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ON GENERALIZED (α, β)-DERIVATIONS IN d-ALGEBRAS

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Abstract. In this paper, we introduce the notions of generalized (α, β)-derivations on d-algebras. Also, the concepts of an α-ideal and G-invariant ideal are defined and relations among them are discussed. Moreover, some results regarding partial ordered edge d-algebras and generelized (α, β)-derivations are proved.

1. Introduction

Algebraic logic emerged as a subdiscipline of algebra in the nineteenth century. In the following years, studies on the relationship between logic and ordered systems such as Boolean algebras, lattice ordered groups, MV-algebras, etc. have been the subject of many researchers. For example, BCK-algebras and BCI-algebras are two classes of logic algebras introduced in 1966 ([**4**], [**5**]) and extensively investigated by many authors. It is known that BCK-algebras form a proper subclass of BCI-algebras. That is, every BCK-algebra is a BCI-algebra but not vice versa ([**3**]). One of the generalizations of BCK-algebras, d-algebras, was presented by Neggers and Kim ([**10**]). After that, d-subalgebra, d-ideal and some related concepts defined and relations among them investigated by Neggers, Jun and Kim ([**11**]).

In the theory of rings, the notion of derivation was given by Posner who establish two very striking results on derivations in prime rings ([**12**]). Jun and Xin ([**6**]) introduced the notion of derivations of BCI-algebras, which was motivated from a lot of work done on the notion of derivations on d-algebras ([**7**], [**8**] and their

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references). In BCI-algebras, (α,β)-derivations studied and obtained some properties by several authors ([**2**], [**9**]). Recently, Al-Omary has introduced the notion of (α,β)-derivations on d-algebras and investigate their relations ([**1**]).

This paper presents the notions of generalized (α,β)-derivations of d-algebras. Also, we obtain some results regarding partial ordered edge d-algebras and generalized (α,β)-derivations. Furthermore, we introduce the concepts of an α-ideal, G-invariant ideal and we investigate their relations.

2. Preliminaries

In this section, we introduce the definition of a d-algebra and properties that important for the study of d-algebras.

DEFINITION 2.1. [**10**] *A non-empty set* $\mathcal{A}=(\mathcal{A};*,0)$ *of type* $(2,0)$ *is called a d-algebra if for all* $a,b\in\mathcal{A}$ *the following conditions hold:*
(1) $a*a=0$,
(2) $0*a=0$,
(3) $a*b=0$ *and* $b*a=0$ *implies* $a=b$.

PROPOSITION 2.1. [**8**] *A d-algebra* $\mathcal{A}$ *has the following properties: for all* $a,b,c\in\mathcal{A}$
(1) $a*(a*b)=b$
(2) $(a*(a*b))*b=0$
(3) $(a*b)*c=(a*c)*b$
(4) $0*(a*b)=(0*a)*(0*b)$
(5) $a\leqslant b$ *implies* $a*c\leqslant b*c$ *and* $c*b\leqslant c*a$
(6) $a*(a*(a*b))=a*b$
(7) $a*0=0$ *implies* $a=0$
(8) $a*b=a*c$ *implies* $b=c$.

DEFINITION 2.2. [**11**] *Suppose* $\mathcal{A}=(\mathcal{A};*,0)$ *be a d-algebra and* $\emptyset\neq I\subseteq\mathcal{A}$. *If* $a*b\in I$ *whenever* $a,b\in I$, *then* I *is called a d-subalgebra of* $\mathcal{A}$. *Also,* I *is called a d-ideal of* $\mathcal{A}$ *if it satisfies:*
(1) $a*b\in I$ *and* $b\in I$ *imply* $a\in I$
(2) $a\in I$ *and* $b\in\mathcal{A}$ *imply* $a*b\in I$, *i.e.,* $I*\mathcal{A}\subseteq I$.

LEMMA 2.1. [**11**] *If* I *is a d-ideal of a d-algebra* $\mathcal{A}$, *then* $0\in I$.

DEFINITION 2.3. [**10**] *Let* $\mathcal{A}$ *be a d-algebra and* $a\in\mathcal{A}$. *Define* $a*\mathcal{A}=\{a*b|\,b\in\mathcal{A}\}$. *If for any* $a\in\mathcal{A}$, $a*\mathcal{A}=\{a,0\}$, *then* $\mathcal{A}$ *is called an edge d-algebra.*

LEMMA 2.2. [**7**] *If* $\mathcal{A}$ *is an edge d-algebra, then* $a*0=a$ *for any* $a\in\mathcal{A}$.

