BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., **12**(3)(2022), 453-460
DOI: 10.7251/BIMVI2203453R

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

ON SOME NANO SETS AND A DECOMPOSITION OF NANO CONTINUITY VIA IDEALIZATION

Ilangovan Rajasekaran

Abstract. We introduce the notions of $n\mathcal{Q}_I$-set and $n\mathcal{SB}_I$-sets. Then we investigate properties of $n\mathcal{SB}_I$-sets. Additionally, we obtain a new decomposition via idealization by using $n\mathcal{SB}_I$-sets.

1. Introduction

An ideal I [**12**] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

(1) $A \in I$ and $B \subset A$ imply $B \in I$ and
(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [**3**]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [**11**] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X, τ, I). We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$.

Some types of new notions in the concept of ideal nanotopological spaces were introduced by Parimala et al. [**5, 6**] and Rajasekaran et al. [**8, 9, 10**].

2010 *Mathematics Subject Classification.* Primary 54C05; Secondary 54C08, 54C10.
Key words and phrases. α-nI-open sets, pre-nI-open sets, b-nI-open sets and β-nI-open sets, $n\mathcal{Q}_I$-sets, $n\mathcal{SB}_I$-sets and $\mathcal{R}$-nI-sets.
Communicated by Daniel A. Romano.

In this article a new notions of $n\mathcal{Q}_I$-set and $n\mathcal{SB}_I$-sets are given. Then we investigate properties of $n\mathcal{SB}_I$-sets. Additionally, we obtain a new decomposition via idealization by using $n\mathcal{SB}_I$-sets.

2. Preliminaries

Definition 2.1. [**7**] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.
(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [**4**] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,
(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

Through out this paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset H of U are denoted by $I_n(H)$ and $C_n(H)$, respectively.

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [**5**] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [**5**] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.3. [**5**] Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^\star_n$ for $A^\star_n(I, \mathcal{N})$.

Theorem 2.1. [**5**] *Let $(U,\mathcal{N},I)$ be a space and A and B be subsets of U. Then*
(1) $A \subseteq B \Rightarrow A^\star_n \subseteq B^\star_n$,
(2) $A^\star_n = n\text{-}cl(A^\star_n) \subseteq n\text{-}cl(A)$ ($A^\star_n$ *is a n-closed subset of n-$cl(A)$*),
(3) $(A^\star_n)^\star_n \subseteq A^\star_n$,
(4) $(A \cup B)^\star_n = A^\star_n \cup B^\star_n$,
(5) $V \in \mathcal{N} \Rightarrow V \cap A^\star_n = V \cap (V \cap A)^\star_n \subseteq (V \cap A)^\star_n$,
(6) $J \in I \Rightarrow (A \cup J)^\star_n = A^\star_n = (A - J)^\star_n$.

Theorem 2.2. [**5**] *Let $(U,\mathcal{N},I)$ be a space with an ideal I and $A \subseteq A^\star_n$, then $A^\star_n = C_n(A^\star_n) = C_n(A)$.*

Definition 2.4. [**5**] Let $(U,\mathcal{N},I)$ be a space. The set operator $C^\star_n$ called a nano $\star$-closure is defined by $C^\star_n(A) = A \cup A^\star_n$ for $A \subseteq U$.

It can be easily observed that $C^\star_n(A) \subseteq C_n(A)$.

Theorem 2.3. [**6**] *In a space $(U,\mathcal{N},I)$, if A and B are subsets of U, then the following results are true for the set operator $C^\star_n$.*
(1) $A \subseteq C^\star_n(A)$,
(2) $C^\star_n(\phi) = \phi$ *and* $C^\star_n(U) = U$,
(3) *If* $A \subset B$, *then* $C^\star_n(A) \subseteq C^\star_n(B)$,
(4) $C^\star_n(A) \cup C^\star_n(B) = C^\star_n(A \cup B)$,
(5) $C^\star_n(C^\star_n(A)) = C^\star_n(A)$.

