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NEARNESS SUBTRACTION ALGEBRAS

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Abstract. Recently, algebraic structures have begun to move from classical set theory to near set theory. Here we will apply subtraction algebras to near set theory.

1. Introduction

B. M. Schein [**21**] considered systems of the form $(\Phi; \circ, \backslash)$, where Φ is a set of functions closed under the composition "$\circ$" of functions (and hence $(\Phi; \circ)$ is a function semigroup) and the set theoretic subtraction "$\backslash$" (and hence $(\Phi; \backslash)$ is a subtraction algebra in the sense of [**1**]). Before these studies, Abbott invented the concept of subtraction algebra in 1967. After that, in the year 1992, Schein developed the concept of subtraction semi group by using the notion of subtraction algebra. Later, in the year of 2007, on near subtraction algebra was developed by Dheena. After that the ideals in near subtraction algebra and some of its properties were studied by Jun et al ([**5**],[**6**],[**7**],[**8**]). In 2013, Peters introduced to near set theory ([**20**]). Prior to this work, Peters has done a lot of work to establish the background of near set theory ([**16**],[**17**],[**18**],[**19**]). Many researchers have begun to move algebraic structures from classical set theory to near set theory (([**3**],[**4**],[**9**],[**10**],[**11**],[**12**],[**13**],[**14**],[**15**]). Here we will apply subtraction algebras to near set theory.

2. Preliminaries

We will give basic definitions and properties on subtraction algebras.

An algebra $(A; -)$ with a single binary operation "$-$" is called a subtraction algebra if the following conditions are provided:

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(i) for all $u, v \in A$, $u - (v - u) = u$,
(ii) for all $u, v \in A$, $u - (u - v) = v - (v - u)$,
(iii) for all $u, v, w \in A$, $(u - v) - w = (u - w) - v$.

The relation $u \leqslant v \Longleftrightarrow u - v = 0$ determines an order relation on A. Herein $0 = u - u$ is an element that doesn't depend on the choice of $u \in A$. The ordered set $(A; \leqslant)$ is a semi-Boolean algebra in sense of [**1**], that is it is a meet semilattice with zero 0 in which ever interval $[0, u]$ is a Boolean algebra with respect to induced order. Here $u \wedge v = u - (u - v)$ and the complement of an element $v \in [0, u]$ is $u - v$.

DEFINITION 2.1. [**5**] *Let A be a subtraction algebra and $\emptyset \neq I$ subset of A. Then I is called an ideal of A if it satisfies*
(1) $0 \in I$,
(2) $u - v \in I \Longrightarrow u \in I$ *for all* $u \in A$, $v \in I$.

Let A be a subtraction algebra and I ideal of A. If $u \leqslant v$ and $v \in I$, then $u \in I$ for any $u, v \in A$.

Let's give the notation in Table1 about nearness sets,

SYMBOL	MEANING
$\mathcal{A}$	a set of perceived objects
$\mathcal{P}$	set of probe functions
$\sim_{B_r}$	indiscernibility relation
$N_r(B)$	collection of partitions
$(\mathcal{A}, \mathcal{P}, \sim_{B_r}, N_r(B))$	A nearness approximation space

Table 1

For all $x, y \in \mathfrak{X}$, $[x]_{\sim_{B_r}} \ominus [y]_{\sim_{B_r}} \subseteq [x \ominus y]_{\sim_{B_r}}$ is true.

DEFINITION 2.2. *For all $x, y \in \mathfrak{X}$, if $[x]_{\sim_{B_r}} \ominus [y]_{\sim_{B_r}} = [x \ominus y]_{\sim_{B_r}}$, then "$\sim_{B_r}$" is called complete indiscernibility relation on $\mathfrak{X}$.*

3. Nearness subtraction algebras

DEFINITION 3.1. *Let $(\mathcal{A}, \mathcal{P}, \sim_{B_r}, N_r(B))$ be a nearness approximation space where 0 be a constant on $\mathcal{A}$, $\mathfrak{X} \neq \emptyset$ and $\mathfrak{X} \subseteq \mathcal{A}$. If all $u, v, w \in \mathfrak{X}$, the following properties in $N_r(B)^{\ominus}\,\mathfrak{X}$ are provided, then $\mathfrak{X}$ is called a nearness subtraction algebra.*
$\mathcal{NS}_1$) $u \ominus (v \ominus u) = u$
$\mathcal{NS}_2$) $u \ominus (u \ominus v) = v \ominus (v \ominus u)$
$\mathcal{NS}_3$) $(u \ominus v) \ominus w = (u \ominus w) \ominus v$
where $\ominus : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ is a well-defined operation.

EXAMPLE 3.1. *Let $\mathcal{A} = \{0, u, v, w, z\}$ be a set of perceived objects, $r = 1$, $B = \{\rho_1, \rho_2\} \subseteq \mathcal{P}$ a set of probe functions, and $\mathfrak{X} = \{v, z\} \subset \mathcal{A}$. Values of the probe functions are given in Table 2.*

$$\rho_1 : \mathcal{A} \to V_1 = \{\vartheta_1, \vartheta_2, \vartheta_3\},$$
$$\rho_2 : \mathcal{A} \to V_2 = \{\vartheta_1, \vartheta_2, \vartheta_3\},$$

	0	u	v	w	z
ρ_1	ϑ_1	ϑ_1	ϑ_2	ϑ_3	ϑ_2
ρ_2	ϑ_1	ϑ_2	ϑ_2	ϑ_3	ϑ_1

Table 2

We can write

$$[0]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(u) = \vartheta_1\} = \{0, u\} = [u]_{\rho_1},$$
$$[v]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(z) = \vartheta_2\} = \{v, z\} = [z]_{\rho_1},$$
$$[w]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(w) = \vartheta_3\} = \{w\} = [w]_{\rho_1}$$

Then we get $\xi_{\rho_1} = \left\{[0]_{\rho_1}, [v]_{\rho_1}, [w]_{\rho_1}\right\}$.

$$[0]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(z) = \vartheta_1\} = \{0, z\} = [z]_{\rho_2},$$
$$[u]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(v) = \vartheta_2\} = \{u, v\} = [v]_{\rho_2},$$
$$[w]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(w) = \vartheta_3\} = \{w\} = [w]_{\rho_2}$$

Then we get $\xi_{\rho_2} = \left\{[0]_{\rho_2}, [u]_{\rho_2}, [w]_{\rho_2}\right\}$.