Notation: Let $\mathcal{A}$ be a d-algebra and $a,b\in\mathcal{A}$. We write $a\wedge b=b*(b*a)$.

DEFINITION 2.4. [**1**] *Let* $\mathcal{A}$ *be a* $d-$*algebra,* $\delta:\mathcal{A}\to\mathcal{A}$ *be a mapping. If* $\delta(a*b)=\delta(a)*\delta(b)$, *then* δ *is called an endomorphism. Note that* $\delta(0)=0$.

REMARK 2.1. [**1**] If $\mathcal{A}$ is an edge d-algebra and α is an endomorphism on $\mathcal{A}$, then $\alpha(a)*0=\alpha(a)$ for all $a\in\mathcal{A}$.

DEFINITION 2.5. [**1**] *Let $\mathcal{A}$ be a d-algebra. A map $d_{(\alpha,\beta)}:\mathcal{A}\to\mathcal{A}$ is called a left-right (α,β)-derivation of $\mathcal{A}$, if it satisfies the identity $d_{(\alpha,\beta)}(ab)=d_{(\alpha,\beta)}(a)\alpha(b)\wedge\beta(a)d_{(\alpha,\beta)}(b)$ for all $a,b\in\mathcal{A}$. If $d_{(\alpha,\beta)}$ satisfies $d_{(\alpha,\beta)}(ab)=\alpha(a)d_{(\alpha,\beta)}(b)\wedge d_{(\alpha,\beta)}(a)\beta(b)$ for all $a,b\in\mathcal{A}$, then $d_{(\alpha,\beta)}$ is called a right-left (α,β)-derivation of $\mathcal{A}$. Also, if $d_{(\alpha,\beta)}$ is a left-right and right-left (α,β)-derivation, then $d_{(\alpha,\beta)}$ is an (α,β)-derivation of $\mathcal{A}$.*

THEOREM 2.1. [**1**] *Suppose $(\mathcal{A},*,0)$ be a d-algebra.*

(1) *If $d_{(\alpha,\beta)}$ is a left-right (α,β)-derivation, then $d_{(\alpha,\beta)}$ is regular.*

(2) *If A is an edge d-algebra and $d_{(\alpha,\beta)}$ is a right-left (α,β)-derivation, then $d_{(\alpha,\beta)}$ is regular.*

3. Generalized (α,β)-derivations on d-algebras

In this section, we introduce the notions of left generalized (α,β)-derivations and right generalized (α,β)-derivations of $\mathcal{A}$ and investigate their basic properties. In what follows, α and β are endomorphisms of a d-algebra $\mathcal{A}$.

DEFINITION 3.1. *Let $\mathcal{A}$ be a d-algebra. A mapping $G:\mathcal{A}\to\mathcal{A}$ is called a left generalized (α,β)-derivation if there exists a left-right (α,β)-derivation $d_{(\alpha,\beta)}$ of $\mathcal{A}$ such that*

$$G(a*b)=(G(a)*\alpha(b))\wedge(\beta(a)*d_{(\alpha,\beta)}(b))$$

for all $a,b\in\mathcal{A}$.

If there exists a right-left (α,β)-derivation $d_{(\alpha,\beta)}$ of A and G satisfies the identity

$$G(a*b)=(\alpha(a)*G(b))\wedge(d_{(\alpha,\beta)}(a)*\beta(b))$$

then G is called a right generalized (α,β)-derivation of A.

Clearly, the notion of left (resp. right) generalized (α,β)-derivation covers the concept of left-right (resp. right-left) (α,β)-derivation when $G=d_{(\alpha,\beta)}$. For simplicity of notations, we write ab instead of $a*b$.

EXAMPLE 3.1. *Consider a d-algebra $\mathcal{A}=\{0,1,2,3,4\}$ with the following Cayley table: ($\mathcal{A}$ is not a BCK-algebra, [**11**])*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	3	0
3	3	3	2	0	3
4	3	3	1	1	0

Define a map

$$d_{(\alpha,\beta)}:\mathcal{A}\to\mathcal{A}, a\mapsto\begin{cases}0, & a=0,1,2,3\\ 1, & a=4\end{cases}$$

and define two endomorphisms

$$\alpha : \mathcal{A} \to \mathcal{A},\ a \mapsto \begin{cases} 0, & a = 0, 1 \\ 2, & a = 2, 3, 4 \end{cases}$$

and

$$\beta : \mathcal{A} \to \mathcal{A},\ a \mapsto \begin{cases} 0, & a = 0, 1 \\ 3, & a = 2, 3, 4 \end{cases}.$$

Then, $d_{(\alpha,\beta)}$ is a left-right (α, β)-derivation. If we define a map

$$G : \mathcal{A} \to \mathcal{A},\ a \longmapsto \begin{cases} 0, & a = 0, 1, 3, 4 \\ 2, & a = 2 \end{cases}$$

then G is a left generalized (α, β)-derivation of A. But G is not a right generalized (α, β)-derivation of A. Because $G(20) = G(2) = 2$, on the other hand $\alpha(2)G(0) \wedge d(2)\beta(0) = 0$.

