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On $\mathbf{N}_m$-$\mathbf{T}_{\frac{1}{2}}$-space

Selvaraj Ganesan

ABSTRACT. In this article, we study different properties of N_mg-closed sets in neutrosophic minimal structure. As applications to N_mg-closed set, we introduce N_m-$\mathrm{T}_{\frac{1}{2}}$-space and obtain some of their basic properties.

1. Introduction

The contribution of mathematics to the present-day technology in reaching to a fast trend cannot be ignored. The theories presented differently from classical methods in studies such as fuzzy set [**17**], intuitionistic set [**4**], soft set [**11**], neutrosophic set [**15**], etc., have great importance in this contribution of mathematics in recent years. Many works have been done on these sets by mathematicians in many areas of mathematics [**1, 2, 3, 6, 7, 8, 13**]. Neutrosophic set is described by three functions : a membership function, indeterminacy function and a nonmembership function that are independently related. The theories of neutrosophic set have achieved greater success in various areas such as medical diagnosis, database, topology, image processing and decision making problems. V. Popa and T. Noiri [**12**] introduced the notions of of minimal structure which is a generalization of a topology on a given nonempty set. And they introduced the notion of $\mathcal{M}$-continuous functions as functions defined between minimal structures. M. Karthika et al [**10**] introduced and studied neutrosophic minimal structure spaces. S. Ganesan and Smarandache [**5**] introduced and studied some new classes of neutrosophic minimal open sets. S. Ganesan et al [**9**] introduced and studied N_mg-closed sets in neutrosophic minimal structure spaces. In this article, we study different properties of

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N_mg-closed set in neutrosophic minimal structure. As applications to N_mg-closed set, we introduce N_m-$T_{\frac{1}{2}}$-space and obtain some of their basic properties.

2. Preliminaries

DEFINITION 2.1. [**12**] A subfamily m_x of the power set $\wp$(X) of a nonempty set X is called a minimal structure (in short, m-structure) on X if $\emptyset \in m_x$ and X $\in m_x$. By (X, m_x), we denote a nonempty set X with a minimal structure m_x on X and call it an m-space.
Each member of m_x is said to be m_x-open (or in short, m-open) and the complement of an m_x-open set is said to be m_x-closed (or in short, m-closed).

DEFINITION 2.2. [**14, 16**] A neutrosophic set (in short ns) K on a set X $\neq \emptyset$ is defined by K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ where P_K : X $\to$ [0,1], Q_K : X $\to$ [0,1] and R_K : X $\to$ [0,1] denotes the membership of an object, indeterminacy and non-membership of an object, for each a $\in$ X to K, respectively and $0 \leqslant P_K(a) + Q_K(a) + R_K(a) \leqslant 3$ for each a $\in$ X.

DEFINITION 2.3. [**13**] Let K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ be a ns. We must introduce the ns $0_\sim$ and $1_\sim$ in X as follows:
$0_\sim$ may be defined as:

(1) $0_\sim = \{\prec x, 0, 0, 1 \succ : x \in X\}$
(2) $0_\sim = \{\prec x, 0, 1, 1 \succ : x \in X\}$
(3) $0_\sim = \{\prec x, 0, 1, 0 \succ : x \in X\}$
(4) $0_\sim = \{\prec x, 0, 0, 0 \succ : x \in X\}$

$1_\sim$ may be defined as:

(1) $1_\sim = \{\prec x, 1, 0, 0 \succ : x \in X\}$
(2) $1_\sim = \{\prec x, 1, 0, 1 \succ : x \in X\}$
(3) $1_\sim = \{\prec x, 1, 1, 0 \succ : x \in X\}$
(4) $1_\sim = \{\prec x, 1, 1, 1 \succ : x \in X\}$

PROPOSITION 2.1. [**13**] *For any ns S, then the following conditions are holds:*
(1) $0_\sim \leqslant S$, $0_\sim \leqslant 0_\sim$.
(2) $S \leqslant 1_\sim$, $1_\sim \leqslant 1_\sim$.

DEFINITION 2.4. [**13**] Let K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ be a ns.

