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CONNECTIONS BETWEEN VARIOUS SUBCLASSES OF PLANAR HARMONIC MAPPINGS ASSOCIATED WITH GENERALIZED DISTRIBUTION SERIES

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Abstract. The purpose of the present paper is to obtain inclusion relation between various subclasses of harmonic univalent functions by applying certain convolution operator associated with generalized distribution series.

1. Introduction

Let $\mathcal{A}$ denote the class of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \ : \ z \in \mathbb{C} \text{ and } |z| < 1\}$ and satisfy the normalization condition $f(0) - f'(0) - 1 = 0$. Let $\mathcal{H}$ be the family of all harmonic functions of the form $f = h + \bar{g}$, where,

$$h(z) = z + \sum_{n=2}^{\infty} A_n z^n, \qquad g(z) = \sum_{n=1}^{\infty} B_n z^n, \qquad |B_1| < 1 \tag{1.2}$$

are in the class $\mathcal{A}$.

Further, we denote by $\mathcal{S}_{\mathcal{H}}$ the subclasses of $\mathcal{H}$, that are univalent and sense preserving in $\mathbb{U}$. We also let the subclasses $\mathcal{S}^0_{\mathcal{H}}$ of $\mathcal{S}_{\mathcal{H}}$ as

$$\mathcal{S}^0_{\mathcal{H}} = \{f = h + \bar{g} \in \mathcal{S}_{\mathcal{H}} : g'(0) = B_1 = 0\}.$$

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The classes $\mathcal{S}_{\mathcal{H}}$ and $\mathcal{S}_{\mathcal{H}}^0$ were first studied by Clunie and Sheil-small [**9**]. Also, we let K_H^0,$\mathcal{S}_{\mathcal{H}}^{*,0}$ and C_H^0 denote the subclasses of $\mathcal{S}_{\mathcal{H}}^0$ of harmonic functions which are respectively convex, starlike and close-to-convex in $\mathbb{U}$. Also let $\mathcal{T}_{\mathcal{H}}^0$ be the class of sense preserving,typically real harmonic functions $f = h + \bar{g}$ in $\mathcal{H}$. For detailed study of these classes one may refer to [**1, 2, 4, 8**].

A function $f(z)$ of the form (1.2) is said to be in the class $\mathcal{N}_{\mathcal{H}}(\gamma)$, if it satisfy the condition

$$\Re\left\{\frac{f'(z)}{z'}\right\} \geqslant \gamma$$

similarly, a function f of the form (1.2) is said to be in the class $G_H(\gamma)$, if it satisfy the condition

$$Re\Big\{(1+e^{-i\alpha})\frac{zf'(z)}{f(z)} - e^{-i\alpha}\Big\} \geqslant \gamma, \quad \gamma \in \mathbb{R},$$

where, $z' = \dfrac{\partial}{\partial\theta}\left(z = re^{i\theta}\right)$ and $f'(z) = \dfrac{\partial}{\partial\theta} f(\gamma e^{i\theta})$.

Define $T\mathcal{N}_{\mathcal{H}}(\gamma) = \mathcal{N}_{\mathcal{H}}(\gamma) \cap T$ and $TG_{\mathcal{H}}(\gamma) = G_{\mathcal{H}}(\gamma) \cap T$, where T consists of the functions $f = h + \bar{g}$ in $\mathcal{S}_{\mathcal{H}}$.So that h and g are of the form:

$$h(z) = z - \sum_{n=2}^{\infty} |A_n| z^n, \qquad g(z) = \sum_{n=1}^{\infty} |B_n| z^n. \tag{1.3}$$

The classes $\mathcal{N}_{\mathcal{H}}(\gamma)$, $T\mathcal{N}_{\mathcal{H}}(\gamma)$, $G_H(\gamma)$ and $TG_H(\gamma)$, were initially introduced and studied by Ahuja and Jahangiri [**4**], Rosy et at.[**20**] respectively.The applications of hypergeometric functions [**18**], confluent hypergeometric functions[**7**], generalized hypergeometric functions [**16**], generalized Bessels functions [**6**], Wright functions [**19**], Fox-wright functions [**8**] are interesting topics of research in Geometric function theory. In 2014, Porwal [**14**] introduced Poisson distribution series and obtain some necessary and sufficient condition for certain classes of univalent functions. This paper establishs a co-relation between distribution function and geometric function theory and opens up a new and interesting direction of research in Geometric function theory. After this appearance of this paper various authors investigated hypergeometric distribution series [**15**], binomial distribution series [**11**], confluent hypergeometric-type distribution series [**17**] and obtain various investigation results on univalent functions. Recently Porwal [**16**] introduced generalized distribution and give a beautiful application on certain classes of univalent functions.