EXAMPLE 3.2. *Consider a d-algebra $\mathcal{A} = \{0, a, b, c\}$ with the following Cayley table:*

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	b
b	b	b	0	0
c	c	c	a	0

Define a map

$$d_{(\alpha,\beta)} : \mathcal{A} \to \mathcal{A},\ x \mapsto \begin{cases} 0, & x = 0, b \\ a, & x = a \\ c, & x = c \end{cases}$$

and define two endomorphisms

$$\alpha : \mathcal{A} \to \mathcal{A},\ x \mapsto \begin{cases} 0, & x = 0 \\ b, & x = a, c \\ a, & x = b \end{cases}$$

and

$$\beta : \mathcal{A} \to \mathcal{A},\ x \mapsto \begin{cases} 0, & x = 0 \\ a, & x = a \\ b, & x = b, c \end{cases}.$$

Then, $d_{(\alpha,\beta)}$ is a right-left (α, β)-derivation. If we define a map

$$G : \mathcal{A} \to \mathcal{A},\ x \longmapsto \begin{cases} 0, & x = 0, b \\ a, & x = a \\ c, & x = c \end{cases}$$

then G is a right generalized (α, β)-derivation of $\mathcal{A}$. Here, G is not a left generalized (α, β)-derivation of $\mathcal{A}$. $G(ca) = G(c) = c$, but $G(c)\alpha(a) \wedge \beta(c)d(a) = 0$.

DEFINITION 3.2. [**1**] *Let G be a self map on a d-algebra $\mathcal{A}$. If $G(0) = 0$, then G is called regular.*

EXAMPLE 3.3. *The left generalized* (α,β)*-derivation* G *and the right generalized* (α,β)*-derivation* G *in* Example 3.1 *and* Example 3.2 *are regular.*

LEMMA 3.1. *Assume that* $\mathcal{A}$ *is a d-algebra and* $G:\mathcal{A}\to\mathcal{A}$ *is a left generalized* (α,β)*-derivation. Hence,* G *is regular.*

PROOF. For any $a\in\mathcal{A}$, we have

$$\begin{aligned} G(0) &= G(0a)=G(0)\alpha(a)\wedge\beta(0)d_{(\alpha,\beta)}(a)\\ &= G(0)\alpha(a)\wedge 0d_{(\alpha,\beta)}(a)\\ &= G(0)\alpha(a)\wedge 0\\ &= 0(0G(0)\alpha(a))=0. \end{aligned}$$

□

LEMMA 3.2. *Assume that* $\mathcal{A}$ *is an edge d-algebra and* $G:\mathcal{A}\to\mathcal{A}$ *is a right generalized* (α,β)*-derivation. Thus,* G *is regular.*

PROOF. Indeed, using Remark 2.1 and $aa=0$, we get

$$\begin{aligned} G(0) &= G(0a)=\alpha(0)G(a)\wedge d_{(\alpha,\beta)}(0)\beta(a)\\ &= 0G(a)\wedge d_{(\alpha,\beta)}(0)\beta(a)=0\wedge d_{(\alpha,\beta)}(0)\beta(a)\\ &= d_{(\alpha,\beta)}(0)\beta(a)(d_{(\alpha,\beta)}(0)\beta(a)0)=0. \end{aligned}$$

□

THEOREM 3.1. *Assume that* $\mathcal{A}$ *is an edge d-algebra.*
(1) *If* G *is a left generalized* (α,β)*-derivation of* A*, then* $G(a)=G(a)\wedge\beta(a)$ *for all* $a\in A$*.*
(2) *If* G *is a right generalized* (α,β)*-derivation of* A*, then* $G(a)=\alpha(a)\wedge d_{(\alpha,\beta)}(a)$*, for all* $a\in A$*.*

