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INTERVAL ROUGH FUZZY IDEALS IN γ-NEAR-RINGS

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Abstract. In this paper we expose interval rough fuzzy ideals in γ-near-rings. Also we prove some interesting properties of γ-ideal and bi-γ-ideal in γ-near-ring.

1. Introduction

γ-near-ring and the ideal theory of γ-near-ring were introduced by Bh. Sathyanaranan[10]. For basic concepts in near-ring we refer to Pilz[8]. Zadeh [16] introduced the concept of fuzzy set in 1965, and it is now a famous area of research with many applications. After that he also introduced the notion of interval valued fuzzy subset in 1975, where the values of membership functions are intervals of numbers instead of a single number as in fuzzy set. The fuzzy set theory has been developed in many directions by the research scholars. Rosenfeld [9] first introduced the fuzzification of the algebraic sturctures and defined fuzzy subgroups. Jun [5]discussed interval-valued Rsubgroups in terms of near-rings. Meenakumari and Tamizh Chelvan[6] discussed fuzzy bi-ideals in γ-near-ring. Pawlak [7]introduced the notion of rough set in his paper. Rough set theory, a new mathematical approach to deal with in exact, uncertain or vague. Knowledge has recently received wide attention on the research areas in both of the real-life applications and theory itself. It has found practical applications in many areas such as knowledge discovery machine learning, data analysis, approximate classification, conflict analysis and so on. Many researchers studied the algebraic approach of rough sets in different algebraic structures. Subha [12] studied the concept of rough fuzzy ideals in γ-near-ring.

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1.1. Preliminaries. This section deals with the basic definitions related to this work.
A near-ring is an algebraic system $(\mathfrak{N}, +, .)$ consisting of a non empty set $\mathfrak{N}$ together with two binary operations called $+$ and . such that $(\mathfrak{N}, +)$ is a group not necessarily abelian and $(\mathfrak{N}, .)$ is a semigroup connected by the following distributive law: $(x+z).y = x.y + x.z$ valid for all $x, y, z \in \mathfrak{N}$.

Definition 1.1. *A γ-near-ring is a triple where $(\mathfrak{N}, +, .)$ (i) $(\mathfrak{N}, +)$ is a group. (ii) γ is nonempty set of binary operators on $\mathfrak{N}$ such that for $i \in \gamma$, $(\mathfrak{N}, +, i)$ is a near ring. (iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in \mathfrak{N}$ and $\alpha, \beta \in \gamma$.*

Definition 1.2. *Let X be a non empty set. A mapping $\bar{A} : X \to D[0,1]$ is called an interval fuzzy subset of X, where $\bar{A}(x) = [A^-(x), A^+(x)]$ $x \in X$, and A^- and A^+ are fuzzy subsets in X such that $A^-(x) \leqslant A^+(x)$, $x \in X$. $D[0,1]$ denotes the set of closed subsets of $[0,1]$.*

Definition 1.3. *Let A be interval fuzzy subset of S and $[\lambda_1, \lambda_2] \in D[0,1]$, we call*

$$A_{[\lambda_1,\lambda_2]} = \{x \in X : A^-(x) \geqslant \lambda_1, A^+(x) \geqslant \lambda_2\} \text{ and}$$
$$A_{(\lambda_1,\lambda_2)} = \{x \in X : A^-(x) > \lambda_1, A^+(x) > \lambda_2\}$$

the $[\lambda_1, \lambda_2]-$ level set of A and $(\lambda_1, \lambda_2)-$ level set of A, respectively, where (λ_1, λ_2) in $A_{(\lambda_1,\lambda_2)}$ is not an interval, it is only a notation, and we may admit $\lambda_1 = \lambda_2$ clearly, $x \in A_{[\lambda_1,\lambda_2]}$ iff $A(x) \geqslant [\lambda_1, \lambda_2]$.