$N_1(B) = \{\xi_{\rho_1}, \xi_{\rho_2}\}$ *and* $\mathfrak{X} = \{v, z\}$

$$Nr(B)^{\ominus}\mathfrak{X} = \bigcup_{[x]_{\rho i} \cap \mathfrak{X} \neq \emptyset} [x]_{\rho_i} = [0]_{\rho_1} \cup [v]_{\rho_1} \cup [0]_{\rho_2} \cup [u]_{\rho_2} = \{0, u, v, z\}$$

"$\ominus$" operation is given by following Table 3 *on* $\mathfrak{X}$

$\ominus$	v	z
v	0	0
z	u	0

Table 3

Since $0 \notin \mathfrak{X}$, $\mathfrak{X}$ *is not subtraction algebra. However* $\mathfrak{X}$ *is a nearness subtraction algebra, because* $(\mathcal{NS}_1)$, $(\mathcal{NS}_2)$, $(\mathcal{NS}_3)$ *are provided on* $Nr(B)^{\ominus}\mathfrak{X}$*with "$\ominus$" operation is given by Table* 4.

$\ominus$	0	u	v	z
0	0	0	0	0
u	u	0	u	0
v	v	v	0	0
z	z	v	u	0

Table 4

EXAMPLE 3.2. *Let* $\mathcal{A} = \{0, u, v, w, z\}$ *be a set of perceived objects,* $r = 1$, $B = \{\rho_1, \rho_2\} \subseteq \mathcal{P}$ *be a set of probe functions and* $\mathfrak{X} = \{u, v\} \subset \mathcal{A}$. *Values of the probe functions are given in Table* 5.

$$\rho_1 : \mathcal{A} \to V_1 = \{\vartheta_1, \vartheta_2, \vartheta_3\},$$
$$\rho_2 : \mathcal{A} \to V_2 = \{\vartheta_1, \vartheta_2, \vartheta_3\},$$

	0	u	v	w
ρ_1	ϑ_1	ϑ_2	ϑ_1	ϑ_3
ρ_2	ϑ_3	ϑ_1	ϑ_2	ϑ_2

Table 5

We can write

$$\begin{aligned}
[0]_{\rho_1} &= \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(v) = \vartheta_1\} = \{0, v\} \\
[u]_{\rho_1} &= \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(u) = \vartheta_2\} = \{u\} \\
[w]_{\rho_1} &= \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(w) = \vartheta_3\} = \{w\}
\end{aligned}$$

$$\begin{aligned}
[0]_{\rho_2} &= \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(0) = \vartheta_3\} = \{0\} \\
[u]_{\rho_2} &= \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(u) = \vartheta_1\} = \{u\} \\
[v]_{\rho_2} &= \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(w) = \vartheta_2\} = \{v, w\} = [w]_{\rho_2}
\end{aligned}$$

Then we get $\xi_{\rho_1} = \left\{[0]_{\rho_1}, [u]_{\rho_1}, [w]_{\rho_1}\right\}$, $\xi_{\rho_2} = \left\{[0]_{\rho_2}, [u]_{\rho_2}, [v]_{\rho_2}\right\}$.

$N_1(B) = \{\xi_{\rho_1}, \xi_{\rho_2}\}$

$$Nr(B)^{\ominus}\mathfrak{X} = \bigcup_{[x]_{\rho i} \cap \mathfrak{X} \neq \emptyset} [x]_{\rho_i} = [0]_{\rho_1} \cup [u]_{\rho_1} \cup [u]_{\rho_2} \cup [v]_{\rho_2} = \{0, v\} \cup \{u\} \cup \{u\} \cup \{v, w\} = \{0, u, v, w\} = \mathcal{A}$$

"$\ominus$" operation is given by following Table 6 *on* $\mathfrak{X}$

$\ominus$	0	u	v	w
0	0	0	0	0
u	u	0	u	0
v	v	v	0	0
w	w	w	w	0

Table 6

$(\mathcal{NS}_1)$ is not provided on $\mathcal{A}$. Because $u \ominus (w \ominus u) = u \ominus w = 0 \neq u$. And so $\mathcal{A}$ is not subtraction algebra. Since $\mathcal{A} = Nr(B)^{\ominus}\mathfrak{X}$, $Nr(B)^{\ominus}\mathfrak{X}$ is not subtraction algebra, too. On the other hand, "$\ominus$" can be defined on $\mathfrak{X}$, as following

$\ominus$	v	z
v	0	0
z	u	0

Table 7

Since $0 \notin \mathfrak{X}$, $\mathfrak{X}$ is not subtraction algebra. Also,

$(\mathcal{NS}_1)$ *For all $x, y \in \mathfrak{X}$, $x \ominus (y \ominus x) = x$ holds on $Nr(B)^{\ominus}\mathfrak{X}$,*

$(\mathcal{NS}_2)$*For all $x, y \in \mathfrak{X}$, $x \ominus (x \ominus y) = y \ominus (y \ominus x)$ holds on $Nr(B)^{\ominus}\mathfrak{X}$,*

$(\mathcal{NS}_3)$ *For all $x, y, z \in \mathfrak{X}$, $x \ominus (y \ominus z) = (x \ominus z) \ominus y$ holds on $Nr(B)^{\ominus}\mathfrak{X}$,*

Thus, $\mathfrak{X}$ is a nearness subtraction algebra.

REMARK 3.1. The relation "$\leqslant$" on $\mathcal{A}$, is defined as following:

$$u \leqslant v \Leftrightarrow u \ominus v = 0$$

Herein $u \ominus u = 0$ and 0 is an element that doesn't depend on the choice of $u \in \mathcal{A}$. Also, $u \wedge v = u \ominus (u \ominus v)$ and the complement of an element $v \in [0, u]$ is $u \ominus v$.