PROOF. (1) Let G be a left generalized (α,β)-derivation of $\mathcal{A}$. Using Theorem 2.1(1), Lemma 2.2 and Remark 2.1, we have

$$\begin{aligned} G(a) &= G(a0)=G(a)\alpha(0)\wedge\beta(a)d_{(\alpha,\beta)}(0)\\ &= G(a)0\wedge\beta(a)0=G(a)\wedge\beta(a). \end{aligned}$$

(2) Let G be a right generalized (α,β)-derivation of $\mathcal{A}$. Using Theorem 2.1(2), Lemma 2.2, Lemma 3.2 and Remark 2.1, we have

$$\begin{aligned} G(a) &= G(a0)=\alpha(a)G(0)\wedge(d_{(\alpha,\beta)}(a)\beta(0))\\ &= \alpha(a)0\wedge d_{(\alpha,\beta)}(a)0=\alpha(a)\wedge d_{(\alpha,\beta)}(a). \end{aligned}$$

□

THEOREM 3.2. *Let* α,β *be two endomorphisms and* G *be a self map on a d-algebra* $\mathcal{A}$*. If* $G(a)=\alpha(a)$ *for all* $a\in\mathcal{A}$*, then* G *is a left (or right) generalized* (α,β)*-derivation on* $\mathcal{A}$*.*

PROOF. Assume G be a self map on a d-algebra $\mathcal{A}$ such that $G(a) = \alpha(a)$ for all $a \in \mathcal{A}$. Since $a(ab) = b$, we have

$$\begin{aligned} G(ab) &= \alpha(ab) = \alpha(a)\alpha(b) \\ &= G(a)\alpha(b) \\ &= (\beta(a)d_{(\alpha,\beta)}(b))((\beta(a)d_{(\alpha,\beta)}(b))(G(a)\alpha(b))) \\ &= G(a)\alpha(b) \wedge \beta(a)d_{(\alpha,\beta)}(b). \end{aligned}$$

Similarly, it can be shown that G is a right generalized (α, β)-derivation on $\mathcal{A}$. □

THEOREM 3.3. *Assume that α, β are two endomorphisms and G is a self map on a d-algebra $\mathcal{A}$. If $G(a) = \alpha(a)$, then $G(ab) = G(a)G(b)$ for all $a, b \in \mathcal{A}$.*

PROOF. Since $a(ab) = b$, using Theorem 3.2, we have

$$\begin{aligned} G(ab) &= G(a)\alpha(b) \wedge \beta(a)d_{(\alpha,\beta)}(b) \\ &= \alpha(a)\alpha(b) \wedge \beta(a)d_{(\alpha,\beta)}(b) \\ &= \beta(a)d_{(\alpha,\beta)}(b)(\beta(a)d_{(\alpha,\beta)}(b)\alpha(a)\alpha(b)) \\ &= \alpha(a)\alpha(b) = G(a)G(b). \end{aligned}$$

□

If $G = d_{(\alpha,\beta)}$ is taken in the two theorems above, the Theorem 3.4. in [**1**] is obtained as a result.

DEFINITION 3.3. [**1**] *Let $\mathcal{A}$ be a d-algebra and G_1, G_2 be two self maps. We define $G_1 \circ G_2 : \mathcal{A} \to \mathcal{A}$ by $(G_1 \circ G_2)(a) = G_1(G_2(a))$ for all $a \in \mathcal{A}$.*

THEOREM 3.4. [**1**] *Let $\mathcal{A}$ be a d-algebra and $d_{1_{(\alpha,\beta)}}$, $d_{2_{(\alpha,\beta)}}$ be two left-right (α, β)-derivations on $\mathcal{A}$ such that $\alpha^2 = \alpha$. Then, $d_{1_{(\alpha,\beta)}} \circ d_{2_{(\alpha,\beta)}}$ is a left-right (α, β)-derivation on $\mathcal{A}$.*

THEOREM 3.5. *Let $\mathcal{A}$ be a d-algebra and G_1, G_2 be two left generalized (α, β)-derivations associated with $d_{1_{(\alpha,\beta)}}$, $d_{2_{(\alpha,\beta)}}$ respectively on $\mathcal{A}$ such that $\alpha^2 = \alpha$. Then, $G_1 \circ G_2$ is a left generalized (α, β)-derivation associated with $d_{1_{(\alpha,\beta)}} \circ d_{2_{(\alpha,\beta)}}$ on $\mathcal{A}$.*