First we recall the definition of generalized distribution. The probability mass function of the generalized distribution is given as

$$p(n) = \frac{t_n}{S}, \; n = 0, \; 1, \; 2, \; \cdots,$$

where $t_n \geqslant 0$ and the series $\sum\limits_{n=0}^{\infty} t_n$ is convergent and

$$S = \sum_{n=0}^{\infty} t_n. \tag{1.4}$$

Further, we introduced the series

$$\phi(x) = \sum_{n=0}^{\infty} t_n x^n \tag{1.5}$$

From (1.4) it is worthy to note that the series given by (1.5) is convergent for $|x| < 1$ and for $x = 1$ it is also convergent. Porwal [**16**] introduced generalized distribution series in the following way :

$$K_\phi(z) = z + \sum_{n=2}^{\infty} \frac{t_{n-1}}{S} z^n. \tag{1.6}$$

Now we define the convolution (or Hadamard product) of two functions $f \in \mathcal{A}$ given by (1.1) and $g \in \mathcal{A}$ given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n$$

as

$$(f * g)(z) = f(z) * g(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n.$$

By using the definition of generalized distribution series $I(\phi_1, \phi_2)$, we introduced the convolution operator by

$$I(\phi_1, \phi_2) f(z) = H(z) + \overline{G(z)}, \tag{1.7}$$

where

$$H(z) = h(z) * K_{\phi_1}(z) \qquad \text{and} \qquad G(z) = g(z) * K_{\phi_2}(z)$$

or equivalently,

$$H(z) = z + \sum_{n=2}^{\infty} \frac{A_n}{S_1} t_{n-1} z^n \qquad \text{and} \qquad G(z) = B_1 z + \sum_{n=2}^{\infty} \frac{B_n}{S_2} \gamma_{n-1} z^n,$$

where,

$$K_{\phi_1}(z) = z + \sum_{n=2}^{\infty} \frac{t_{n-1}}{S_1} z^n \qquad \text{and} \qquad K_{\phi_2}(z) = z + \sum_{n=2}^{\infty} \frac{\gamma_{n-1}}{S_2} z^n$$

and

$$\phi_1(z) = \sum_{n=0}^{\infty} t_n z^n \qquad \text{and} \qquad \phi_2(z) = \sum_{n=0}^{\infty} \gamma_n z^n.$$

Here $S_1 = \phi_1(1)$ and $S_2 = \phi_2(1)$.

Motivated by the results on the connections between various subclasses of analytic and harmonic univalent functions by using hypergeometric functions [**2, 15**], generalized Bessels functions [**6**], distribution series [**17**], we obtain a number of inclusion relation between the classes $G_{\mathcal{H}}(\gamma)$, $K_{\mathcal{H}}^0$, $\mathcal{S}_{\mathcal{H}}^{*,0}$, $C_{\mathcal{H}}^0$ and $\mathcal{N}_{\mathcal{H}}(\beta)$ by applying he convolution operator I.

2. Main results

In order to establish our main results we shall requires the following lemmas.

LEMMA 2.1. [**10**] *If* $f = h + \bar{g} \in K^0_{\mathcal{H}}$*, where* g *and* h *are given by (1.2) with* $B_1 = 0$*, then*

$$|A_n| < \frac{n+1}{2}, \qquad |B_n| < \frac{n-1}{2}.$$

LEMMA 2.2. [**20**] *Let* $f = h + \bar{g}$ *be given by (1.2). If* $0 < \gamma < 1$ *and*

$$\sum_{n=2}^{\infty}(2n-1-\gamma)|A_n| + \sum_{n=1}^{\infty}(2n+1+\gamma)|B_n| \leqslant 1-\gamma. \tag{2.1}$$

Then f *is sense-preserving Goodman-Rønning-type harmonic univalent functions in* $\mathbb{U}$ *and* $f \in G_{\mathcal{H}}(\gamma)$

