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SOFT BI-INTERIOR IDEALS OVER $\Gamma-$SEMIRINGS

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Abstract. In this paper, we introduce the notion of a soft bi-interior ideal over $\Gamma-$semiring. we characterize regular $\Gamma-$semiing by using their soft ideals and soft regular $\Gamma-$semiring are characterized by soft right(left) ideals.

1. Introduction

The notion of a $\Gamma-$semiring was introduced by Murali Krishna Rao[**10**] in 1995, not only generalizes the notion of a semiring and a $\Gamma-$ring but also the notion of a ternary semiring. The notion of a semiring is an algebraic structure with two associative binary operations where one distributes over the other, was first introduced by Vandiver[**23**] in 1934 but semirings had appeared earlier in studies on the theory of ideals of rings. Herniksen[**4**] defined $k-$ideals in semirings to obtain analogous of ring results for a semiring. The notion of ideals was introduced by Dedekind for the theory of algebraic numbers, was generalized by E. Noether for associative rings. The one and two sided ideals introduced by her, are still central concepts in ring theory. We know that the notion of a one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left and right ideals whereas the bi-ideals are generalization of quasi ideals. The notions of bi-ideals and interior ideals in semigroups were introduced by Lajos. Iseki[**5**] introduced the concept of a quasi ideal for a semiring. Quasi ideals in $\Gamma-$semirings studied by Jagtap and Pawar[**8**]. As a further generalization of ideals, Steinfeld[**22**] first introduced the notion of quasi ideals for semigroups and then for rings. We know that the notion of a bi-ideal in semirings is a special case of (m, n) ideal introduced by S. Lajos. The concept of bi-ideals was first introduced by R. A. Good and D. R. Hughes[**3**] for a semigroup. Murali Krishna Rao[**11, 13, 14, 15**] introduced

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the notion of a left (right) bi-quasi ideal, a bi-interior ideal and a bi-quasi-interior ideal of a semiring, a $\Gamma-$semiring, a $\Gamma-$semigroup and studied their properties. The notion of a $\Gamma-$ring was introduced by Nobusawa[**20**] as a generalization of ring in 1964. Sen[**21**] introduced the notion of a $\Gamma-$semigroup in 1981. The notion of a ternary algebraic system was introduced by Lehmer in 1932.

Molodtsov[**9**] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties, only partially resolves the problem is that objects in universal set often does not precisely satisfy the parameters associated to each of the elements in the set. Feng et al. studied soft semirings by using the soft set theory[**2**]. In this paper, we introduce the notion of a soft bi-interior ideals over $\Gamma-$semirings and study some of their algebraical properties. We characterizes regular $\Gamma-$semirings by using soft bi-interior ideals.

2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier. In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

DEFINITION 2.1. [1] *A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a semiring provided*

(i) *addition is a commutative operation.*
(ii) *multiplication distributes over addition both from the left and from the right.*
(iii) *there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.*

DEFINITION 2.2. *Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M a Γ-semiring, if there exists a mapping $M \times \Gamma \times M \to M$ (images of (x, α, y) will be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$) such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$*

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z.$

Every semiring R is a Γ-semiring with $\Gamma = R$ and ternary operation $x\gamma y$ defined as the usual semiring multiplication.

EXAMPLE 2.1. *Let S be a semiring and $M_{p,q}(S)$ denote the additive abelian semigroup of all $p \times q$ matrices with identity element whose entries are from S. Then $M_{p,q}(S)$ is a Γ- semiring with $\Gamma = M_{p,q}(S)$ ternary operation is defined by $x\alpha z = x(\alpha^t)z$ as the usual matrix multiplication, where α^t denote the transpose of the matrix α; for all x, y and $\alpha \in M_{p,q}(S)$.*

EXAMPLE 2.2. *Let M be the set of all natural numbers. Then $(M,$ max$,$ min$)$ is a semiring. If $\Gamma = M$, then M is a $\Gamma-$semiring.*

EXAMPLE 2.3. *Let M be the additive semigroup of all $m \times n$ matrices over the set of non negative rational numbers and Γ be the additive semigroup of all $n \times m$ matrices over the set of non negative integers.Then with respect to usual matrix multiplication M is a Γ-semiring.*