Since for all $u, v \in \mathfrak{X}$,

$$\begin{aligned}(u \wedge v) \ominus u &= (u \ominus (u \ominus v)) \ominus u \\ &= (u \ominus u) \ominus (u \ominus v) \\ &= 0 \ominus (u \ominus v) = 0\end{aligned}$$

and

$$\begin{aligned}(u \wedge v) \ominus v &= (u \ominus (u \ominus v)) \ominus v \\ &= (u \ominus v) \ominus (u \ominus v) = 0\end{aligned}$$

we get $u \wedge v \leqslant u$ and $u \wedge v \leqslant v$.Let $z \leqslant u$ and $z \leqslant v$, $z \in \mathfrak{X}$. Since $z \leqslant v$,

$$u \ominus v \leqslant u \ominus z$$

and

$$u \ominus (u \ominus z) \leqslant u \ominus (u \ominus v).$$

From here we get

$$\begin{aligned}z \ominus (u \ominus (u \ominus v)) &\leqslant z \ominus (u \ominus (u \ominus z)) \\ = z \ominus (z \ominus (z \ominus u)) &= z \ominus (z \ominus 0) = z \ominus z = 0\end{aligned}$$

Thus, we obtain that $u \wedge v$ is lower bound for u and v.

Let $(\mathcal{A}, \mathcal{P}, \sim_{B_r}, N_r(B))$ be a nearness approximation space $\emptyset \neq \mathfrak{X} \subseteq \mathcal{A}$, and "$\ominus$" be a binary operation on $\mathcal{A}$ and 0 be a constant element of $\mathcal{A}$. In this case 0 must be in $Nr(B)^{\ominus}\mathfrak{X}$ and "$\ominus$" must be a binary operation on $Nr(B)^{\ominus}\mathfrak{X}$ for which $\mathfrak{X}$ be a nearness subtraction algebra.

LEMMA 3.1. *In nearness subtraction algebra $\mathfrak{X}$, the following statements are provided:*

(1) $(u \ominus v) \ominus v = u \ominus v$

(2) $u \ominus 0 = u$ *and* $0 \ominus u = 0$

(3) $(u \ominus v) \ominus u = 0$

(4) $u \ominus (u \ominus v) \leqslant v$

(5) $(u \ominus v) \ominus (v \ominus u) = u \ominus v$

(6) $u \ominus (u \ominus (u \ominus v)) = u \ominus v$

(7) $(u \ominus v) \ominus (w \ominus v) \leqslant u \ominus w$

(8) $u \leqslant v \Leftrightarrow$*For some* $w \in Nr(B)^{\ominus}\mathfrak{X}$, $u = v \ominus w$

(9) $u \leqslant v$ *implies for all* $w \in \mathfrak{X}$, $u \ominus w \leqslant v \ominus w$ *and* $w \ominus v \leqslant w \ominus u$

(10) $u, v \leqslant w$ *implies* $u \ominus v = u \wedge (w \ominus v)$.

Proof. (1) From $(\mathcal{NS}_1)$, we have $u \ominus (v \ominus u) = u$. Replacing $u \ominus v$ by u, we get

$$(u \ominus v) \ominus (v \ominus (u \ominus v)) = u \ominus v$$

Again by $(\mathcal{NS}_1)$ we obtain $(u \ominus v) \ominus v = u \ominus v$.

(2) From $(\mathcal{NS}_1)$, we have $u \ominus (v \ominus u) = u$. Replacing u by v, we get

$$u \ominus (u \ominus u) = u$$

That is, $u \ominus 0 = u$. In $(\mathcal{NS}_1)$, if we write 0 for u, then we get $0 \ominus (v \ominus 0) = 0$. Since $v \ominus 0 = v$, we obtain $0 \ominus v = 0$ for all $v \in \mathfrak{X}$.

(3) For all $v \in \mathfrak{X}$, we have $0 \ominus v = 0$ from (2) and for all $u \in \mathfrak{X}$, we have $(u \ominus u) = 0$. Therefore $(u \ominus u) \ominus v = 0$ for all u,$v \in \mathfrak{X}$. From $(\mathcal{NS}_3)$, we get $(u \ominus v) \ominus u = 0$.

(4) From $(\mathcal{NS}_3)$, we have $(u \ominus (u \ominus v)) \ominus v = (u \ominus v) \ominus (u \ominus v) = 0$. Thus we get $u \ominus (u \ominus v) \leqslant v$.

(5) From $(\mathcal{NS}_3)$, we can write $(u \ominus v) \ominus (v \ominus u) = (u \ominus (v \ominus u)) \ominus v = u \ominus v$. Thus we get $u \ominus (u \ominus v) \leqslant v$ by $(\mathcal{NS}_1)$.

(6) From $(\mathcal{NS}_2)$, we have $(u \ominus (u \ominus v)) = (v \ominus (v \ominus u))$ for all u,$v \in \mathfrak{X}$. Replacing $u \ominus v$ by v, we have

$$\begin{aligned} u \ominus (u \ominus (u \ominus v)) &= (u \ominus v) \ominus ((u \ominus v) \ominus u) \\ &= (u \ominus v) \ominus ((u \ominus u) \ominus v) \\ &= (u \ominus v) \ominus (0 \ominus v) \\ &= (u \ominus v) \ominus 0 = (u \ominus v). \end{aligned}$$

(7) From $(\mathcal{NS}_3)$ we get

$$\begin{aligned} &((u \ominus v) \ominus (w \ominus v)) \ominus (u \ominus w) \\ &= ((u \ominus v) \ominus (u \ominus w)) \ominus (w \ominus v) \\ &= ((u \ominus (u \ominus w)) \ominus v) \ominus (w \ominus v) \\ &= ((w \ominus (w \ominus u)) \ominus v) \ominus (w \ominus v) \\ &= ((w \ominus v) \ominus (w \ominus u)) \ominus (w \ominus v) \\ &= ((w \ominus v) \ominus (w \ominus v)) \ominus (w \ominus u) = 0 \ominus (w \ominus u) = 0. \end{aligned}$$

Therefore $(u \ominus v) \ominus (w \ominus v) \leqslant u \ominus w$.