Definition 1.4. *An interval fuzzy set η^i in $\mathfrak{N}$ is called an interval sub γ-near-ring of $\mathfrak{N}$ if*
(i) $\eta^i(x-y) \geqslant \eta^i(x) \wedge \eta^i(y)$
(ii) $\eta^i(x\alpha y) \geqslant \eta^i(x) \wedge \eta^i(y)$, for all $x, y \in \mathfrak{N}$ and $\alpha \in \gamma$.

Definition 1.5. *An interval fuzzy set η^i in $\mathfrak{N}$ is called an interval left(resp.right) γ-ideal of $\mathfrak{N}$ if*
(i) $\eta^i(x-y) \geqslant \eta^i(x) \wedge \eta^i(y)$
(ii) $\eta^i(y+x-y) \geqslant \eta^i(x)$
(iii) $\eta^i(a\alpha(x+b) - a\alpha b) \geqslant \eta^i(x)(resp., \eta^i(x\alpha a) \geqslant \eta^i(a)$, for all $x, y, a, b \in \mathfrak{N}$ and $\alpha \in \gamma$.

Definition 1.6. *An interval fuzzy set η^i in $\mathfrak{N}$ is called an interval bi- γ-ideal of $\mathfrak{N}$ if*
(i) $\eta^i(x-y) \geqslant \eta^i(x) \wedge \eta^i(y)$
(ii) $\eta^i(x\alpha y\beta z) \geqslant \eta^i(x) \wedge \eta^i(y)$, for all $x, y, a, b \in \mathfrak{N}$ and $\alpha \in \gamma$.

Theorem 1.1. *Let η^i be a interval fuzzy subset of $\mathfrak{N}$ if and only if $\eta^i_{[t_1,t_2]}$, $[t_1, t_2] \in D[0,1]$ is an ideal of $\mathfrak{N}$.*

Let $\mathfrak{N}$ be the universal set. For an equivalence relation $\mho$ on $\mathfrak{N}$, the equivalence class of x and is denoted by $[x]_{\mho}$.

DEFINITION 1.7. *Let $\mathfrak{N}$ be universal set and $\mho$ congruence relation on $\mathfrak{N}$. The pair $(\mathfrak{N}, \mho)$ is called an approximation space. Let Ω be any nonempty subset of $\mathfrak{N}$. The sets*

$$\mho_*(\Omega) = \{x \in \mathfrak{N}/[x]_\mho \subseteq \Omega\} \text{ and}$$
$$\mho^*(\Omega) = \{x \in \mathfrak{N}/[x]_\mho \cap \Omega \neq \phi\}$$

are called the lower and upper approximations of Ω.
Then $\mho(\Omega) = (\mho_*(\Omega), \mho^*(\Omega))$ is called rough set in $(\mathfrak{N}, \mho)$.

DEFINITION 1.8. *Let Ω be a fuzzy subset of $\mathfrak{N}$. The fuzzy subsets of $\mathfrak{N}$ defined by*

$$\mho^*(\Omega)(x) = \bigvee_{a\in[x]_\mho} \Omega(a) \text{ and } \mho_*(\Omega)(x) = \bigwedge_{a\in[x]_\mho} \Omega(a)$$

are called respectively, the upper and lower approximations of the fuzzy set Ω. $\mho(\Omega) = (\mho_*(\Omega), \mho^*(\Omega))$ is called a rough fuzzy set of Ω with respect to $\mho$ if $\mho_*(\Omega) \neq \mho^*(\Omega)$.

THEOREM 1.2. *Let η^i an interval fuzzy subset of fuzzy subset of $\mathfrak{N}$, η^i is an interval fuzzy left (right) ideal of $\mathfrak{N}$ if and only if $\eta^i_{[a_1,a_2]}$ left (right) ideal of $\mathfrak{N}$; for all $[a_1, a_2] \in D[0; 1]$*

2. Interval rough fuzzy ideals in -near-rings

In this section we studied the new concept interval rough fuzzy ideals in near rings. We combine the rough set and interval fuzzy sets. Also investigated the interval rough fuzzy sub-γ-near-ring, interval rough fuzzy γ-ideal and interval rough fuzzy bi-γ-ideal in near-ring. Some interesting properties of these ideals are proved.