DEFINITION 2.3. *A Γ-semiring M is said to be commutative Γ-semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$.*

DEFINITION 2.4. *Let M be a Γ-semiring. An element $1 \in M$ is said to be unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$.*

DEFINITION 2.5. *In a Γ-semiring M with unity 1, an element $a \in M$ is said to be left invertible (right invertible) if there exist $b \in M, \alpha \in \Gamma$ such that $b\alpha a = 1(a\alpha b = 1)$.*

DEFINITION 2.6. *In a Γ-semiring M with unity 1, an element $a \in M$ is said to be invertible if there exist $b \in M, \alpha \in \Gamma$ such that $a\alpha b = b\alpha a = 1$.*

DEFINITION 2.7. *A Γ-semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$, for all $x \in M, \alpha \in \Gamma$.*

DEFINITION 2.8. *An element a in a Γ-semiring M is said to be idempotent if there exists $\alpha \in \Gamma$ such that $a = a\alpha a$.*

DEFINITION 2.9. *Every element of M, is an idempotent of M then M is said to be idempotent $\Gamma-$semiring.*

DEFINITION 2.10. *A Γ-semiring M is called a division Γ-semiring if for each non-zero element of M has multiplication inverse.*

DEFINITION 2.11. *A non-empty subset A of a Γ-semiring M is called*

(i) *a Γ-subsemiring of M if (A is a subsemigroup of $(M, +)$ and $A\Gamma A \subseteq A$.*
(ii) *a quasi ideal of M if A is a Γ-subsemigroup of $(M, +)$ and $A\Gamma M \cap M\Gamma A \subseteq A$.*
(iii) *a bi-ideal of M if A is a Γ-subsemiring of M and $A\Gamma M\Gamma A \subseteq A$.*
(iv) *an interior ideal of M if A is a Γ-subsemiring of M and $M\Gamma A\Gamma M \subseteq A$.*
(v) *a left (right) ideal of M if A is a Γ-subsemigroup of $(M, +)$ and $M\Gamma A \subseteq A(A\Gamma M \subseteq A)$.*
(vi) *an ideal if A is a Γ-subsemigroup of $(M, +)$, $A\Gamma M \subseteq A$ and $M\Gamma A \subseteq A$.*
(vii) *a $k-$ideal if A is a Γ-subsemigroup of $(M, +)$, $A\Gamma M \subseteq A, M\Gamma A \subseteq A$ and $x \in M,\ x + y \in A, y \in A$ then $x \in A$.*
(viii) *a bi-interior ideal of M if A is a Γ-subsemiring of M and $M\Gamma A\Gamma M \cap A\Gamma M\Gamma A \subseteq A$.*
(ix) *a left bi-quasi ideal (right bi-quasi ideal) of M if A is a subsemigroup of $(M, +)$ and $M\Gamma A \cap A\Gamma M\Gamma A \subseteq A$ $(A\Gamma M \cap A\Gamma M\Gamma A \subseteq A)$.*
(x) *a left quasi-interior ideal (right quasi-interior ideal) of M if A is a Γ-subsemiring of M and $M\Gamma A\Gamma M\Gamma A \subseteq A$ $(A\Gamma M\Gamma A\Gamma M \subseteq A)$.*
(xi) *a bi-quasi-interior ideal of M if A is a Γ-subsemiring of M and $A\Gamma M\Gamma A\Gamma M\Gamma A \subseteq A$.*

(xii) *a left tri- ideal (right tri- ideal) of M if A is a Γ-subsemiring of M and $A\Gamma M\Gamma A\Gamma A \subseteq A$ $(A\Gamma A\Gamma M\Gamma A \subseteq A)$.*
(xiii) *a tri-ideal of M if A is a Γ-subsemiring of M and $A\Gamma M\Gamma A\Gamma A \subseteq A$ and $A\Gamma A\Gamma M\Gamma A \subseteq A$.*
(xiv) *a left(right) weak-interior ideal of M if A is a $\Gamma-$subsemiring of M and $M\Gamma A\Gamma A \subseteq A($ $A\Gamma A\Gamma M \subseteq A)$.*
(xv) *a weak-interior ideal of M if A is a $\Gamma-$subsemiring of M and $A$is a left weak-interior ideal and a right weak-interior ideal of M.*

DEFINITION 2.12. *A $\Gamma-$semiring M is said to be bi-interior simple $\Gamma-$semiring if M has no bi-interior ideals other than M itself.*

In the following theorem, we mention some important properties and we omit the proofs since proofs are straight forward.