PROOF. Let $a, b \in \mathcal{A}$. Since $a(ab) = b$, we have

$$\begin{aligned} (G_1 \circ G_2)(ab) &= G_1(G_2(ab)) \\ &= G_1(G_2(a)\alpha(b) \wedge \beta(a)d_{2_{(\alpha,\beta)}}(b)) \\ &= G_1(G_2(a)\alpha(b)) \\ &= G_1(G_2(a))\alpha^2(b) \wedge \beta(G_2(a))d_{1_{(\alpha,\beta)}}(\alpha(b)) \\ &= G_1(G_2(a))\alpha(b) \\ &= (\beta(a)(d_{1_{(\alpha,\beta)}} \circ d_{2_{(\alpha,\beta)}})(b))(\beta(a)(d_{1_{(\alpha,\beta)}} \circ d_{2_{(\alpha,\beta)}})(b)(G_1 \circ G_2)(a)\alpha(b)) \\ &= (G_1 \circ G_2)(a)\alpha(b) \wedge \beta(a)(d_{1_{(\alpha,\beta)}} \circ d_{2_{(\alpha,\beta)}})(b). \end{aligned}$$

□

DEFINITION 3.4. [1] *Let $\mathcal{A}$ be a d-algebra. Define a binary relation $\leqslant$ on $\mathcal{A}$ by $a \leqslant b$ iff $ab = 0$. Thus, $(\mathcal{A}, \leqslant)$ is a partially ordered set.*

LEMMA 3.3. *Suppose that $\mathcal{A}$ is an edge d-algebra with partial order $\leqslant$. If G is a right generalized (α,β)-derivation on $\mathcal{A}$, then $G(a) \leqslant \alpha(a)$ for all $a \in \mathcal{A}$.*

PROOF. Let G be a right generalized (α,β)-derivation on $\mathcal{A}$. It follows from Theorem 3.1 (2) that

$$\begin{aligned} G(a) &= \alpha(a) \wedge d_{(\alpha,\beta)}(a) \\ &= d_{(\alpha,\beta)}(a)(d_{(\alpha,\beta)}(a)\alpha(a)) \end{aligned}$$

and so

$$G(a)\alpha(a) = (d_{(\alpha,\beta)}(a)(d_{(\alpha,\beta)}(a)\alpha(a)))\alpha(a).$$

Since $(a(ab))b = 0$, we have $G(a)\alpha(a) = 0$ and so $G(a) \leqslant \alpha(a)$. □

LEMMA 3.4. *Let $\mathcal{A}$ be a d-algebra with partial order $\leqslant$ and let G be a right generalized (α,β)-derivation on $\mathcal{A}$. Then, $G(ab) \leqslant \alpha(a)G(b)$ for all $a, b \in \mathcal{A}$.*

PROOF. We have

$$\begin{aligned} G(ab) &= \alpha(a)G(b) \wedge d_{(\alpha,\beta)}(a)\beta(b) \\ &= (d_{(\alpha,\beta)}(a)\beta(b))((d_{(\alpha,\beta)}(a)\beta(b))(\alpha(a)G(b))). \end{aligned}$$

Since $(a(ab))b = 0$, we get

$$\begin{aligned} G(ab)(\alpha(a)G(b)) &= (d_{(\alpha,\beta)}(a)\beta(b))((d_{(\alpha,\beta)}(a)\beta(b))(\alpha(a)G(b)))(\alpha(a)G(b)) \\ &= 0 \end{aligned}$$

and this completes the proof. □

LEMMA 3.5. *Assume that $\mathcal{A}$ is an edge d-algebra with partial order $\leqslant$ and G is a right generalized (α,β)-derivation on $\mathcal{A}$.*

(1) *For all $a \in A$, $G(G(a)) \leqslant \alpha(G(a))$.*

(2) *If $\alpha(a) = 0$ for all $a \in A$, then*

$$G^{-1}(0) = \{ a \in \mathcal{A} | \, G(a) = 0\}$$

is a d-subalgebra of A.

PROOF. (1) Assume that G is a right generalized (α,β)-derivation on $\mathcal{A}$. By Theorem 3.1 (2), we have

$$\begin{aligned} G(G(a)) &= \alpha(G(a)) \wedge d_{(\alpha,\beta)}(G(a)) \\ &= d_{(\alpha,\beta)}(G(a))(d_{(\alpha,\beta)}(G(a))\alpha(G(a))). \end{aligned}$$

Multiplying both sides from the right by $\alpha(G(a))$, we get $G(G(a))\alpha(G(a)) = 0$ and so we have required result.