DEFINITION 2.1. *Let $\mho$ be congruence relation on $\mathfrak{N}$. An interval fuzzy subγ-near-ring η is said to be an upper and lower interval rough fuzzy subγ-near-ring of $\mathfrak{N}$ if the following condition are holds:*
1. $\mho^*$ is an interval fuzzy sub-γ-near-ring of $\mathfrak{N}$.
2. $\mho_*$ is an interval fuzzy sub-γ-near-ring of $\mathfrak{N}$.

THEOREM 2.1. *An interval fuzzy sub-γ-near-ring of $\mathfrak{N}$ is a upper interval rough fuzzy subγ-near-ring of $\mathfrak{N}$.*

PROOF. Let η^i be an interval fuzzy sub-γ-near-ring of $\mathfrak{N}$. Then for all $\alpha, \beta \in \mathfrak{N}$ we consider

$$\begin{aligned}
\mho^*(\eta^i)(\alpha - \beta) &= \bigvee_{\nu\in[\alpha-\beta]_\mho} \eta^i(\nu) \\
&= \bigvee_{\nu\in[\alpha]_\mho+[-\beta]_\mho} \eta^i(\nu) \\
&= \bigvee_{\kappa\in[\alpha]_\mho,\lambda\in[-\beta]_\mho} \eta^i(\nu) \\
&\geqslant min\left\{\bigvee_{\kappa\in[\alpha]_\mho} \eta^i(\kappa), \bigvee_{\lambda\in[-\beta]_\mho} \eta^i(\lambda)\right\} \\
&= min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(-\beta)\right\} \\
&\geqslant min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\beta)\right\}
\end{aligned}$$

Also

$$\begin{aligned}\mho^*(\eta^i)(\alpha c\beta) &= \bigvee_{\nu\in[\alpha c\beta]_\mho} \eta^i(\nu)\\ &= \bigvee_{\nu\in[\alpha]_\mho\gamma[\beta]_\mho} \eta^i(\nu)\\ &= \bigvee_{\kappa\in[\alpha]_\mho,\lambda\in[\beta]_\mho} \eta^i(\kappa\gamma\lambda)\\ &\geqslant min\left\{\bigvee_{\kappa\in[\alpha]_\mho}\eta^i(\kappa), \bigvee_{\lambda\in[\beta]_\mho}\eta^i(\lambda)\right\}\\ &= min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\beta)\right\}\\ &\geqslant min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\beta)\right\}\end{aligned}$$

Hence proved. □

THEOREM 2.2. *An interval fuzzy sub-γ-near-ring of $\mathfrak{N}$ is a lower interval rough fuzzy sub-γ-near-ring of $\mathfrak{N}$.*

PROOF. Let η^i be an interval fuzzy sub-γ-near-ring of $\mathfrak{N}$. Then for all $\alpha, \beta \in \mathfrak{N}$ we consider

$$\begin{aligned}\mho_*(\eta^i)(\alpha-\beta) &= \bigwedge_{\nu\in[\alpha-\beta]_\mho} \eta^i(\nu)\\ &= \bigwedge_{\nu\in[\alpha]_\mho+[-\beta]_\mho} \eta^i(\nu)\\ &= \bigwedge_{\kappa\in[\alpha]_\mho,\lambda\in[-\beta]_\mho} \eta^i(\nu)\\ &\geqslant min\left\{\bigwedge_{\kappa\in[\alpha]_\mho}\eta^i(\kappa), \bigwedge_{\lambda\in[-\beta]_\mho}\eta^i(\lambda)\right\}\\ &= min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(-\beta)\right\}\\ &\geqslant min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(\beta)\right\}\end{aligned}$$