THEOREM 2.1. *Let M be a $\Gamma-$semiring.Then the following are hold*

(1) *Every left ideal is a bi-interior ideal of M.*
(2) *Every right ideal is a bi-interior ideal of M.*
(3) *Every quasi ideal is a bi-interior ideal of M.*
(4) *If A and B are bi-interior ideals of M,then $A\Gamma B$ and $B\Gamma A$ are bi-interior ideals of M.*
(5) *Every ideal is a bi-interior ideal of M.*
(6) *If B is a bi-interior ideal of M, then $B\Gamma M$ and $M\Gamma B$ are bi-interior ideals of M.*

DEFINITION 2.13. *Let S be a $\Gamma-$semiring, E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to P(S)$ where $P(S)$ is the power set of S. Then (f, A) is called a soft $\Gamma-$semiring over S if and only if for each $a \in A$, $f(a)$ is $\Gamma-$subsemiring of S. i.e. (i) $x, y \in S \Rightarrow\ x+y \in f(a)$ (ii) $x, y \in S, \alpha \in \Gamma \Rightarrow\ x\alpha y \in f(a)$.*

DEFINITION 2.14. *Let S be a $\Gamma-$semiring, E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to [0,1]^S$ where $[0,1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft $\Gamma-$semiring over S if and only if for each $a \in A, f(a) = f_a$ is the fuzzy $\Gamma-$subsemiring of S. i.e.,(i) $f_a(x + y) \geqslant min\{f_a(x), f_a(y)\}$ (ii) $f_a(x\alpha y) \geqslant min\{f_a(x), f_a(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.*

DEFINITION 2.15. *Let S be a $\Gamma-$semiring, E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to P(S)$. Then (f, A) is called a soft left(right) ideal over S if and only if for each $a \in A, f(a)$ is a left(right) ideal of S. i.e., (i) $x, y \in f(a) \Rightarrow\ x + y \in f(a)$ (ii) $x, y \in f(a), \alpha \in \Gamma, r \in S \Rightarrow\ r\alpha x(x\alpha r) \in f(a)$.*

DEFINITION 2.16. *Let S be a $\Gamma-$semiring, E be a parameter set, $A \subseteq E$ and $f : A \to\ P(R)$. Then (f, A) is called a soft ideal over S if and only if for each $a \in A, f(a)$ is an ideal of S. i.e., (i) $x, y \in f(a) \Rightarrow\ x + y \in f(a)$ (ii) $x \in f(a), \alpha \in \Gamma, r \in S \Rightarrow\ r\alpha x \in f(a)$ and $x\alpha r \in f(a)$.*

3. Soft bi-interior ideals over $\Gamma-$semirings

In this section, we introduce the notion of a soft bi-interior ideal over $\Gamma-$semiring. We characterize soft regular $\Gamma-$semiing by using soft right(left) ideals and soft bi-interior ideals over $\Gamma-$semiring

Definition 3.1. *Let E be a set, then we call a pair $(F,\ E)$ is a soft set (over E) if $F : E \longrightarrow \mathcal{P}(U)$, where $\mathcal{P}(U)$ is the set of all subsets of U.*

Definition 3.2. *Let $(F\ A)$ and $(G,\ B)$ be two soft sets over a $\Gamma-$semiring M. Then (G, B) is called a soft subset over $(F,\ A)$, if $B \subseteq A$ and $G(b) \subseteq F(b)$, for all $b \in B$. It is denoted by $(G,\ B) \subseteq (F,\ A)$.*

Definition 3.3. *A soft set $(F\ A)$ over a $\Gamma-$semiring M is called a soft set with cover if $\bigcup\limits_{a\in A} F(a) = M$.*