(8) If $u \leqslant v$, then $u \ominus v = 0$. If we take $w = v \ominus u$, then $u = u \ominus 0 = u \ominus (u \ominus v) = v \ominus (v \ominus u) = v \ominus w$.

Let $u = v \ominus w$ for some $w \in Nr(B)^{\ominus}\mathfrak{X}$. Since $u \ominus v = (v \ominus w) \ominus v = (v \ominus v) \ominus w = 0 \ominus w = 0$, we obtain $u \leqslant v$.

(9) Let $u \leqslant v$. That is, $u \ominus v = 0$. From (7),

$$\begin{aligned} ((u \ominus w) \ominus (v \ominus w)) \ominus (u \ominus v) &= 0 \\ ((u \ominus w) \ominus (v \ominus w)) \ominus 0 &= 0 \\ (u \ominus w) \ominus (v \ominus w) &= 0 \end{aligned}$$

Thus, we get $w \ominus v \leqslant w \ominus u$.

(10) Let $u, v \leqslant w$. Since $u \leqslant w$, we get $u \ominus v \leqslant u \ominus w$. On the other hand, $u \ominus v \leqslant u$. Since $u \wedge (w \ominus v)$ is the lower bound for a and $w \ominus v$, we get $u \ominus v \leqslant u \wedge (w \ominus v)$.

$$
\begin{aligned}
&(u \wedge (w \ominus v)) \ominus (u \ominus v) \\
&(u \wedge (w \ominus v) \ominus 0) \ominus (u \ominus v) \\
&= [(u \wedge (w \ominus v)) \ominus \{(u \wedge (w \ominus v)) \ominus (u \wedge (w \ominus v)) \ominus v\}] \ominus (u \ominus v) \\
&= [(u \wedge (w \ominus v)) \ominus v] \ominus (u \ominus v) \\
&= [(u \ominus (u \ominus (u \wedge (w \ominus v))) \ominus v)] \ominus (u \ominus v) \\
&= ((u \ominus v) \ominus (u \ominus (u \wedge (w \ominus v))) \ominus (u \ominus v) \\
&= ((u \ominus v) \ominus (u \ominus v)) \ominus (u \ominus (u \wedge (w \ominus v)) \\
&= 0 \ominus (u \ominus (u \wedge (w \ominus v))) = 0.
\end{aligned}
$$

And so, $u \wedge (w \ominus v \leqslant (u \ominus v)$. Hence $u \ominus v = u \wedge (w \ominus v)$. $\lceil (u \wedge (w \ominus v)) \ominus [(u \wedge (w \ominus v)) \ominus v] = (u \wedge (w \ominus v)) \wedge v = 0. \rfloor$ □

THEOREM 3.1. *Let $\mathfrak{X}$ be a nearness subtraction algebra We have $u \ominus (v \wedge u) = u \ominus v$ for all $u, v \in \mathfrak{X}$.*

PROOF. For $u, v \in \mathfrak{X}$, $u \ominus (v \wedge u) = u \ominus (v \ominus (v \ominus u))$. From $(\mathcal{NS}_2)$, we get $u \ominus (v \wedge u)) = u \ominus (u \ominus (u \ominus v)) = (u \ominus v)$ by (6). □

THEOREM 3.2. *Let $\mathfrak{X}$ be a nearness subtraction algebra For $u, v, w \in \mathfrak{X}$, $u \leqslant v$ and $v \leqslant w$ imply $u \leqslant w$.*

PROOF. Let $u \leqslant v$ and $v \leqslant w$.We get $u \ominus v = 0$ and $v \ominus w = 0$. From $(\mathcal{NS}_2)$, $(\mathcal{NS}_3)$

$$
\begin{aligned}
u \ominus w = (u \ominus w) \ominus 0 &= (u \ominus w) \ominus (u \ominus v) \\
&= (u \ominus (u \ominus v)) \ominus w \\
&= (v \ominus (v \ominus u)) \ominus w = (v \ominus w) \ominus (v \ominus u) \\
&= 0 \ominus (v \ominus u) = 0.
\end{aligned}
$$

Thus, $u \leqslant w$. □

THEOREM 3.3. *Let $\mathfrak{X}$ be a nearness subtraction algebra For $u, v, w \in \mathfrak{X}$, we have that $u \ominus v \leqslant w$ implies $u \ominus w \leqslant v$.*

PROOF. Let $u \ominus w \leqslant v$ for all $u, v, w \in \mathfrak{X}$. This means that $(u \ominus v) \ominus w = 0$. From $(\mathcal{NS}_3)$, we get $(u \ominus w) \ominus v = 0$. That is, we obtain that $u \ominus w \leqslant v$. □

DEFINITION 3.2. *Let $\mathfrak{X}$ be a nearness subtraction algebra, and $\emptyset \neq \mathcal{S}$ be a subset of $\mathfrak{X}$. If $u \ominus v \in N_r(B)^{\ominus} \mathcal{S}$ for all $u, v \in \mathcal{S}$, then, $\mathcal{S}$ is called a nearness subtraction subalgebra of $\mathfrak{X}$.*

Besides, if $\emptyset \neq \mathcal{U} \subset \mathfrak{X}$ and $u \ominus v \in N_r(B)^{\ominus} \mathcal{U}$ for all $u, b \in N_r(B)^{\ominus} \mathcal{U}$, then $\mathcal{U}$ is called an upper near subtraction subalgebra of $\mathfrak{X}$.