Also

$$\begin{aligned}\mho_*(\eta^i)(\alpha c\beta) &= \bigwedge_{\nu\in[\alpha c\beta]_\mho} \eta^i(\nu)\\ &= \bigwedge_{\nu\in[\alpha]_\mho\gamma[\beta]_\mho} \eta^i(\nu)\\ &= \bigwedge_{\kappa\in[\alpha]_\mho,\lambda\in[\beta]_\mho} \eta^i(\kappa\gamma\lambda)\\ &\geqslant min\left\{\bigwedge_{\kappa\in[\alpha]_\mho}\eta^i(\kappa), \bigwedge_{\lambda\in[\beta]_\mho}\eta^i(\lambda)\right\}\\ &= min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(\beta)\right\}\\ &\geqslant min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(\beta)\right\}\end{aligned}$$

Hence proved. □

DEFINITION 2.2. *If $\mho$ be congruence relation on $\mathfrak{N}$. An interval fuzzy ideal η^i is said to be an upper and lower interval rough fuzzy ideal of $\mathfrak{N}$ if the following condition are holds:*
1. $\mho^*$ is an interval fuzzy ideal of $\mathfrak{N}$.
2. $\mho_*$ is an interval fuzzy ideal of $\mathfrak{N}$.

THEOREM 2.3. *Let η^i an interval valued fuzzy subset of $\mathfrak{N}$, η^i is an interval fuzzy left (right) ideal of $\mathfrak{N}$ if and only if $\eta^i_{(a_1,a_2)}$ left (right) ideal of $\mathfrak{N}$; for all $(a_1,a_2) \in D[0,1]$*

LEMMA 2.1. *If η^i be an interval fuzzy subset of $\mathfrak{N}$ and $[a_1,a_2] \in [0,1]$ then*
(i) $\big(\mho_*(\eta^i)\big)_{[a_1,a_2]} = \mho_*(\eta^i_{[a_1,a_2]})$
(ii) $\big(\mho^*(\eta^i)\big)_{(a_1,a_2)} = \mho^*(\eta^i_{(a_1,a_2)})$

PROOF. (i)Let
$$\begin{aligned} k \in \big(\mho_*(\eta^i)\big)_{[a_1,a_2]} &\iff \mho_*(\eta^i)(k) \geqslant [a_1,a_2] \\ &\iff \bigwedge_{j\in[k]_\mho} (\eta^i)(j) \geqslant [a_1,a_2] \\ &\iff \forall j \in [k]_\mho, \eta^i(j) \geqslant [a_1,a_2] \\ &\iff [k]_\mho \subseteq \eta^i_{[a_1,a_2]} \\ &\iff k \in \mho_*(\eta^i_{[a_1,a_2]}) \end{aligned}$$

(ii) Let
$$\begin{aligned} k \in \big(\mho^*(\eta^i)\big)_{(a_1,a_2)} &\iff \mho^*(\eta^i)(k) > (a_1,a_2) \\ &\iff \bigvee_{j\in[k]_\mho} \eta^i(j) > (a_1,a_2) \\ &\iff \exists j \in [k]_\mho, \eta^i(j) > (a_1,a_2) \\ &\iff [k]_\mho \cap \eta^i_{(a_1,a_2)} \neq \emptyset \\ &\iff k \in \mho^*(\eta^i_{(a_1,a_2)}) \end{aligned}$$

□

THEOREM 2.4. *Let $\mho$ be a congruence relation on $\mathfrak{N}$ and let η^i be an interval fuzzy set of $\mathfrak{N}$. If η^i is an interval fuzzy γ-ideal of $\mathfrak{N}$ then η^i is an interval fuzzy ideal of $\mathfrak{N}$.*

Proof is obvious by applying Theorem 1 and Lemma 1(i).

THEOREM 2.5. *Let $\mho$ be a congruence relation on $\mathfrak{N}$ and let η^i be an interval fuzzy set of $\mathfrak{N}$. If η^i is an interval fuzzy γ-ideal of $\mathfrak{N}$ then η^i is an interval fuzzy ideal of $\mathfrak{N}$.*

Proof is obvious by applying Theorem 1 and Lemma 1(ii).