REMARK 2.1. In [**20**], it is also shown that $f = h + \bar{g}$ be given by (1.3) is in the family $TG_{\mathcal{H}}(\gamma)$,if and only if the coefficients condition (2.1) holds. Moreover, if $f \in TG_{\mathcal{H}}(\gamma)$, then

$$|A_n| = \frac{1-\gamma}{2n-1-\gamma}, \quad n \geqslant 2 \qquad \text{and} \qquad |B_n| = \frac{1-\gamma}{2n+1+\gamma}, \quad n \geqslant 1.$$

LEMMA 2.3. [**10**] *Let* $f = h + \bar{g} \in \mathcal{S}^{*,0}_{\mathcal{H}}$ *or* $C^0_{\mathcal{H}}$*, where* $h(z)$ *and* $g(z)$ *are of the form (1.2) with* $B_1 = 0$*. Then*

$$|A_n| \leqslant \frac{(2n+1)(n+1)}{6} \qquad \text{and} \qquad |B_n| \leqslant \frac{(2n-1)(n-1)}{6}$$

LEMMA 2.4. [**4**] *Let* $f = h + \bar{g}$*, where* $h(z)$ *and* $g(z)$ *are given by (1.2) with* $B_1 = 0$*, and suppose that* $0 \leqslant \beta \leqslant 1$*. Then* $f \in T\mathcal{N}_{\mathcal{H}}(\beta)$ *if and only if*

$$\sum_{n=2}^{\infty}|A_n|n + \sum_{n=1}^{\infty}|B_n|n \leqslant 1-\beta. \tag{2.2}$$

REMARK 2.2. If $f \in T\mathcal{N}_{\mathcal{H}}(\beta)$ then $|A_n| \leqslant \dfrac{1-\beta}{n}$ and $|B_n| \leqslant \dfrac{1-\beta}{n}$, $n \geqslant 2$.

Throughout this paper, we will frequently use the notation $I(\phi_1, \phi_2) \equiv I$.

THEOREM 2.1. *Let* $f = h + \bar{g} \in H$ *be of the form (1.2) with* $B_1 = 0$ *and the inequality*

$$\frac{1}{S_1}\Big(2\phi_1''(1) + (7-\gamma)\phi_1'(1)\Big) + \frac{1}{S_2}\Big(2\phi_2''(1) + (5+\gamma)\phi_2'(1)\Big) \leqslant \frac{2(1-\gamma)}{S_1}\phi_1(0) \tag{2.3}$$

is satisfied. Then $I(K^0_{\mathcal{H}}) \subset G_{\mathcal{H}}(\gamma)$.

PROOF. Let $f = h + \bar{g} \in K^0_{\mathcal{H}}$ where h and g are given by (1.2) with $B_1 = 0$. We have to prove that $I(f) = H + \bar{G} \in G_{\mathcal{H}}(\gamma)$, where G and H are given in (1.7) with $B_1 = 0$ are analytical functions in $\mathbb{U}$. In virtue of Lemma 2.2, we need to prove that $p_1 \leqslant 1-\gamma$, where,

$$p_1 = \sum_{n=2}^{\infty}(2n-1-\gamma)\frac{t_{n-1}}{S_1}|A_n| + \sum_{n=2}^{\infty}(2n+1+\gamma)\frac{\gamma_{n-1}}{S_2}|B_n| \tag{2.4}$$

By using Lemma 2.1

$$\begin{aligned}
p_1 &\leqslant \frac{1}{2}\left[\sum_{n=2}^{\infty}(n+1)(2n-1-\gamma)\frac{t_{n-1}}{S_1}+\sum_{n=2}^{\infty}(n-1)(2n+1+\gamma)\frac{\gamma_{n-1}}{S_2}\right]\\
&=\frac{1}{2}\left[\sum_{n=2}^{\infty}\Big\{2(n-1)(n-2)+(7-\gamma)(n-1)+2(1-\gamma)\Big\}\frac{t_{n-1}}{S_1}\right]\\
&\quad+\frac{1}{2}\left[\sum_{n=2}^{\infty}\Big\{2(n-1)(n-2)+(5+\gamma)(n-1)\Big\}\frac{\gamma_{n-1}}{S_2}\right]\\
&=\frac{1}{2}\left[\sum_{n=1}^{\infty}\Big\{2n(n-1)+(7-\gamma)n+2(1-\gamma)\Big\}\frac{t_n}{S_1}\right.\\
&\qquad\left.+\sum_{n=1}^{\infty}\Big\{2n(n-1)+(5+\gamma)n\Big\}\frac{\gamma_n}{S_2}\right]\\
&=\frac{1}{2}\left[\frac{1}{S_1}\Big\{2\phi_1''(1)+(7-\gamma)\phi_1'(1)+2(1-\gamma)(S_1-\phi_1(0))\Big\}\right.\\
&\qquad\left.+\frac{1}{S_2}\Big\{2\phi_2''(1)+(5+\gamma)\phi_2'(1)\Big\}\right]\\
&\leqslant 1-\gamma
\end{aligned}$$