(1) A ns K is an empty set i.e., K = $0_\sim$ if 0 is membership of an object and 0 is an indeterminacy and 1 is an non-membership of an object respectively. i.e., $0_\sim = \{x, (0, 0, 1) : x \in X\}$
(2) A ns K is a universal set i.e., K = $1_\sim$ if 1 is membership of an object and 1 is an indeterminacy and 0 is an non-membership of an object respectively. $1_\sim = \{x, (1, 1, 0) : x \in X\}$
(3) $K_1 \cup K_2$ = $\{$a, max $\{P_{K_1}(a), P_{K_2}(a)\}$, max $\{Q_{K_1}(a), Q_{K_2}(a)\}$, min $\{R_{K_1}(a), R_{K_2}(a)\}$: a $\in$ X$\}$

(4) $K_1 \cap K_2 = \{a, \min \{P_{K_1}(a), P_{K_2}(a)\}, \min \{Q_{K_1}(a), Q_{K_2}(a)\}, \max \{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$
(5) $K^C = \{\prec a, R_K(a), 1 - Q_K(a), P_K(a) \succ : a \in X\}$

Definition 2.5. [**13**] A neutrosophic topology (nt) in Salama's sense on a nonempty set X is a family τ of ns in X satisfying three axioms:

(1) Empty set ($0_\sim$) and universal set ($1_\sim$) are members of τ.
(2) $K_1 \cap K_2 \in \tau$ where $K_1, K_2 \in \tau$.
(3) $\cup K_\delta \in \tau$ for every $\{K_\delta : \delta \in \Delta\} \leqslant \tau$.

Each ns in nt are called neutrosophic open sets. Its complements are called neutrosophic closed sets.

Definition 2.6. [**10**] Let the neutrosophic minimal structure space over a universal set X be denoted by N_m. N_m is said to be neutrosophic minimal structure space (in short, nms) over X if it satisfying following the axiom: $0_\sim, 1_\sim \in N_m$. A family of neutrosophic minimal structure space is denoted by (X, N_{mX}).
Note that neutrosophic empty set and neutrosophic universal set can form a topology and itis known as neutrosophic minimal structure space.
Each ns in nms is neutrosophic minimal open set. The complement of neutrosophic minimal open set is neutrosophic minimal closed set.

Remark 2.1. [**10**] Each ns in nms is neutrosophic minimal open set.
The complement of neutrosophic minimal open set is neutrosophic minimal closed set.

Definition 2.7. [**10**] A is N_m-closed if and only if $N_m cl(A) = A$. Similarly, A is a N_m-open if and only if $N_m int(A) = A$.

Definition 2.8. [**10**] Let N_m be any nms and A be any neutrosophic set. Then

(1) Every $A \in N_m$ is open and its complement is closed.
(2) N_m-closure of A = min {F : F is a neutrosophic minimal closed set and $F \geqslant A$} and it is denoted by $N_m cl(A)$.
(3) N_m-interior of A = max {F : F is a neutrosophic minimal open set and F $\leqslant A$} and it is denoted by $N_m int(A)$.

In general $N_m int(A)$ is subset of A and A is a subset of $N_m cl(A)$.

Proposition 2.2. [**10**] *Let R and S are any ns of nms N_m over X. Then*

(1) $N_m^C = \{0, 1, R_i^C\}$ *where* R_i^c *is a complement of ns* R_i.
(2) $X - N_m int(S) = N_m cl(X - S)$.
(3) $X - N_m cl(S) = N_m int(X - S)$.
(4) $N_m cl(R^C) = (N_m\ cl(R))^C = N_m int(R)$.
(5) *N_m closure of an empty set is an empty set and N_m closure of a universal set is a universal set. Similarly, N_m interior of an empty set and universal set respectively an empty and a universal set.*
(6) *If S is a subset of R then* $N_m cl(S) \leqslant N_m cl(R)$ *and* $N_m int(S) \leqslant N_m int(R)$.
(7) $N_m cl(N_m cl(R)) = N_m cl(R)$ *and* $N_m int(N_m int(R)) = N_m int(R)$.
(8) $N_m cl(R \vee S) = N_m cl(R) \vee N_m cl(S)$.

(9) $N_m cl(R \wedge S) = N_m cl(R) \wedge N_m cl(S)$.

DEFINITION 2.9. [**10**] Let (X, N_{mX}) be nms.

(1) Arbitrary union of neutrosophic minimal open sets in (X, N_{mX}) is neutrosophic minimal open. (Union Property).
(2) Finite intersection of neutrosophic minimal open sets in (X, N_{mX}) is neutrosophic minimal open. (intersection Property).