COROLLARY 2.1. *Let $\mho$ be a congruence relation on $\mathfrak{N}$ and let η^i be an interval fuzzy set of $\mathfrak{N}$. If η^i is an interval fuzzy γ-ideal of $\mathfrak{N}$ then η^i is a rough interval fuzzy γ-ideal of $\mathfrak{N}$.*

THEOREM 2.6. *Intersection of two interval rough fuzzy γ-ideals is also an interval rough fuzzy γ-ideal.*

PROOF. Let η^i and ξ^i be an interval rough fuzzy *gamma*-ideals in $\mathfrak{N}$.
Then by Theorem 6 and 7 $\mho_*(\eta^i)$, $\mho_*(\xi^i)$ and $\mho^*(\eta^i)$, $\mho^*(\xi^i)$ are fuzzy ideals of $\mathfrak{N}$.
Then for all $\alpha, \beta \in \mathfrak{N}$ we have
$$\begin{aligned} &\mho^*(\eta^i) \cap \mho^*(\xi^i)(\alpha-\beta) \\ &\quad = min\left\{\mho^*(\eta^i)(\alpha-\beta), \mho^*(\xi^i)(\alpha-\beta)\right\} \\ &\quad = min\left\{min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\beta)\right\}, min\left\{\mho^*(\xi^i)(\alpha), \mho^*(\xi^i)(\beta)\right\}\right\} \\ &\quad = min\left\{min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\xi^i)(\alpha)\right\}, min\left\{\mho^*(\eta^i)(\beta), \mho^*(\xi^i)(\beta)\right\}\right\} \end{aligned}$$

$$\geqslant min\left\{\mho^*(\eta^i)\cap\mho^*(\xi^i)(\alpha), \mho^*(\eta^i)\cap\mho^*(\xi^i)(\beta)\right\}.$$

Also

$$\begin{aligned}\mho^*(\eta^i)\cap\mho^*(\xi^i)(\alpha+\beta-\alpha) \\ &= min\left\{\mho^*(\eta^i)(\alpha+\beta-\alpha), \mho^*(\xi^i)(\alpha+\beta-\alpha)\right\} \\ &\geqslant min\left\{\mho^*(\eta^i)(\beta), \mho^*(\xi^i)(\beta)\right\} \\ &\geqslant \mho^*(\eta^i)\cap\mho^*(\xi^i)(\beta)\end{aligned}$$

More over

$$\begin{aligned}\mho^*(\eta^i)\cap\mho^*(\xi^i)(\alpha a(\nu+\beta)-\alpha a\beta) \\ &= min\left\{\mho^*(\eta^i)(\alpha a(\nu+\beta)-\alpha a\beta), \mho^*(\xi^i)(\alpha a(\nu+\beta)-\alpha a\beta)\right\} \\ &\geqslant min\left\{\mho^*(\eta^i)(\nu), \mho^*(\xi^i)(\nu)\right\} \\ &\geqslant (\mho^*(\eta^i)\cap\mho^*(\xi^i))(\nu)\end{aligned}$$

Similarly we can prove for the other case. Hence proved. □

THEOREM 2.7. *Let $\mho(\eta^i_k)$ be a family of interval rough valued fuzzy γ-ideals in $\mathfrak{N}$ then $\bigcap\mho(\eta^i_k)$ is also an interval rough valued fuzzy γ-ideal in $\mathfrak{N}$.*

PROOF. Since $\mho(\eta^i_k)$ be a family of interval rough valued fuzzy γ-ideals in $\mathfrak{N}$. Let $\alpha, \beta, \nu \in \mathfrak{N}$ and $a, b \in \gamma$. Then

$$\bigcap_{k\in I}\mho(\eta^i_k)\alpha = (\inf_{k\in I}\mho(\eta^i_k))(\alpha) = \inf_{k\in I}\mho(\eta^i_k)(\alpha)$$

i.e.,

$$\bigcap_{k\in I}\mho^*(\eta^i_k)\alpha = (\inf_{k\in I}\mho^*(\eta^i_k))(\alpha) = \inf_{k\in I}\mho^*(\eta^i_k)(\alpha)$$

and

$$\bigcap_{k\in I}\mho_*(\eta^i_k)\alpha = (\inf_{k\in I}\mho_*(\eta^i_k))(\alpha) = \inf_{k\in I}\mho_*(\eta^i_k)(\alpha).$$