Definition 3.4. *Two soft sets $(F,\ A)$ and $(G,\ B)$ over a $\Gamma-$semiring M are said to be equal if $(F,\ A) \subseteq (G,\ B)$ and $(G,\ B) \subseteq (F,\ A)$.*

Definition 3.5. *A soft set $(F,\ A)$ over a $\Gamma-$semiring M is called a soft (left, right) ideal over a $\Gamma-$semiring M if $F(a)$ is an (left, right) ideal of M, when $F(a) \neq \emptyset$, for all $a \in A$.*

Definition 3.6. *A soft set $(F,\ A)$over a $\Gamma-$semiring M is called a soft bi-interior ideal over M if $F(a)$ is a bi=interior ideal of a $\Gamma-$semiring M, when $F(a) \neq \emptyset$.*

Definition 3.7. *Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over M. Then $(F,\ A) \wedge (G,\ B) = (H,\ A \times B)$,is defined by $H(a, b) = F(a)\Gamma G(b)$, for all $(a,\ b) \in A \times B$.*

Definition 3.8. *Let $(G,\ B)$ be a soft subset of a soft set $(F,\ A)$ over a $\Gamma-$semiring M . Then $(G,\ B)$ is said to be soft bi-interior ideal of $(F,\ A)$ if $G(b) \neq \emptyset$ is a bi-interior ideal of $F(b)$, for all $b \in B$.*

Definition 3.9. *A soft set $(B,\ A)$ over M is called a soft bi-interior ideal over M, if $B(a) \neq \emptyset$ and $B(a)$ is a bi-interior ideal of $\Gamma-$semiring M, for all $a \in A$.*

Theorem 3.1. *Let $(R,\ A)$ be a soft right ideal over a $\Gamma-$semiring M. Then $(R,\ A)$ is a soft bi-interior ideal of $\Gamma-$semiring M.*

Proof. Suppose $(R,\ A)$ be a soft right ideal over M.
Then $R(a)$ is a right ideal of M, for all $a \in A$.
If $R(a)$ is a right ideal of M, then $R(a)\Gamma M\Gamma R(a) \subseteq R(a)\Gamma R(a) \subseteq R(a)$. That implies $R(a)\Gamma M\Gamma R(a) \cap M\Gamma R(a)\Gamma M \subseteq R(a)\Gamma M\Gamma R(a) \subseteq R(a)$, for all $a \in M$. Hence $R(a)$ is a soft bi-interior ideal of the $\Gamma-$semiring M. Therefore $(R,\ A)$ is a soft bi-interior ideal of M. □

Corollary 3.1. *Let $(R,\ A)$ be a soft left ideal over a $\Gamma-$semiring M. Then $(R,\ A)$ is a soft bi-interior ideal of $\Gamma-$semiring M.*

Corollary 3.2. *Let $(R,\ A)$ be a soft ideal over a $\Gamma-$semiring M. Then $(R,\ A)$ is a soft bi-interior ideal of $\Gamma-$semiring M.*

THEOREM 3.2. *Let $(R,\ A)$ be a soft right ideal and $(L,\ A)$ be a soft left ideal over a $\Gamma-$semiring M. Then $(R,\ A) \wedge (L,\ B)$ is a soft bi-interior ideal over M.*

PROOF. By definition $(R,\ A) \wedge (L,\ B) = (H,\ C)$, where $c = A \times B$ and $H(a,\ b) = R(a)\Gamma L(b)$.
Therefore, $R(a)\Gamma L(b)$ is a bi-interior ideal, for all $(a,\ b) \in A \times B$.
Hence, $(R,\ A) \wedge (L,\ B)$ is a soft bi-interior ideal over M. □

THEOREM 3.3. *Every soft bi-ideal over $\Gamma-$semiring M is a soft bi-interior ideal over M.*

PROOF. Let $(F,\ A)$ be soft bi-ideal over $\Gamma-$semiring M.
Then $F(a)$ is a bi-ideal over M. By theorem[2.1], $F(a)$ is a bi-interior ideal of M.
Hence,$(F,\ A)$ is a soft bi-interior ideal over M. □