by the given hypothesis,this completes the proof of the Theorem 2.1. □

THEOREM 2.2. *Let $f=h+\bar{g}\in\mathcal{S}_{\mathcal{H}}^{*,0}$ or $C_{\mathcal{H}}^{0}$, where h and g are given by (1.2) with $B_1=0$ and the inequality*

$$\begin{aligned}
&\frac{1}{S_1}(4\phi_1'''(1)+(28-2\gamma)\phi_1''(1)+(39-9\gamma)\phi_1'(1))\\
&\qquad+\frac{1}{S_2}(4\phi_2'''(1)+(20+2\gamma)\phi_2''(1)+(15+3\gamma)\phi_2'(1))\leqslant\frac{6(1-\gamma)}{S_1}\phi_1(0)
\end{aligned}$$

satisfied. Then $I(\mathcal{S}_{\mathcal{H}}^{*,0})\subseteq G_{\mathcal{H}}(\gamma)$, or $I(C_{\mathcal{H}}^{0})\subseteq G_{\mathcal{H}}(\gamma)$.

PROOF. Let $f=h+\bar{g}\in\mathcal{H}$, where h and g are given by (1.2) with $B_1=0$, we need to prove that $p_1\leqslant 1-\gamma$, where p_1 is given by (2.4), the virtue of Lemma 2.3, we have,

$$\begin{aligned}
p_1 &\leqslant \frac{1}{6}\left[\sum_{n=2}^{\infty}(n+1)(2n+1)(2n-1-\gamma)\frac{t_{n-1}}{S_1}+\sum_{n=2}^{\infty}(n-1)(2n-1)(2n+1+\gamma)\frac{\gamma_{n-1}}{S_2}\right]\\
&= \frac{1}{6}\left[\frac{1}{S_1}\sum_{n=2}^{\infty}\{4(n-1)(n-2)(n-3)+(28-2\gamma)(n-1)(n-2)+(39-9\gamma)(n-1)+6(1-\gamma)\}(t_{n-1})\right]\\
&\quad+\frac{1}{6}\left[\frac{1}{S_2}\sum_{n=2}^{\infty}\{4(n-1)(n-2)(n-3)+(20+2\gamma)(n-1)(n-2)+(15+3\gamma)(n-1)\}(\gamma_{n-1})\right]\\
&= \frac{1}{6}\left[\frac{1}{S_1}\sum_{n=1}^{\infty}\{4n(n-1)(n-2)+(28-2\gamma)n(n-1)+(39-9\gamma)n+6(1-\gamma)\}t_n\right]\\
&\quad+\frac{1}{6}\left[\frac{1}{S_2}\sum_{n=1}^{\infty}\{4n(n-1)(n-2)+(20+2\gamma)n(n-1)+(15+3\gamma)n\}\gamma_n\right]\\
&= \frac{1}{6}\left[\frac{1}{S_1}\{4\phi_1'''(1)+(28-2\gamma)\phi_1''(1)+(39-9\gamma)\phi_1'(1)+6(1-\gamma)(S_1-\phi_1(0))\}\right]\\
&\quad+\frac{1}{6}\left[\frac{1}{S_2}\{4\phi_2'''(1)+(20+2\gamma)\phi_2''(1)+(15+3\gamma)\phi_2'(1)\}\right]\leqslant 1-\gamma
\end{aligned}$$

by the given hypothesis, this establish the proof of Theorem 2.3. □

In our next theorem,we establish connections between $TG_{\mathcal{H}}(\gamma)$ and $G_{\mathcal{H}}(\gamma)$.