Consider

$$\begin{aligned}\bigcap_{k\in I}\mho^*(\eta^i_k)(\alpha-\beta) &= \inf_{k\in I}\mho^*(\eta^i_k)(\alpha-\beta) \\ &\geqslant \inf_{k\in I} min\left\{\mho^*(\eta^i_k)(\alpha), \mho^*(\eta^i_k)(\beta)\right\} \\ &= min\left\{\inf_{k\in I}\mho^*(\eta^i_k)(\alpha), \inf_{k\in I}\mho^*(\eta^i_k)(\beta)\right\} \\ &= min\left\{\bigcap_{k\in I}\mho^*(\eta^i_k)(\alpha), \bigcap_{k\in I}\mho^*(\eta^i_k)(\beta)\right\}\end{aligned}$$

also

$$\begin{aligned}\bigcap_{k\in I}\mho^*(\eta^i_k)(\alpha+\beta-\alpha) &= \inf_{k\in I}\mho^*(\eta^i_k)(\alpha+\beta-\alpha) \\ &\geqslant \inf_{k\in I}\mho^*(\eta^i_k)(\beta) \\ &= \bigcap_{k\in I}\mho^*(\eta^i_k)(\beta)\end{aligned}$$

moreover

$$\begin{aligned}\bigcap_{k\in I}\mho^*(\eta^i_k)((\alpha+\beta)a\nu-\alpha a\nu) &= \inf_{k\in I}\mho^*(\eta^i_k)((\alpha+\beta)a\nu-\alpha a\nu) \\ &\geqslant \inf_{k\in I}\mho^*(\eta^i_k)(\beta)\end{aligned}$$

$$= \bigcap_{k\in I} \mho^*(\eta_k^i)(\beta)$$

Finally

$$\begin{aligned}\bigcap_{k\in I} \mho^*(\eta_k^i)(\alpha a\beta) &= \inf_{k\in I} \mho^*(\eta_k^i)(\alpha a\beta)\\ &\geqslant \inf_{k\in I} \mho^*(\eta_k^i)(\beta)\\ &= \bigcap_{k\in I} \mho^*(\eta_k^i)(\beta)\end{aligned}$$

Similarly we can prove for other case.
Hence proved. □

THEOREM 2.8. *Let $\mho(\eta_k^i)$ be a family of rough interval fuzzy ideals in $\mathfrak{N}$ then $\bigcup \mho(\eta_k^i)$ is also an rough interval valued fuzzy ideals in $\mathfrak{N}$.*

PROOF. Since $\mho(\eta_k^i)$ be a family of rough interval fuzzy ideals in $\mathfrak{N}$. Let $\alpha, \beta, \nu \in \mathfrak{N}$ and $a, b \in \gamma$. Then

$$\bigcup_{k\in I} \mho(\eta_k^i)\alpha = (\sup_{k\in I} \mho(\eta_k^i))(\alpha) = \sup_{k\in I} \mho(\eta_k^i)(\alpha)$$

i.e.,

$$\bigcup_{k\in I} \mho^*(\eta_k^i)\alpha = (\sup_{k\in I} \mho^*(\eta_k^i))(\alpha) = \sup_{k\in I} \mho^*(\eta_k^i)(\alpha)$$

and

$$\bigcup_{k\in I} \mho_*(\eta_k^i)\alpha = (\sup_{k\in I} \mho_*(\eta_k^i))(\alpha) = \sup_{k\in I} \mho_*(\eta_k^i)(\alpha).$$