THEOREM 3.4. *Every soft quasi interior ideal over a $\Gamma-$semiring M is a soft bi-interior ideal over M.*

PROOF. Let $(F,\ A)$ be a soft quasi interior over the $\Gamma-$semiring M.
Then $F(a)$ is a quasi ideal of M, for all $a \in A$. By theorem[2.1], $F(a)$ is a bi-interior ideal of M. Hence, $(F,\ A)$ is a soft bi-interior ideal over M. □

THEOREM 3.5. *Let $(R,\ A)$, $(L,\ B)$ be soft right ideal over a soft left ideal over a $\Gamma-$semiring M respectively. Then $(R,\ A) \cap (L,\ B) = (H,\ C)$, where $C = A \cap B \neq \emptyset$.*

PROOF. Let $c \in C$. Then $H(c) = R(c) \cap L(c)$. Since $R(c)$ and $L(c)$ are a right ideal and a left ideal of the $\Gamma-$semiring M. Therefore, by theorem[3.2], we get that $R(c) \cap L(c)$ is a bi-interior ideal of $\Gamma-$semiring M. Hence $(R,\ A) \cap (L,\ A)$ is a soft bi-interior ideal of the $\Gamma-$semiring M. □

THEOREM 3.6. *If $(F,\ A)$ and $(G,\ B)$ are two soft bi-interior ideals over a $\Gamma-$semiring M then the following statements hold*
(i) $(F,\ A) \cap (G,\ B)$ *is a soft bi-interior ideal over M, where $A \cap B \neq \emptyset$*
(ii) $(F,\ A) \cup (G,\ B)$ *is a soft bi-interior ideal over $\Gamma-$semiring M, where $A \cap B \neq \emptyset$.*

DEFINITION 3.10. *Let $(G,\ B)$ be a soft subset of a soft $\Gamma-$semiring $(F,\ A)$ over M. Then $(G,\ B)$ is said to be soft bi-interior ideal of $(F,\ A)$ if $G(a) \neq \emptyset$ is a bi-interior ideal of $F(b)$, for all $b \in B$.*

THEOREM 3.7. *Let $(F,\ A)$ and $(G,\ B)$ be two soft bi-interior ideals over a regular $\Gamma-$semiring M. Then $(F,\ A)\Gamma(G,\ B)$ is a soft bi-ideal over M.*

PROOF. It is known that $(F,\ A)\Gamma(G,\ B) = (H,\ A \times B)$, where H is a function from $A \times B$ to $\mathcal{P}(S)$.
Define $H(a,\ b) = F(a)\Gamma G(b)$. Since $f(a)$ and $G(b)$ are bi-interior ideals of M, for all $a \in A,\ b \in B$.
Now, $F(a)\Gamma G(b)\Gamma F(a)\Gamma G(b) \subseteq F(a)\Gamma G(b)\Gamma M\Gamma F(a)\Gamma G(b) \subseteq F(a)\Gamma G(b)$ (since M is a regular $\Gamma-$semiring). Thus $F(a)\Gamma G(b)$ is a $\Gamma-$subsemiring of M.
Therefore, $(H,\ A \times B)$ is a soft $\Gamma-$subsemiring of M.
Now $(F(a)\Gamma G(b))\Gamma M\Gamma(F(a)\Gamma G(b)) \cap (M\Gamma F(a)\Gamma G(b)\Gamma M)$

$\subseteq (F(a)\Gamma G(b))\Gamma M\Gamma(F(a)\Gamma G(b)) \subseteq F(a)\Gamma(G(b))$.
Therefore, $F(a)\Gamma G(b)$ is a bi-interior ideal of $\Gamma-$semiring M. Thus $(F,\ A)\Gamma(G,\ B)$ is a soft bi-ideal over M. □

4. Characterization of regular and soft regular Γ−semirings

We characterize the regular $\Gamma-$semiring using soft right ideals and soft left ideals over $\Gamma-$semiring.