EXAMPLE 3.3. *Let $\mathcal{A} = \{0, u, v, w, z\}$ be a set of perceived objects, $r = 1$, $B = \{\rho_1, \rho_2\} \subseteq \mathcal{P}$ be a set of probe functions and $\mathfrak{X} = \{u, v, w\} \subset \mathcal{A}$. Values of the probe functions are given in Table 8*

$$\rho_1 : \mathcal{A} \to V_1 = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}$$
$$\rho_2 : \mathcal{A} \to V_2 = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}$$

	0	u	v	w	z
ρ_1	ϑ_1	ϑ_1	ϑ_2	ϑ_3	ϑ_4
ρ_2	ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_1

Table 8

We can write

$$[0]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(u) = \vartheta_1\} = \{0, u\} = [u]_{\rho_1},$$
$$[v]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(v) = \vartheta_2\} = \{v\}$$
$$[w]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(w) = \vartheta_3\} = \{w\}$$
$$[z]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(z) = \vartheta_4\} = \{z\}$$

and

$$[0]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(0) = \vartheta_1\} = \{0, z\} = [z]_{\rho_2}$$
$$[u]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(u) = \vartheta_2\} = \{u\}$$
$$[v]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(v) = \vartheta_3\} = \{v\}$$
$$[w]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(w) = \vartheta_4\} = \{w\}$$

Hence $\xi_{\rho_1} = \left\{[0]_{\rho_1}, [v]_{\rho_1}, [w]_{\rho_1}, [z]_{\rho_1}\right\}$ *and* $\xi_{\rho_2} = \left\{[0]_{\rho_2}, [u]_{\rho_2}, [v]_{\rho_2}, [w]_{\rho_2}\right\}$

$Nr(B)^{\ominus}\mathfrak{X} = \bigcup\limits_{[x]_{\rho_i} \cap \mathfrak{X} \neq \emptyset} [x]_{\rho_i} = [0]_{\rho_1} \cup [v]_{\rho_1} \cup [w]_{\rho_1} \cup [u]_{\rho_2} \cup [v]_{\rho_2} \cup [w]_{\rho_2} = \{0, u, v, w\}$

"$\ominus$" operation is given by following Table 9 on $\mathfrak{X}$

$\ominus$	0	u	v	w	z
0	0	0	0	0	0
u	u	0	u	u	0
v	v	v	0	v	0
w	w	w	w	0	w
z	z	v	u	z	0

Table 9

Since $(\mathcal{NS}_1)$, $(\mathcal{NS}_2)$ and $(\mathcal{NS}_3)$ are provided in $\mathfrak{X}$.

$\mathfrak{X}$ is a nearness subtraction algebra. Let us take $S = \{u, v\} \subseteq \mathfrak{X}$. In this case, we get $Nr(B)^{\ominus}S = \bigcup\limits_{[x]_{\rho_i} \cap S \neq \emptyset} [x]_{\rho_i} = [0]_{\rho_1} \cup [v]_{\rho_1} \cup [u]_{\rho_2} \cup [v]_{\rho_2} = \{0, u, v\}$ *We can easly check that for all $u, v \in S$, $u \ominus v \in Nr(B)^{\ominus}S$. Thus S is nearness subtraction subalgebra of $\mathfrak{X}$ by Definition 3.2.*

EXAMPLE 3.4. *Let* $\mathcal{A} = \{0, u, v, w, z\}$ *be a set of perceived objects,* $r = 1$, $B = \{\rho_1, \rho_2\} \subseteq \mathcal{P}$ *be a set of probe functions and* $\mathfrak{X} = \{u, v, z\} \subset \mathcal{A}$*. Values of the probe functions are given in* $Table 10$

$$\rho_1 : \mathcal{A} \to V_1 = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}$$
$$\rho_2 : \mathcal{A} \to V_2 = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\}$$

	0	u	v	w	z
ρ_1	ϑ_1	ϑ_2	ϑ_1	ϑ_3	ϑ_4
ρ_2	ϑ_1	ϑ_1	ϑ_2	ϑ_3	ϑ_4

$Table 10$

We can write

$$[0]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(v) = \vartheta_1\} = \{0, v\} = [v]_{\rho_1},$$
$$[u]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(u) = \vartheta_2\} = \{u\}$$
$$[w]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(w) = \vartheta_3\} = \{w\}$$
$$[z]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(z) = \vartheta_4\} = \{z\}$$

and

$$[0]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(u) = \vartheta_1\} = \{0, u\}$$
$$[v]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(v) = \vartheta_2\} = \{v\}$$
$$[w]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(w) = \vartheta_3\} = \{w\}$$
$$[z]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(z) = \vartheta_4\} = \{z\}$$

Hence $\xi_{\rho_1} = \left\{[0]_{\rho_1}, [u]_{\rho_1}, [w]_{\rho_1}, [z]_{\rho_1}\right\}$ *and* $\xi_{\rho_2} = \left\{[0]_{\rho_2}, [v]_{\rho_2}, [w]_{\rho_2}, [z]_{\rho_2}\right\}$

$$Nr(B)^{\ominus}\mathfrak{X} = \bigcup_{[x]_{\rho_i} \cap \mathfrak{X} \neq \emptyset} [x]_{\rho_i} = [0]_{\rho_1} \cup [u]_{\rho_1} \cup [z]_{\rho_1} \cup [0]_{\rho_2} \cup [v]_{\rho_2} \cup [z]_{\rho_2} = \{0, u, v, z\}$$

$\emptyset \neq \mathcal{U} \subset \mathfrak{X}$, $\mathcal{U} = \{u, v\}$ *and according to* $Table$ 11 *on* $\mathfrak{X}$

$\ominus$	0	u	v	w	z
0	0	0	0	0	0
u	u	0	u	u	0
v	v	v	0	v	0
w	w	w	w	0	w
z	z	v	u	z	0

$Table$ 11

$$Nr(B)^{\ominus}\mathcal{U} = \bigcup_{[x]_{\rho_i} \cap \mathfrak{X} \neq \emptyset} [x]_{\rho_i} = [0]_{\rho_1} \cup [u]_{\rho_1} \cup [0]_{\rho_2} \cup [b]_{\rho_2} = \{0, u, v\}$$

Thus $\mathcal{U}$ *is upper near subtraction subalgebra of* $\mathfrak{X}$ *by Definition 3.2.*

Definition 3.3. *Let $\mathfrak{X}$ be a nearness subtraction algebra on $\mathcal{A}$, $B_r \subseteq \mathcal{P}$ where $r \leqslant\mid B \mid$ and $B \subseteq \mathcal{P}$, $\sim_{B_r}$ be indiscernibility relation on $\mathcal{A}$. If for $x, y \in \mathfrak{X}$, $x \sim_{B_r} y$, implies $(x \ominus u) \sim_{B_r} (y \ominus u)$, and $(v \ominus x) \sim_{B_r} (v \ominus y)$ for all $u, v \in \mathfrak{X}$, then $\sim_{B_r}$ is called a congruence indiscernibility relation on $\mathfrak{X}$.*

Let $\mathfrak{X}$ be a nearness subtraction algebra. Let $X \ominus Y = \{x \ominus y \mid x \in X \text{ and } y \in Y\}$ where X and Y are subset of $\mathfrak{X}$.