Consider

$$\begin{aligned}\bigcup_{k\in I} \mho^*(\eta_k^i)(\alpha-\beta) &= \sup_{k\in I} \mho^*(\eta_k^i)(\alpha-\beta)\\ &\geqslant \sup_{k\in I} min\left\{\mho^*(\eta_k^i)(\alpha), \mho^*(\eta_k^i)(\beta)\right\}\\ &= min\left\{\sup_{k\in I} \mho^*(\eta_k^i)(\alpha), \sup_{k\in I} \mho^*(\eta_k^i)(\beta)\right\}\\ &= min\left\{\bigcap_{k\in I} \mho^*(\eta_k^i)(\alpha), \bigcup_{k\in I} \mho^*(\eta_k^i)(\beta)\right\}\end{aligned}$$

also

$$\begin{aligned}\bigcup_{k\in I} \mho^*(\eta_k^i)(\alpha+\beta-\alpha) &= \sup_{k\in I} \mho^*(\eta_k^i)(\alpha+\beta-\alpha)\\ &\geqslant \sup_{k\in I} \mho^*(\eta_k^i)(\beta)\\ &= \bigcup_{k\in I} \mho^*(\eta_k^i)(\beta)\end{aligned}$$

moreover

$$\begin{aligned}\bigcup_{k\in I} \mho^*(\eta_k^i)((\alpha+\beta)a\nu-\alpha a\nu) &= \sup_{k\in I} \mho^*(\eta_k^i)((\alpha+\beta)a\nu-\alpha a\nu)\\ &\geqslant \sup_{k\in I} \mho^*(\eta_k^i)(\beta)\\ &= \bigcup_{k\in I} \mho^*(\eta_k^i)(\beta)\end{aligned}$$

Finally

$$\begin{aligned}\bigcup_{k\in I}\mho^*(\eta_k^i)(\alpha a\beta) &= \sup_{k\in I}\mho^*(\eta_k^i)(\alpha a\beta)\\ &\geqslant \sup_{k\in I}\mho^*(\eta_k^i)(\beta)\\ &= \bigcup_{k\in I}\mho^*(\eta_k^i)(\beta)\end{aligned}$$

Similarly we can prove for other case.
Hence proved. □

DEFINITION 2.3. *If $\mho$ be congruence relation on $\mathfrak{N}$. An interval fuzzy bi-γ-ideal η^i is said to be an upper and lower interval rough fuzzy ideal of $\mathfrak{N}$ if the following condition are holds:*
1. $\mho^*$ is an interval fuzzy bi-γ-ideal of $\mathfrak{N}$.
2. $\mho_*$ is an interval fuzzy bi-γ-ideal of $\mathfrak{N}$.

THEOREM 2.9. *An interval fuzzy bi-γ-ideal of $\mathfrak{N}$ is an upper interval rough fuzzy bi-γ-ideal of $\mathfrak{N}$.*

PROOF. Let η^i be an interval fuzzy bi-γ-ideal of $\mathfrak{N}$. Then for all $\alpha, \beta, \nu \in \mathfrak{N}$ and $i, j \in \gamma$. Obviously

$$\mho^*(\eta^i)(\alpha-\beta) \geqslant min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\beta)\right\}$$

Consider

$$\begin{aligned}\mho^*(\eta^i)(\alpha i\beta j\nu) &= \bigvee_{\lambda\in[\alpha i\beta j\nu]_\mho}\eta^i(\nu)\\ &= \bigvee_{\nu\in[\alpha]_\mho\gamma[\beta]_\mho\gamma[\nu]_\mho}\eta^i(\lambda)\\ &= \bigvee_{a\in[\alpha]_\mho,b\in[\beta]_\mho,c\in[\nu]_\mho}\eta^i(a\gamma b\gamma c)\\ &\geqslant min\left\{\bigvee_{a\in[\alpha]_\mho}\eta^i(a), \bigvee_{c\in[\nu]_\mho}\eta^i(\nu)\right\}\\ &= min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\nu)\right\}\\ &\geqslant min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\nu)\right\}\end{aligned}$$