Definition 2.5. A subset H of nano space is said to be a
(1) nano semi-open (resp. ns-open) [**4**] if $H \subseteq C_n(I_n(H))$.
(2) nano pre-open (resp. np-open) [**4**] if $H \subseteq I_n(C_n(H))$.
(3) nano α-open (resp. $n\alpha$-open) [**4**] if $H \subseteq I_n(C_n(I_n(H)))$.
(4) nano β-open (resp. $n\beta$-open) [**4**] if $H \subseteq C_n(I_n(C_n(H)))$.
(5) nano t-set (resp. nt-set) if [**2**] $I_n(H) = I_n(C_n(H))$.
(6) nano $\mathcal{B}$-set (resp. $n\mathcal{B}$-set) if [**2**] $H = H_1 \cap H_2$, where H_1 is n-open and H_2 is nt-set.
(7) nano $\mathcal{Q}$-set (resp. $n\mathcal{Q}$-set) [**9**] if $I_n(C_n(H)) = C_n(I_n(H))$.
(8) nano strong $\mathcal{B}$-set (resp. $n\mathcal{SB}$-set) [**9**] if $H = H_1 \cap H_2$, where H_1 is n-open and H_2 is both nt-set and $n\mathcal{Q}$-set.

Definition 2.6. [**6**] A subset H of a ideal nano topological space $(U,\mathcal{N},I)$ is said to be nano-I-open (resp. nI-open) if $H \subseteq I_n(H^\star_n)$.

Definition 2.7. [**8**] A subset H of ideal nano topological space $(U,\mathcal{N},I)$ is said to be a
(1) nano α-I-open (resp. α-nI-open) if $H \subseteq I_n(C^\star_n(I_n(H)))$,
(2) nano semi-I-open (resp. semi-nI-open) if $H \subseteq C^\star_n(I_n(H))$,
(3) nano pre-I-open (resp. *pre*-nI-open) if $H \subseteq I_n(C^\star_n(H))$,
(4) nano β-I-open (resp. β-nI-open) if $H \subset C_n(I_n(C^\star_n(H)))$.

Definition 2.8. [**10**] A subset H of ideal nano topological space is said to be a
(1) nano t-I-set (resp. t-nI-set) if $I_n(H) = I_n(C^\star_n(H))$.

(2) nano $\mathcal{R}$-I-set (resp. $\mathcal{R}$-nI-set) if $H = H_1 \cap H_2$, where H_1 is n-open and H_2 is t-nI-set.

LEMMA 2.1. [8] *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space and O a subset of U. If H is n-open in $(U, \mathcal{N}, I)$, then $H \cap C_n^{\star}(O) \subseteq C_n^{\star}(H \cap O)$.*

DEFINITION 2.9. [1] A function $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ is said to be pre-nI-continuous (resp. semi-nI-continuous, α-nI-continuous and β-nI-continuous) if for every $H_2 \in \mathcal{N}'$, $f^{-1}(H_2)$ is pre-nI-open (resp. semi-nI-open, α-nI-open and β-nI-open) in $(U, \mathcal{N}, I)$.

3. Some nano sets in ideal nano space

DEFINITION 3.1. A subset H of ideal nano topological space is said to be

(1) nano $\mathcal{Q}_I$-set (resp. $n\mathcal{Q}_I$-set) if $I_n(C_n^{\star}(H)) = C_n(I_n(H))$.
(2) nano strong $\mathcal{B}_I$-set (resp. $n\mathcal{SB}_I$-set) if $H = H_1 \cap H_2$, where H_1 is n-open and H_2 is both t-nI-set and $n\mathcal{Q}_I$-set.

Note :

(1) nano $\mathcal{Z}$-set (resp. n$\mathcal{Z}$-set) if H is both nt-set and $n\mathcal{Q}$-set.
(2) nano $\mathcal{Z}_I$-set (resp. n$\mathcal{Z}_I$-set) if H is both t-nI-set and $\mathcal{Q}_I$-set.

REMARK 3.1. In an ideal nano topological space, t-nI-sets and $n\mathcal{Q}_I$-sets are independent.