Proposition 3.1. *Let P and Q be non-empty subsets of $\mathfrak{X}$. In this case we have*

(i) $(Nr(B)^{\ominus}P) \ominus (Nr(B)^{\ominus}Q) \subseteq Nr(B)^{\ominus}(P \ominus Q)$

(ii) If $\sim_{B_r}$ is complete indiscernibility relation, then

$$(Nr(B)_{\ominus}P) \ominus (Nr(B)_{\ominus}Q) \subseteq Nr(B)_{\ominus}(P \ominus Q).$$

Proof. (i) Let $w \in (Nr(B)^{\ominus}P) \ominus (Nr(B)^{\ominus}Q)$. Then, we can write

$$w = u \ominus v \ , u \in (Nr(B)^{\ominus}P)$$

and

$$v \in Nr(B)^{\ominus}Q.$$

Therefore

$$p \in [u]_{B_r} \cap P \text{ and } q \in [v]_{B_r} \cap Q.$$

Since $p \in [u]_{B_r}$ and $q \in [v]_{B_r}$. We have

$$p \ominus q \in [u]_{B_r} \ominus [v]_{B_r} \subseteq [u \ominus v]_{B_r} .$$

Thus we get

$$[u \ominus v]_{B_r} \cap (P \ominus Q) \neq \emptyset.$$

That is,

$$w = u \ominus v \in Nr(B)^{\ominus}(P \ominus Q).$$

(ii) Let $w \in (Nr(B)_{\ominus}P) \ominus (Nr(B)_{\ominus}Q)$. Then, there exist

$$u \in Nr(B)_{\ominus}P \text{ and } v \in Nr(B)_{\ominus}Q$$

such that $w = u \ominus v$. Since

$$u \in Nr(B)_{\ominus}P \text{ and } v \in Nr(B)_{\ominus}Q,$$

we get $[u]_{B_r} \subseteq P$ and $[v]_{B_r} \subseteq Q$. From hypothesis, we get

$$[u \ominus v]_{Br} = [u]_{B_r} \ominus [v]_{B_r} \subseteq P \ominus Q.$$

Thus we get $w \in Nr(B)_{\ominus}(P \ominus Q)$. □

Theorem 3.4. *Let $\mathfrak{X}$ be a nearness subtraction algebra, $\sim_{B_r}$ be a congruence indiscernibility relation on $\mathfrak{X}$ and $\mathcal{S} \subseteq \mathfrak{X}$. İf $u \ominus v \in \mathcal{S}$ for all $u, v \in \mathcal{S}$, the $\mathcal{S}$ is an upper near subtraction subalgebra of $\mathfrak{X}$.*

Proof. Let us take $u, v \in Nr(B)^{\ominus}\mathcal{S}$. Therefore we get $x \in [u]_{B_r} \cap \mathcal{S}$ and $y \in [v]_{B_r} \cap \mathcal{S}$. Since $x \in [u]_{B_r}$ and $y \in [v]_{B_r}$, we have $x \sim_{B_r} u$ and $y \sim_{B_r} v$. From hypothesis $(x \ominus y) \sim_{B_r} (u \ominus v)$. This means that $x \ominus y \in [u \ominus v]_{B_r}$. Also, since $u \ominus v \in \mathcal{S}$ we have $[u \ominus v]_{B_r} \cap \mathcal{S} \neq \emptyset$. That is $u \ominus v \in Nr(B)^{\ominus}\mathcal{S}$. □

4. Nearness subtraction-ideals in nearness subtraction-algebras

DEFINITION 4.1. *Let* $(\mathfrak{X}, \ominus, 0)$ *be a nearness subtraction algebra, and* $\emptyset \neq \mathcal{I} \subseteq \mathfrak{X}$. *Then,*

i) A set $\mathcal{I}$ *is called a nearness subtraction ideal of* $\mathfrak{X}$ *if it satisfies:*
$\mathcal{NI}_1$*)* $0 \in Nr(B)^{\ominus}\mathcal{I}$
$\mathcal{NI}_2$*)* $u \ominus v \in Nr(B)^{\ominus}\mathcal{I}$ *and* $v \in \mathcal{I}$ *imply* $u \in \mathcal{I}$ *for all* $u, v \in \mathfrak{X}$,
ii) A set $\mathcal{I}$ *is called an upper near subtraction ideal of* $\mathfrak{X}$ *if it satisfies:*
$\mathcal{UNI}_1)0 \in Nr(B)^{\ominus}\mathcal{I}$
$\mathcal{UNI}_2)$ $u \ominus v \in Nr(B)^{\ominus}\mathcal{I}$ *and* $v \in \mathcal{I}$ *means* $u \in \mathcal{I}$ *for all* $u, v \in \mathfrak{X}$,
$\mathcal{UNI}_3)u \in Nr(B)^{\ominus}\mathcal{I}$ *and* $v \in Nr(B)^{\ominus}\mathfrak{X}$ *means* $u \ominus v \in Nr(B)^{\ominus}\mathcal{I}$

EXAMPLE 4.1. *Let* $\mathcal{A} = \{0, u, v, w, z\}$ *be a set of perceived objects,* $r = 1$, $B = \{\rho_1, \rho_2\} \subseteq \mathcal{P}$ *be a set of probe functions, and* $\mathfrak{X} = \{u, v, w\} \subset \mathcal{A}$. *Values of the probe functions are given in Table* 12.

$$\rho_1 : \mathcal{A} \to V_1 = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\},$$
$$\rho_2 : \mathcal{A} \to V_2 = \{\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4\},$$