Hence proved. □

THEOREM 2.10. *An interval fuzzy bi-γ-ideal of $\mathfrak{N}$ is a upper interval rough fuzzy bi-γ-ideal of $\mathfrak{N}$.*

PROOF. Let η^i be an interval fuzzy bi-γ-ideal of $\mathfrak{N}$. Then for all $\alpha, \beta, \nu \in \mathfrak{N}$ and $i, j \in \gamma$. Obviously

$$\mho_*(\eta^i)(\alpha-\beta) \geqslant min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(\beta)\right\}$$

Consider

$$\begin{aligned}\mho_*(\eta^i)(\alpha i\beta j\nu) &= \bigwedge_{\lambda\in[\alpha i\beta j\nu]_\mho}\eta^i(\nu)\\ &= \bigwedge_{\nu\in[\alpha]_\mho\gamma[\beta]_\mho\gamma[\nu]_\mho}\eta^i(\lambda)\\ &= \bigwedge_{a\in[\alpha]_\mho,b\in[\beta]_\mho,c\in[\nu]_\mho}\eta^i(a\gamma b\gamma c)\end{aligned}$$

$$\geqslant min\left\{\bigwedge_{a\in[\alpha]_{\mho}}\eta^i(a), \bigwedge_{c\in[\nu]_{\mho}}\eta^i(\nu)\right\}$$
$$= min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(\nu)\right\}$$
$$\geqslant min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(\nu)\right\}$$

Hence proved. □

THEOREM 2.11. *Intersection of two interval rough fuzzy bi-γ-ideals is also an interval rough fuzzy bi-γ-ideal.*

PROOF. Let η^i and ξ^i be an interval rough fuzzy bi-γ-ideals in $\mathfrak{N}$. Then by $\mho_*(\eta^i)$, $\mho_*(\xi^i)$ and $\mho^*(\eta^i)$, $\mho^*(\xi^i)$ are fuzzy bi-γ-ideals of $\mathfrak{N}$. Then for all $\alpha, \beta, i, j \in \mathfrak{N}$ we have

$$\mho^*(\eta^i) \cap \mho^*(\xi^i)(\alpha i\beta j\nu)$$
$$= min\left\{\mho^*(\eta^i)(\alpha i\beta j\nu), \mho^*(\xi^i)(\alpha i\beta j\nu)\right\}$$
$$= min\left\{min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\eta^i)(\nu)\right\}, min\left\{\mho^*(\xi^i)(\alpha), \mho^*(\xi^i)(\nu)\right\}\right\}$$
$$= min\left\{min\left\{\mho^*(\eta^i)(\alpha), \mho^*(\xi^i)(\alpha)\right\}, min\left\{\mho^*(\eta^i)(\nu), \mho^*(\xi^i)(\nu)\right\}\right\}$$
$$\geqslant min\left\{\mho^*(\eta^i) \cap \mho^*(\xi^i)(\alpha), \mho^*(\eta^i) \cap \mho^*(\xi^i)(\nu)\right\}.$$

Similarly we have

$$\mho_*(\eta^i) \cap \mho_*(\xi^i)(\alpha i\beta j\nu)$$
$$= min\left\{\mho_*(\eta^i)(\alpha i\beta j\nu), \mho_*(\xi^i)(\alpha i\beta j\nu)\right\}$$
$$= min\left\{min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\eta^i)(\nu)\right\}, min\left\{\mho_*(\xi^i)(\alpha), \mho_*(\xi^i)(\nu)\right\}\right\}$$
$$= min\left\{min\left\{\mho_*(\eta^i)(\alpha), \mho_*(\xi^i)(\alpha)\right\}, min\left\{\mho_*(\eta^i)(\nu), \mho_*(\xi^i)(\nu)\right\}\right\}$$
$$\geqslant min\left\{\mho_*(\eta^i) \cap \mho_*(\xi^i)(\alpha), \mho_*(\eta^i) \cap \mho^*(\xi^i)(\nu)\right\}.$$

Hence proved. □

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