EXAMPLE 3.1. Let $U = \{10, 20, 30, 40\}$ with $U/R = \{\{10, 20, 30\}, \{40\}\}$ and $X = \{10, 30, 40\}$. Then $\mathcal{N} = \{\phi, U, \{40\}, \{10, 30\}, \{10, 30, 40\}\}$ if the ideal $I = \{\phi, \{30\}, \{40\}, \{30, 40\}\}$.

(1) The set $H = \{10, 30, 40\}$ is $n\mathcal{Q}_I$-set which is not a t-nI-set. For $H = \{10, 30, 40\} \subseteq U$, since $H_n^{\star} = \{10, 20, 30\}$ and $C_n^{\star}(H) = H \cup H_n^{\star} = U$, we have $I_n(C_n^{\star}(H)) = U$. Furthermore, sine $I_n(H) = H = \{10, 30, 40\}$, we have $C_n(I_n(H)) = C_n(H) = U$. Consequently, $I_n(C_n^{\star}(H)) = U = C_n(I_n(H))$ and hence H is a $n\mathcal{Q}_I$-set. On the other hand, since $I_n(C_n^{\star}(H)) = U \neq \{10, 30, 40\} = I_n(H)$, H is not a t-nI-set.
(2) The set $H = \{20, 40\}$. Then H is a t-nI-set which is not a $n\mathcal{Q}_I$-set. For $H = \{20, 40\} \subseteq U$, since $H_n^{\star} = \{20\}$ and $C_n^{\star}(H) = H \cup H_n^{\star} = \{20, 40\}$, we have $I_n(C_n^{\star}(H)) = \{20\} = I_n(H)$ and hence H is a t-nI-set. On the other hand, since $C_n(I_n(H)) = \{20, 40\} \neq \{40\} = I_n(C_n^{\star}(H))$, H is not a $n\mathcal{Q}_I$-set. Therefore, H is not a $n\mathcal{SB}_I$-set.

PROPOSITION 3.1. *For a subset H of an ideal nano topological space. The next conditions are hold:*

(1) *If H is n$\mathcal{Z}_I$-set, then H is $n\mathcal{SB}_I$-set.*
(2) *If H is $n\mathcal{SB}_I$-set, then H is $\mathcal{R}$-nI-set.*
(3) *If H is n-open set, then H is $n\mathcal{SB}_I$-set.*

PROOF.
(2) The proof is obvious.
(1) , (3) Since $U \in \mathcal{N} \cap$ (family of t-nI-set) $\cap$ (family of $n\mathcal{Q}_I$-set),

the proof is obvious. □

PROPOSITION 3.2. *For a subset H of an ideal nano topological space. The next conditions are hold:*

1. *If H is $n\mathcal{Z}$-set, then H is $n\mathcal{Z}_I$-set.*
2. *If H is $n\mathcal{SB}$-set, then H is $n\mathcal{SB}_I$-set.*
3. *If H is $n\mathcal{B}$-set, then H is $\mathcal{R}$-nI-set.*

PROOF.

1. Let H be a $n\mathcal{Z}$-set. Then $I_n(C_n^\star(H)) \subseteq I_n(C_n(H)) = I_n(H)$ and hence $I_n(C_n^\star(H)) = I_n(H)$. Moreover, we have $C_n(I_n(H)) = I_n(C_n(H)) = I_n(H) = I_n(C_n^\star(H))$ and hence H is a $n\mathcal{Z}_I$-set.
2. Let H be a $n\mathcal{SB}$-set. Then, $H = H_1 \cap H_2$, where H_1 is n-open and H_2 is $n\mathcal{Z}$-set. By (1), H_2 is a $n\mathcal{Z}_I$-set and H is a $n\mathcal{SB}_I$-set.
3. The proof is every nt-set is t-nI-set for let H be a nt-set. Then, we have $I_n(C_N^\star(H)) = I_n(H_n^\star \cup H) \subseteq I_n(C_n(H) \cup H) = I_n(C_n(H)) = I_n(H)$, $C_n^\star(H) \supseteq H$ and $I_n(C_n^\star(H)) \supseteq I_n(H)$.
   Therefore, we obtain $I_n(C_n^\star(H)) = I_n(H)$.

