.1), we obtain (3.5), (3.12) and (3.17). Since C is compact, for each $t \in \Omega$, there exists $\{g_{n(s)}(t)\} \subset \{g_n(t)\}$ such that $\{g_{n(s)}(t)\}$is convergent. This choice of $\{g_{n(s)}(t)\}$depends on t; for different values of t, the choice of the subsequence may be different.
Let, for$t \in \Omega$,

$$\lim_{n\to\infty} g_{n(s)}(t) = k(t) \tag{3.21}$$

From the above it does not follows that $k\,(t)$ is a measurable function. Afterwards, we will prove it to be so.
By (3.12) and (3.16),

$$\lim_{s\to\infty} T(t, g_{n(s)}(t)) = \lim_{s\to\infty} S(t, g_{n(s)}(t)) = k(t). \tag{3.22}$$

Then for $t \in \Omega$,

$$\begin{aligned}
\|T(t,k(t)) - k(t)\| &\leqslant \|T(t,k(t)) - S(t,g_{n(s)}(t))\| + \|S(t,g_{n(s)}(t)) - k(t)\| \\
&\leqslant \|S(t,g_{n(s)}(t)) - k(t)\| \\
&\quad + \max\{\frac{\|g_{n(s)}(t) - S(t,g_{n(s)}(t))\| + \|k(t) - T(t,k(t))\|}{2}, \\
&\quad \frac{\|g_{n(s)}(t) - T(t,k(t))\| + \|k(t) - S(t,g_{n(s)}(t))\|}{3}, \\
&\quad \frac{\|g_{n(s)}(t) - k(t)\| + \|g_{n(s)}(t) - S(t,g_{n(s)}(t))\| + \|k(t) - T(t,k(t))\|}{3}\}
\end{aligned}$$

taking limit as $n \to \infty$ and using (3.21) & (3.22) we obtain

$$\begin{aligned}
\|T(t,k(t)) - k(t)\| &\leqslant 0 + \max\{\frac{\|T(t,k(t)) - k(t)\|}{2}, \frac{\|k(t) - T(t,k(t))\|}{3}, \\
&\quad \frac{\|k(t) - T(t,k(t))\|}{3}\} \\
&\leqslant \frac{\|k(t) - T(t,k(t))\|}{2}
\end{aligned}$$

implies that

$$\|k(t) - T(t,k(t))\| = 0 \quad \textit{or} \quad T(t,k(t)) = k(t). \tag{3.23}$$

In an exactly similar way we prove that, for all $t \in \Omega$,

$$S(t, k(t)) = k(t). \tag{3.24}$$

Next we show that $g_n(t) \to k(t)$ as $n \to \infty$ for all $t \in \Omega$.
By lemma (3.1) for all $t \in \Omega$,

$$\|S(t, g_n(t)) - k(t)\| \leqslant \|g_n(t) - k(t)\|, \tag{3.25}$$

$$\|T(t, g_n(t)) - k(t)\| \leqslant \|g_n(t) - k(t)\|. \tag{3.26}$$

Then, for all $t \in \Omega$,

$$\begin{aligned}
\|g_{n+1}(t) - k(t)\| &= \|(1-\alpha_n)g_n(t) + \alpha_n h_n(t) - k(t)\| \\
&\leqslant (1-\alpha_n)\|g_n(t) - k(t)\| + \alpha_n\|h_n(t) - k(t)\| \\
&\leqslant (1-\alpha_n)\|g_n(t) - k(t)\| \\
&\quad + \alpha_n\{\|\beta_n S(t, g_n(t)) + (1-\beta_n)T(t, g_n(t)) - k(t)\|\} \\
&\leqslant (1-\alpha_n)\|g_n(t) - k(t)\| \\
&\quad + \alpha_n\{\beta_n\|S(t, g_n(t)) - k(t)\| + (1-\beta_n)\|T(t, g_n(t)) - k(t)\|\} \\
&\leqslant (1-\alpha_n)\|g_n(t) - k(t)\| \\
&\quad + \alpha_n\{\beta_n\|g_n(t) - k(t)\| + (1-\beta_n)\|g_n(t) - k(t)\|\}
\end{aligned}$$

Or, for all $t \in \Omega$,

$$\|g_{n+1}(t) - k(t)\| \leqslant \|g_n(t) - k(t)\|. \tag{3.27}$$

Formulas (3.22) and (3.27) jointly imply that for all $t \in \Omega$,

$$\lim_{n\to\infty} g_n(t) = k(t). \tag{3.28}$$

Since for all $t \in \Omega$, $\{g_n(t)\} \subset C$ and C is compact, it follows from (3.28) that $k(t) \in C$, for all $t \in \Omega$.
Also, by (3.7), $k(t)$ is the limit of a sequence of measurable functions and hence is itself measurable. From (3.23), (3.24), (3.28) and the above mentioned fact establish that $k : \Omega \to C$ is a common random fixed point of the random operators S and T and the random iteration scheme converges to $k\,(t)$.
This completes the proof of the theorem. □

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