THEOREM 2.3. *Let $f = h + \bar{g} \in TG_{\mathcal{H}}(\gamma)$ where h and g are given by (1.3) if for some $\gamma\,(0 \leqslant \gamma \leqslant 1)$ the inequality $\phi_1(0) \geqslant (1-\gamma)S_1$ is satisfied. Then $I(TG_{\mathcal{H}}(\gamma)) \subseteq G_{\mathcal{H}}(\gamma)$.*

PROOF. Let $f = h + \bar{g} \in TG_{\mathcal{H}}(\gamma)$ be given by (1.5). We have to prove that $p_2 \leqslant 1-\gamma$, where

$$p_2 = \sum_{n=2}^{\infty}(2n-1-\gamma)\frac{t_{n-1}}{S_1}|A_n| + \sum_{n=1}^{\infty}(2n+1+\gamma)\frac{\gamma_{n-1}}{S_2}|B_n|. \tag{2.5}$$

Using Remark 2.1, we have

$$\begin{aligned}
p_2 &\leqslant (1-\gamma)\sum_{n=2}^{\infty}\frac{t_{n-1}}{S_1} + (1-\gamma)\sum_{n=1}^{\infty}\frac{\gamma_{n-1}}{S_2}\\
&= (1-\gamma)\sum_{n=1}^{\infty}\frac{t_n}{S_1} + (1-\gamma)\sum_{n=0}^{\infty}\frac{\gamma_n}{S_2}\\
&= (1-\gamma)\left[\frac{S_1-\phi_1(0)}{S_1}\right] + (1-\gamma)\frac{S_2}{S_2} = 2(1-\gamma) - \frac{\phi_1(0)}{S_1}\\
&\leqslant 1-\gamma
\end{aligned}$$

by the given hypothesis, this completes the proof of Theorem 2.3. □

THEOREM 2.4. *Let $f = h + \bar{g} \in T\mathcal{N}_{\mathcal{H}}(\beta)$ for some $(0 \leqslant \beta \leqslant 1)$ be of the form (1.2) and the inequality*

$$(1-\beta)\left[4 + (1+\gamma)\int_0^1\left(\frac{\phi_2(t)}{S_2} - \frac{\phi_1(t)}{S_1}\right)dt - \frac{\phi_1(0)}{S_1}\right] \leqslant (1-\gamma)$$

is satisfied. Then $I(T\mathcal{N}_{\mathcal{H}}(\beta)) \subseteq G_{\mathcal{H}}(\gamma)$.

PROOF. Let $f = h + \bar{g} \in T\mathcal{N}_{\mathcal{H}}(\beta)$, where h and g are given by (1.3). In view of Lemma 2.2, we have to prove that $p_2 \leqslant 1-\gamma$ where p_2 is given by (2.5). Using Remark 2.2, we have

$$\begin{aligned}
p_2 &\leqslant (1-\beta)\left[\sum_{n=2}^{\infty}\left\{2-\frac{(1-\gamma)}{n}\right\}\frac{t_{n-1}}{S_1}+\sum_{n=1}^{\infty}\left\{2+\frac{(1+\gamma)}{n}\right\}\frac{\gamma_{n-1}}{S_2}\right]\\
&= (1-\beta)\left[\sum_{n=1}^{\infty}\left\{2-\frac{(1-\gamma)}{n}\right\}\frac{t_{n}}{S_1}+\sum_{n=0}^{\infty}\left\{2+\frac{(1+\gamma)}{n}\right\}\frac{\gamma_{n}}{S_2}\right]\\
&= (1-\beta)\left[\frac{2}{S_1}(S_1-\phi_1(0))+2+(1+\gamma)\left\{\frac{1}{S_2}\int_0^1\phi_2(t)dt-\frac{1}{S_1}\int_0^1\phi_1(t)dt+\frac{\phi_1(0)}{S_1}\right\}\right]\\
&= (1-\beta)\left[4+(1+\gamma)\int_0^1\left(\frac{\phi_2(t)}{S_2}-\frac{\phi_1(t)}{S_1}\right)dt-\frac{\phi_1(0)}{S_1}\right]\\
&\leqslant (1-\gamma)
\end{aligned}$$

by the given hypothesis,this completes the proof of Theorem 2.4. □

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