Theorem 4.1. *A $\Gamma-$semiring M is a regular if and only if $(R,\ A)\Gamma(L,\ B) = (R,\ A) \cap (L,\ B)$, for every soft right ideal $(R,\ A)$ and soft left ideal $(L,\ B)$ over $\Gamma-$semiring M.*

Proof. Suppose $(R,\ A)\Gamma(L,\ B) = (H,\ A \times B)$, where H is a function from $A \times B$ to $\mathcal{P}(S)$ defined by $H(a,\ b) = R(a)\Gamma L(b)$, for all $(a,\ b) \in A \times B$.
We define $(R,\ A) \cap (L,\ B) = (G,\ A \times B)$, where G is a function from $A \times B$ to $\mathcal{P}(M)$ such that $G(a, `b) = R(a) \cap L(b)$.
We have $R(a)\Gamma L(b) \subseteq R(a)\Gamma M \subseteq R(a)$ and $R(a)\Gamma L(b) \subseteq M\Gamma L(b) \subseteq L(b)$.
Hence $R(a)\Gamma L(b) \subseteq R(a) \cap L(b)$, for all $a \in A,\ b \in B$.
Therefore $(R,\ A)\Gamma(L,\ B) \subseteq (R,\ A) \cap (L,\ B)$.
Suppose that $x \in R(a) \cap L(b)$. Since M is a regular and $x \in M$, there exist $x \in M$ and $\alpha, \beta \in \Gamma$ such that $x = u\alpha x\beta u$. That implies $u \in R(a)$ and $x\beta u \in L(b)$. Therefore $u \in R(a)\Gamma L(b)$. Hence $R(a) \cap L(b) \subseteq R(a)\Gamma L(b)$.
Therefore $(R,\ A)\Gamma(L,\ B) = (R,\ A) \cap (L,\ B)$.
Sufficient condition: Suppose that $A = B = M$ define R a function from A to $\mathcal{P}(S)$, and L is a function from B to $\mathcal{P}(M)$, $x \in M$. Define $R(u) = u\alpha M$, for all $\alpha \in \Gamma$ and $L(u) = M\alpha u$, for all $\alpha \in \Gamma$. Then $(R,\ M)$ and $(L,\ M)$ are soft right ideal and a soft left ideal over M respectively. We have that $R(u) \cap L(u) = R(u)\Gamma L(u)$.
Now $x \in R(u) \cap L(u) = R(u)\Gamma L(u) = u\Gamma M\Gamma M\Gamma \leqslant u\Gamma M\Gamma u$. Therefore $u \in u\Gamma M\Gamma u$. Hence M is a regular Γsemiring. □

Theorem 4.2. *Let M be a $\Gamma-$semiring. Then M is regular if and only if $(Q,\ A) \cap (L,\ B) \subseteq (Q,\ A)\Gamma(L,\ B)$, for every soft quasi ideal $(Q,\ A)$ and every soft left ideal $(L,\ B)$ over M.*

Proof. Suppose $(a,\ b) \in A\times B$, M is a regular Γsemiring and $x \in Q(a)\cap L(b)$. Since $x \in M$ and M is regular there exist $\alpha, \beta \in \Gamma$ and $v \in M$ such that
$u = u\alpha v\beta u \in Q(a)\Gamma M\Gamma L(b) \subseteq Q(a)\Gamma L(b)$.
Therefore $Q(a) \cap L(b) \subseteq Q(a)\Gamma L(b)$. Hence $(Q,\ A) \cap (L,\ B) \subseteq (Q,\ A)\Gamma(L,\ B)$.
Sufficiently, suppose that $A = B = M$. Define $Q : A \longrightarrow \mathcal{P}(M)$ and
$L : B \longrightarrow \mathcal{P}(M)$ by $Q(u) = u\alpha M$ and $L(u) = M\beta u$, for all $u \in M,\ \alpha, \beta \in \Gamma$. Then $(Q,\ A)$ and $(L,\ B)$ are soft quasi ideal and soft left ideal over M respectively. Therefore, $x \in Q(u) \cap L(u) \subseteq Q(u)\Gamma L(b) = u\alpha M\gamma M\beta u \subseteq u\alpha M\beta u$.
Hence, M is a regula $\Gamma-$semiring. □

Definition 4.1. *A soft $\Gamma-$semiring $(F,\ A)$ over a $\Gamma-$semiring M is called a soft regular $\Gamma-$semiring if for each $a \in A,\ F(a)$ is regular $\Gamma-$semiring.*