	0	u	v	w	z
ρ_1	ϑ_1	ϑ_2	ϑ_1	ϑ_3	ϑ_4
ρ_2	ϑ_1	ϑ_1	ϑ_2	ϑ_3	ϑ_4

Table 12

We can write

$$[0]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(v) = \vartheta_1\} = \{0, v\} = [v]_{\rho_1},$$
$$[u]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(u) = \vartheta_2\} = \{u\} = [u]_{\rho_1},$$
$$[w]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(w) = \vartheta_3\} = \{w\} = [w]_{\rho_1}$$
$$[z]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(z) = \vartheta_4\} = \{z\} = [z]_{\rho_1}$$

Then $\xi_{\rho_1} = \left\{[0]_{\rho_1}, [u]_{\rho_1}, [w]_{\rho_1}, [z]_{\rho_1}\right\}$ *and*

$$[0]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(0) = \rho_2(u) = \vartheta_1\} = \{0, u\} = [u]_{\rho_2},$$
$$[v]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(v) = \vartheta_2\} = \{v\} = [v]_{\rho_2},$$
$$[w]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(w) = \vartheta_3\} = \{w\} = [w]_{\rho_2}$$
$$[z]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(z) = \vartheta_4\} = \{z\} = [z]_{\rho_2}$$
$$N_1(B)^{-}\mathfrak{X} = \bigcup_{[x]_{\rho_i} \cap \mathfrak{X} \neq \varnothing} [x]_{\rho_i} = \{0, u, v, w\}.$$
$$N_1(B)^{-}\mathcal{I} = \bigcup_{[x]_{\rho_i} \cap \mathcal{I} \neq \varnothing} [x]_{\rho_i} = \{0, u, v\}.$$

*Considering the Table*11 *of operation, then* $\mathcal{I} := \{u, v\} \subseteq \mathfrak{X}$ *is a nearness subtraction ideal of* $\mathfrak{X}$ *by Definition 4.1 and* $\mathcal{I}=\{u, v\} \subseteq \mathfrak{X}$ *is a upper near subtraction ideal of* $\mathfrak{X}$ *by Definition 4.1.*

EXAMPLE 4.2. *Let* $\mathcal{A} = \{0, u, v, w, z\}$ *be a set of perceived objects,* $r = 1$, $B = \{\rho_1, \rho_2\} \subseteq \mathcal{P}$ *be a set of probe functions, and* $\mathfrak{X} = \{u, v, w\} \subset \mathcal{A}$. *Values of the probe functions are given in* $Table$ 13.

$$\rho_1 : \mathcal{A} \to V_1 = \{\vartheta_1, \vartheta_2, \vartheta_3\},$$
$$\rho_2 : \mathcal{A} \to V_2 = \{\vartheta_1, \vartheta_2\},$$

	0	u	v	w	z
ρ_1	ϑ_1	ϑ_1	ϑ_2	ϑ_3	ϑ_3
ρ_2	ϑ_1	ϑ_2	ϑ_1	ϑ_2	ϑ_1

Table 13

We can write

$$[0]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(u) = \vartheta_1\} = \{0, u\} = [u]_{\rho_1},$$
$$[v]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(v) = \vartheta_2\} = \{v\} = [v]_{\rho_1},$$
$$[w]_{\rho_1} = \{x' \in \mathcal{A} \mid \rho_1(x') = \rho_1(w) = \rho_1(z) = \vartheta_3\} = \{w, z\} = [z]_{\rho_1}$$

Then $\xi_{\rho_1} = \left\{[0]_{\rho_1}, [v]_{\rho_1}, [w]_{\rho_1}\right\}$ *and*

$$[0]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(0) = \rho_2(v) = \rho_2(z) = \vartheta_1\} = \{0, v, z\} = [v]_{\rho_2} = [z]_{\rho_2},$$
$$[u]_{\rho_2} = \{x' \in \mathcal{A} \mid \rho_2(x') = \rho_2(u) = \rho_2(w) = \vartheta_2\} = \{u, w\} = [w]_{\rho_2}$$

$\xi_{\rho_2} = \left\{[0]_{\rho_2}, [u]_{\rho_2}\right\}$

$$N_1(B)^- \mathfrak{X} = \bigcup_{[x]_{\rho_i} \cap \mathfrak{X} \neq \varnothing} [x]_{\rho_i} = \{0, u, v, w, z\}.$$

and $\mathcal{I} := \{u, w\} \subseteq \mathfrak{X}$

$$N_1(B)^- \mathcal{I} = \bigcup_{[x]_{\rho_i} \cap \mathcal{I} \neq \varnothing} [x]_{\rho_i} = \{0, u, w, z\}.$$

Considering $Table$11 *of operation,* $\mathcal{I} := \{u, w\} \subseteq \mathfrak{X}$ *is a nearness subtraction ideal of* $\mathfrak{X}$ *by Definition 4.1 but* $\mathcal{I}$=$\{u, w\} \subseteq \mathfrak{X}$ *is not a upper near subtraction ideal of* $\mathfrak{X}$ *by Definition 4.1.*

COROLLARY 4.1. *Let* $(\mathfrak{X}, \ominus, 0)$ *be a nearness subtraction algebra and* $\mathcal{I} \subseteq \mathfrak{X}$ *and* $\mathcal{I} \neq \emptyset$. $\mathcal{I}$ *is an upper-near subtraction ideal of* $\mathfrak{X}$ *then* $\mathcal{I}$ *is a nearness subtraction ideal of* $\mathfrak{X}$.

DEFINITION 4.2. *Let* $\mathfrak{X} \subseteq \mathcal{A}$ *and* $(\mathcal{A}, \mathcal{P}, \mathfrak{X}, \sim B_r, N_r(B))$ *be a nearness approximation spaces,* $B_r \subseteq \mathcal{P}$ *where* $r \leqslant \mid B \mid$ *and* $B \subseteq \mathcal{P}$, $\sim_{B_r}$ *be an indiscernibility relation on* $\mathcal{A}$. *If* $[u]_{B_r} \ominus [v]_{B_r} = [u \ominus v]_{B_r}$ *for all* $u, v \in \mathfrak{X}$, *then,* $\sim_{B_r}$ *is called a complete congruence indiscernibility relation on nearness subtraction* $\mathfrak{X}$.