DEFINITION 2.10. [**9**] Let (X, N_{mX}) be a nms and A $\leqslant$ X is called an Neutrosophic minimal generalized closed set (in short, N_m-g-closed set) if $N_m$cl(A) $\leqslant$ U whenever A $\leqslant$ U and U is N_m-open.
The complement of an N_m-g-closed set is called an N_m-g-open set.

3. N_m-$T_{\frac{1}{2}}$-spaces

DEFINITION 3.1. Let (X, N_{mX}) be a nms. Then, X is said to be N_m-$T_{\frac{1}{2}}$-space if every N_mg-closed in X is N_m-closed.

EXAMPLE 3.1. Let X = {a} with $N_m = \{0_\sim, A, 1_\sim\}$ and $N_m^C = \{1_\sim, B, 0_\sim\}$ where
$A = \prec (0.9, 0.4, 0.7) \succ$; $B = \prec (0.7, 0.6, 0.1) \succ$
We know that $0_\sim = \{\prec x, 0, 0, 1 \succ : x \in X\}$, $1_\sim = \{\prec x, 1, 1, 0 \succ : x \in X\}$ and $0_\sim^C = \{\prec x, 1, 1, 0 \succ : x \in X\}$, $1_\sim^C = \{\prec x, 0, 0, 1 \succ : x \in X\}$.
Now N_mg-closed sets is $V = \prec (0.7, 0.6, 0.1) \succ$
Thus, (X, N_{mX}) is a N_m-$T_{\frac{1}{2}}$-spaces.

THEOREM 3.1. *If $N_m cl(\{x\}) \wedge A \neq 0_\sim$ holds for every $x \in N_m cl(A)$, then $N_m cl(A) \setminus A$ does not contain a non empty N_m-closed set.*

PROOF. Suppose there exists a non empty N_m-closed set F such that F $\leqslant$ $N_m$cl(A) $\setminus$ A. Let x $\in$ F, then x $\in$ $N_m$cl(A). It follows that F $\wedge$ A = $N_m$cl(F) $\wedge$ A $\geqslant$ $N_m$cl({x}) $\wedge$ A $\neq 0_\sim$. Hence, F $\wedge$ A $\neq 0_\sim$. This is a contradiction. Thus, F = $0_\sim$. □

THEOREM 3.2. *Let (X, N_{mX}) be a nms and A $\leqslant$ X. Then*

(1) *$x \in N_m cl(A)$ if and only if $A \wedge V \neq 0_\sim$ for every N_m-open set V containing x.*
(2) *$x \in N_m int(A)$ if and only if there exists an N_m-open set U such that U $\leqslant$ A.*

PROOF. (1) Suppose there is an N_m-open set V containing x such that A $\wedge$ V = $0_\sim$. Then X − V is an N_m-closed set such that A $\leqslant$ X − V, x $\notin$ X − V. This implies x $\notin$ $N_m$cl(A).
The reverse relation is obvious.
(2) Obvious. □

THEOREM 3.3. *Let (X, N_{mX}) be a nms. Then, for each x $\in$ X, either {x} is N_m-closed or X $\setminus$ {x} is N_mg-closed.*

PROOF. Suppose that {x} is not N_m-closed, then by Definition 2.8 (1), X ⟍ {x} is not N_m-open. Let L be any N_m-open set such that X ⟍ {x} $\leqslant$ L, so L = X. Hence, $N_m$cl(X ⟍ {x}) $\leqslant$ L. Thus, X ⟍ {x} is N_mg-closed. □

THEOREM 3.4. *Let (X, N_{mX}) be a nms. Then, X is a N_m-$T_{\frac{1}{2}}$ if and only if {x} is N_m-closed or N_m-open, for each $x \in X$.*

PROOF. Suppose {x} is not N_m-closed. Then it follows from assumption and Theorem 3.3, that {x} is N_m-open.
Conversely, let F be N_mg-closed set in X and x be any point in $N_m$cl(F), then {x} is N_m-open or N_m-closed.

(1) Suppose {x} is N_m-open. Then by Theorem 3.2(1), we have {x} $\wedge$ F $\neq 0_\sim$ and hence x $\in$ F. This implies $N_m$cl(F) $\leqslant$ F, therefore F is N_m-closed.
(2) Suppose {x} is N_m-closed. Assume x $\notin$ F, then x $\in$ $N_m$cl(F) ⟍ F. This is not possible by Theorem 3.1. Thus, we have x $\in$ F. Therefore, $N_m$cl(F) = F and hence F is N_m-closed.