(2) By Lemma 3.2, we have G is a regular. Hence, $G^{-1}(0) \neq \emptyset$. Let $a, b \in G^{-1}(0)$. By Lemma 3.4, we have $G(ab) \leqslant \alpha(a)G(b) = 0$, since $\alpha(a) = 0 = G(b)$. Hence, $G(ab) = 0$ and so $ab \in G^{-1}(0)$. Therefore, $G^{-1}(0)$ is a d-subalgebra of A. □

THEOREM 3.6. *Assume that $\mathcal{A}$ is an edge d-algebra with partial order $\leqslant$ and G is a right generalized (α,β)-derivation on $\mathcal{A}$ such that $\alpha^2=\alpha$. Then,*

$$G^n(G^{n-1}(...(G^2(G(a))))...)\leqslant\alpha(a)$$

for all $a\in\mathcal{A}$.

PROOF. Let's prove the theorem using the mathematical induction method. Let $n=1$. By Lemma 3.3, we have $G(a)\leqslant\alpha(a)$. Assume that

$$G^n(G^{n-1}(...(G^2(G(a))))...)\leqslant\alpha(a)$$

for any $n\in\mathbb{N}$. Let $\nabla_n=G^n(G^{n-1}(...(G^2(G(a))))...)$, that is $\nabla_n\leqslant\alpha(a)$. Hence,

$$\begin{array}{rcl} G^{n+1}(\nabla_n) & = & G^{n+1}(\nabla_n 0) \\ & = & \alpha(\nabla_n)G^{n+1}(0)\wedge d^{n+1}_{(\alpha,\beta)}(\nabla_n)\beta(0) \\ & = & \alpha(\nabla_n)\wedge d^{n+1}_{(\alpha,\beta)}(\nabla_n) \\ & = & d^{n+1}_{(\alpha,\beta)}(\nabla_n)(d^{n+1}_{(\alpha,\beta)}(\nabla_n)\alpha(\nabla_n)) \end{array}$$

which implies that

$$G^{n+1}(\nabla_n)\alpha(\nabla_n)=((d^{n+1}_{(\alpha,\beta)}(\nabla_n)(d^{n+1}_{(\alpha,\beta)}(\nabla_n)\alpha(\nabla_n))\alpha(\nabla_n)=0,$$

since $(a(ab))b=0$. Thus, $G^{n+1}(\nabla_n)\leqslant\alpha(\nabla_n)\leqslant\alpha(a)$. This completes the proof. □

DEFINITION 3.5. *Let G be a generalized (α,β)-derivation of a d-algebra $\mathcal{A}$. We say that a d-ideal I of $\mathcal{A}$ is an α-ideal (resp. β-ideal) if $\alpha(I)\subseteq I$ (resp. $\beta(I)\subseteq I$).*

DEFINITION 3.6. *Let G be a generalized (α,β)-derivation of a d-algebra $\mathcal{A}$. We say that a d-ideal I of $\mathcal{A}$ is G-invariant if $G(I)\subseteq I$.*

EXAMPLE 3.4. *Let G be a left generalized (α,β)-derivation on $\mathcal{A}$ which described in* Example 3.1 *and $I=\{0,1\}$. Then, I is both an α-ideal and β-ideal. Moreover, it is a G-invariant of $\mathcal{A}$.*

EXAMPLE 3.5. *Let G be a right generalized (α,β)-derivation on $\mathcal{A}$ which described in* Example 3.2 *and $I=\{0,b\}$. It is clear that I is a β-ideal, but I is not an α-ideal. Also, I is a G-invariant of $\mathcal{A}$.*

THEOREM 3.7. *Suppose that $\mathcal{A}$ is an edge d-algebra, I is a d-ideal of $\mathcal{A}$ and G is a right generalized (α,β)-derivation of $\mathcal{A}$. Then, G is regular iff every α-ideal of A is G-invariant.*

PROOF. Suppose that I is an α-ideal of $\mathcal{A}$ and G is regular. Let $b\in G(I)$. Then, we have $b=G(a)$ for some $a\in I$. By Lemma 3.3, we have $G(a)\leqslant\alpha(a)$, that is $G(a)\alpha(a)=0\in I$. Hence, $\alpha(a)\in\alpha(I)\subseteq I$ which implies that $b\in I$. Thus, $G(I)\subseteq I$ which implies that I is a G-invariant. Now, assume that every α-ideal of $\mathcal{A}$ is G-invariant. The ideal $\{0\}$ is both α-ideal and G-invariant. Thus, we have $G(0)\subseteq\{0\}$ and so $G(0)=0$. It implies that G is regular.

□

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