THEOREM 4.1. *Let* $\mathfrak{X}$ *be a nearness subtraction algebra, and* $\mathcal{I} \subseteq \mathfrak{X}$ *and* $\mathcal{I} \neq \emptyset$. *Then* $\mathcal{I}$ *is an upper near subtraction ideal of* $\mathfrak{X}$, *if* $\sim_{B_r}$ *be a complete congruence indiscernibility relation on* $\mathfrak{X}$, *and* $\mathcal{I}$ *is a nearness subtraction ideal of* $\mathfrak{X}$.

PROOF. Let u,$v \in \mathfrak{X}$ so that $v, u \ominus v \in N_r(B)^{\ominus}\mathcal{I}$. Then such that their intersection is different from the empty set $[u \ominus v]_{B_r} \cap \mathcal{I} \neq \emptyset$ and since $\sim_{B_r}$is a complete congruence indiscernibility relation , we have such that their intersection is different from the empty set $[u]_{B_r} \ominus [v]_{B_r} \cap \mathcal{I} \neq \emptyset$ and $[v]_{B_r} \cap \mathcal{I} \neq \emptyset$. In that case, there is $w, z \in \mathcal{I}$ so that $w \in [u]_{B_r} \ominus [v]_{B_r} \cap \mathcal{I} \neq \emptyset$ and $z \in [v]_{v_r} \cap \mathcal{I} \neq \emptyset$. In this way, we get $w = x \ominus y$; $x \in [u]_{B_r}$, $y \in [v]_{B_r}$, and thus we take $u \sim_{B_r} x$ and $v \sim_{B_r} y$ and also $v \sim_{B_r} z$. From here, we have $y \sim_{B_r} z$ and so $[y]_{B_r} = [z]_{B_r}$. Since $z \in \mathcal{I}$, we have $y \in \mathcal{I}$. Joining $w = x \ominus y \in \mathcal{I}$ and using $y \in \mathcal{I}$, we have $x \in \mathcal{I}$ by hypotesis. Therefore, $[u]_{B_r} \cap \mathcal{I} \neq \emptyset$ means $u \in N_r(B)^{\ominus}\mathcal{I}$. Similarly, $\mathcal{UNI}_3$ is obtained. On the other we have $u \ominus v = 0 \in N_r(B)^{\ominus}\mathcal{I}$ by hypotesis. □

References

[1] J. C. Abbott, Sets, Lattices and Boolean Algebras, Allyn and Bacon, Boston, (1969).
[2] P. Dheena and G. Satheesh Kumar, On strongly regular near-subtraction semigroups, *Communications of the Korean MathematicalSociety*, **22**(3) (2007), 323–330.
[3] E. İnan and M. A. Öztürk, Near groups on nearness approximation spaces, *Hacet. J. Math. Stat.*, **41**(4) (2012), 545–558.
[4] E. İnan and M. A. Öztürk, Near semigroups on nearness approximation spaces, *Ann. Fuzzy Math. Inform.*, **10**(2) (2015), 287–297.
[5] Y. B. Jun, H. S. Kim, and E. H. Roh, Ideal theory of subtraction algebras *Sci. Math. Jpn: Online e-2004* (2004), 397–402.
[6] Y. B. Jun, Y. H. Kim, and K. A. Oh, Subtarction algebras with additional conditions, *Commun. Korean Math. Soc.*, **22**(1) (2007), 1–7.
[7] Y. B. Jun and K. H. Kim, Prime and irreducible ideals in subtraction algebras, *International Mathematical Forum*, **3**(10) (2008), 457–462.
[8] Y. H. Kim and H. S. Kim, Subtraction algebras and BCK-algebras, *Math. Bohemica*, **128** (2003), 21–24.
[9] M. A. Öztürk, M. Uçkun, and E. İnan, Near groups of weak cosets on nearness approximation spaces, *Fund. Inform.* **133** (2014), 433-448.
[10] M. A. Öztürk, İ. Çelik Siner, and Y. B. Jun, Nearness *BCK*-algebras, *Int. J. Open Probl. Comput. Sci. Math.*, **8**(4) (2015), 37–57.
[11] M. A. Öztürk, Semirings on weak nearness approximation spaces, *Ann. Fuzzy Math. Inform.* **15**(3) (2018), 227–241.
[12] M. A. Öztürk, Y. B. Jun, and A. İz, Gamma semigroups on weak nearness approximation spaces, *J. Int. Math. Virtual Inst.* **9** (2019), 53–72.
[13] M. A. Öztürk, Prime ideals of gamma semigroups on weak nearness approximation spaces, *Asian-Eur. J. Math.*, **12**(5) (2019), 1950080 (10 pages).
[14] M. A. Öztürk, Nearness *d*-Algebras, *Journal of Algebraic Hyperstructures and Logical Algebras* **2**(3) (2021), 73–84.
[15] M. A. Öztürk, Y. B. Jun, and G. Muhiuddin, Quotient nearness *BCK*-algebras, *Discrete Mathematics, Algorithms and Applications*, p.2250124, https://doi.org/10.1142/S1793830922501245.
[16] J. F. Peters, Near sets: General theory about nearness of objects, *Appl. Math. Sci.* **1**(53-56) (2007), 2609–2629.
[17] J. F. Peters, Near sets: Special theory about nearness of objects, *Fund. Inform.* **75**(1-4) (2007), 407–433.
[18] J. F. Peters, Classification of perceptual objects by means of features, *Int. J. Info. Technol. Intell. Comput.*, **3**(2) (2008), 1–35.
[19] J. F. Peters and P. Wasilewski, Foundations of near sets, *Inform. Sci.*, **179**(18) (2009), 3091–3109.

[20] J. F. Peters, Near sets: An introduction, *Math. Comput. Sci.* **7**(1) (2013), 3–9.
[21] B. M. Schein, Difference Semigroups, *Comm. Algebra*, **20** (1992), 2153–2169.

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