□

THEOREM 3.5. *A subset A is N_mg-closed then $N_m$cl(A) − A contains no nonempty N_m-closed set.*

PROOF. Necessity. Suppose that A is N_mg-closed. Let S be a N_m-closed subset of $N_m$cl(A) − A. Then A $\leqslant S^c$. Since A is N_m-closed, we have $N_m$cl(A) $\leqslant S^c$. Consequently, S $\leqslant (N_m\text{cl(A)})^c$. Hence, S $\leqslant$ $N_m$cl(A) $\wedge$ $(N_m\text{cl(A)})^c = 0_\sim$. Therefore S is empty. □

PROPOSITION 3.1. *If A is N_mg-closed in (X, N_{mX}) and A $\leqslant$ B $\leqslant$ $N_m$cl(A), then B is also a N_mg-closed in (X, N_{mX}).*

PROOF. Let U be a N_m-open set of (X, N_{mX}) such that B $\leqslant$ U. Then A $\leqslant$ U. Since A is N_mg-closed, we get, $N_m$cl(A) $\leqslant$ U. Now $N_m$cl(B) $\leqslant$ $N_m$cl($N_m$cl(A)) = $N_m$cl(A) $\leqslant$ U. Therefore, B is also a N_mg-closed in (X, N_{mX}). □

PROPOSITION 3.2. *Let A $\leqslant$ L $\leqslant$ X and suppose that A is N_mg-closed in (X, N_{mX}). Then A is N_mg-closed relative to L.*

PROOF. Let A $\leqslant$ L $\wedge$ U, where U is N_m-open in (X, N_{mX}). Then A $\leqslant$ U and hence $N_m$cl(A) $\leqslant$ U. This implies that L $\wedge$ $N_m$cl(A) $\leqslant$ L $\wedge$ U. Thus A is N_mg-closed relative to L. □

DEFINITION 3.2. The intersection of all N_m-open subsets of (X, N_{mX}) containing A is called the N_m-kernel of A and denoted by N_m-ker(A).

LEMMA 3.1. *A subset A of (X, N_{mX}) is N_mg-closed if and only if $N_m$cl(A) $\leqslant$ N_m-ker(A).*

PROOF. Suppose that A is N_mg-closed. Then $N_m$cl(A) $\leqslant$ U whenever A $\leqslant$ U and U is N_m-open. Let x $\in$ $N_m$cl(A). If x $\notin$ N_m-ker(A), then there is a N_m-open set U containing A such that x $\notin$ U. Since U is a N_m-open set containing A, we have x $\notin$ $N_m$cl(A) and this is a contradiction.

Conversely, let $N_m cl(A) \leqslant N_m$-ker(A). If U is any N_m-open set containing A, then $N_m cl(A) \leqslant N_m$-ker(A) $\leqslant$ U. Therefore, A is N_mg-closed. □

THEOREM 3.6. *For a space (X, N_{mX}) the following properties are equivalent:*

(1) *(X, N_{mX}) is a N_m-$T_{\frac{1}{2}}$-space.*
(2) *Every singleton subset of (X, N_{mX}) is either N_m-closed or N_m-open.*

PROOF. (1) → (2). Assume that for some x ∈ X, the set {x} is not a N_m-closed in (X, N_{mX}). Then the only N_m-open set containing $\{x\}^c$ is X and so $\{x\}^c$ is N_mg-closed in (X, N_{mX}). By assumption $\{x\}^c$ is N_m-closed in (X, N_{mX}) or equivalently {x} is N_m-open.

(2) → (1). Let A be a N_mg-closed subset of (X, N_{mX}) and let x ∈ $N_m$cl(A). By assumption {x} is either N_m-closed or N_m-open.

Case (a) Suppose that {x} is N_m-closed. If x ∉ A, then $N_m$cl(A) − A contains a nonempty N_m-closed set {x}, which is a contradiction to Theorem 3.5. Therefore x ∈ A.

Case (b) Suppose that {x} is N_m-open. Since x ∈ $N_m$cl(A), $\{x\} \wedge A \neq \phi$ and so x ∈ A. Thus in both case, x ∈ A and therefore $N_m cl(A) \leqslant A$ or equivalently A is a N_m-closed set of (X, N_{mX}). □

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Selvaraj Ganesan, Assistant Professor, PG & Research Department of Mathematics, Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India. (Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India)

Email address: sgsgsgsgsg77@gmail.com