Theorem 4.3. *Let* $(F,\ A)$ *be a soft regular* $\Gamma-$*semiring* M *with cover over a* $\Gamma-$*semiring* M. *Then* M *is a regular* $\Gamma-$*semiring.*

Proof. Suppose $(F,\ A)$ is a soft regular $\Gamma-$semiring and $x \in M$.. Then $F(a)$ is a regular $\Gamma-$semiring M, for all $a \in A$. Then $M = \bigcup\limits_{a \in A} F(a)$. Then there exists $b \in A$ such that $x \in F(b)$. Since $F(b)$ is regular, there exist $y \in F(b),\ \alpha, \beta \in \Gamma$ such that $x = x\alpha y\beta x$. That implies $y \in F(b) \subseteq \bigcup\limits_{a \in A} F(a) = M$. Therefore M is a regular $\Gamma-$semiring. □

Proof of the following theorem is similar to Theorem[4.2].

Theorem 4.4. *Let* M *be a* $\Gamma-$*semiring if and only if* $(B,\ A) \cap (L,\ B) \subseteq (B,\ A)\Gamma(L,\ B)$, *for every soft bi-ideal* $(B,\ A)$ *and every left ideal* $(L,\ B)$ *over* B.

Theorem 4.5. $(F,\ A)$ *is a regular soft* $\Gamma-$*semiring over* $\Gamma-$*semiring* M, *if and only if* $(R,\ B) \circ (L,\ C) = (R,\ B) \cap (L,\ C)$, *for all soft right ideal* $(R,\ B)$ *and soft left ideal* $(L,\ C)$ *over* M.

Proof. Suppose $(R,\ B) \circ (L,\ C) = (H,\ B \cap C)$, where $H : B \cap C \longrightarrow \mathcal{P}(M)$ defined by $H(a) = R(a)\Gamma L(a)$, for all $a \in B \cap C$.
$(R,\ B) \cap (L,\ C) = (G,\ B \cap C)$, where $G : B \cap C \longrightarrow \mathcal{P}(M)$ defined by
$G(a) = R(a) \cap L(a)$, for all $a \in B \cap C$.
We have $R(a)\Gamma L(a) \subseteq R(a)\Gamma M \subseteq R(a)$ and $R(a)\Gamma L(a) \subseteq M\Gamma L(a) \subseteq L(a)$. That implies $R(a)\Gamma L(a) \subseteq R(a) \cap L(a)$.
Therefore, $(H,\ B \cap C) \subseteq (G,\ B \cap C)$. Suppose $x \in R(a) \cap L(a)$.
Since $u \in F(a)$ and $F(a)$ is a regular $\Gamma-$semiring, there exist $\alpha, \beta \in \Gamma,\ v \in M$ such that $u = \alpha v\beta u \in R(a)\Gamma L(a)$. This shows that $R(a) \cap L(a) \subseteq R(a)\Gamma L(a)$.
Hence, $(R,\ B) \circ (L,\ C) = (R,\ B)\Gamma(L,\ C)$.
Conversely suppose that if $(R,\ B) \circ (L,\ C) = (R,\ B) \cap (L,\ C)$, for all soft right ideals $(R,\ B)$ and soft left ideals $(L,\ C)$ over $F(a)$, for all $a \in B \cap C \neq \emptyset$. Suppose that $B = C = F(a)$. R is a function from B to $\mathcal{P}(F(a))$ defined by $R(u) = u\alpha F(a)$, for all $u \in F(a),\ \alpha \in \Gamma$ and L is a function from $F(a)$ to $\mathcal{P}(F(a))$ defined by $L(u) = F(a)\beta u$, for all $u \in F(a),\ \beta \in \Gamma$.
Thus $(R,\ B)$ and $(L,\ C)$ are soft right ideal and soft left ideal respectively over $F(a)$. Then $R(a) \cap L(a) = F(a)\Gamma L(a) = u\alpha F(a)\Gamma F(a)\beta u \subseteq F(a)$.
Hence, $(F,\ A)$ is a regular soft $\Gamma-$semiring over M. □

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