□

REMARK 3.2. The following diagram, where none of the implications is reversible. as shown by Examples below.

$$\begin{array}{ccccc} n\mathcal{Z}\textbf{-set} & \longrightarrow & n\mathcal{SB}\textbf{-set} & \longrightarrow & n\mathcal{B}\textbf{-set} \\ \downarrow & & \downarrow & & \downarrow \\ n\mathcal{Z}_I\textbf{-set} & \longrightarrow & n\mathcal{SB}_I\textbf{-set} & \longrightarrow & \mathcal{R}\textbf{-}nI\textbf{-set} \end{array}$$

EXAMPLE 3.2. Let U=$\{10, 20, 30, 40, 50\}$ with $U/R = \{\{10\}, \{20, 30\}, \{40, 50\}\}$ and $X = \{10, 20\}$. Then $\mathcal{N} = \{\phi, U, \{10\}, \{20, 30\}, \{10, 20, 30\}\}$ if the ideal $I = \{\phi, \{20\}\}$.

1. then the set $\{10\}$ is $n\mathcal{SB}$-set is not $n\mathcal{Z}_I$-set.
2. then the set $\{20, 40, 50\}$ is $n\mathcal{Z}_I$-set is not $n\mathcal{B}$-set.

REMARK 3.3. The following diagram, where none of the implications is reversible.

$$\begin{array}{ccccc} n\textbf{-open} & \longrightarrow & n\mathcal{SB}_I\textbf{-set} & \longrightarrow & \mathcal{R}\textbf{-}nI\textbf{-set} \\ \downarrow & & & & \\ \alpha\textbf{-}nI\textbf{-open} & \longrightarrow & \textbf{pre-}nI\textbf{-open} & & \\ \downarrow & & \downarrow & & \\ \textbf{semi-}nI\textbf{-open} & \longrightarrow & \beta\textbf{-}nI\textbf{-open} & & \end{array}$$

EXAMPLE 3.3. In the Example 3.2,

1. then the set $\{20\}$ is $\mathcal{R}$-nI-set is not $n\mathcal{SB}_I$-set.
2. then the set $\{20, 30\}$ is $n\mathcal{SB}_I$-set is not n-open.

PROPOSITION 3.3. *For a subset H of an ideal nano topological space. The next conditions are equivalent:*

1. *H is n-open set.*

(2) *H is α-nI-open set and $n\mathcal{SB}_I$-set.*
(3) *H is pre-nI-open (or semi-nI-open) set and $n\mathcal{SB}_I$-set.*
(4) *H is β-nI-open set and $n\mathcal{SB}_I$-set.*

Proof.
We prove only the implication (4) $\Longrightarrow$ (1), the other implications (1) $\Longrightarrow$ (2), (2) $\Longrightarrow$ (3) and (3) $\Longrightarrow$ (4) being obvious from Diagram by Remark 3.3.

(4) $\Longrightarrow$ (1) Let H be β-nI-open and a $n\mathcal{SB}_I$-set.

Then $H = H_1 \cap H_2$, where H_1 is n-open and H_2 is (family of t-nI-set) $\cap$ (family of $n\mathcal{Q}_I$-set).

$H \subseteq C_n(I_n(C_n^\star(H))) = C_n(I_n(C_n^\star(H_1 \cap H_2))) \subseteq C_n(I_n(C_n^\star(H_1) \cap C_n^\star(H_2))) = C_n(I_n(C_n^\star(H_1)) \cap I_n(C_n^\star(H_2))) \subseteq C_n(I_n(C_n^\star(H_1))) \cap C_n(I_n(C_n^\star(H_2))) = C_n(I_n(C_n^\star(H_1))) \cap C_n(I_n(H_2)) = C_n(I_n(C_n^\star(H_1))) \cap I_n(H_2)$.

Hence $H \subseteq H_1$ and $I_n(H) \subseteq H \subseteq H_1 \cap I_n(H_2) = I_n(H)$. Thus we obtain $H \in \mathcal{N}$.

□

4. Decomposition of continuity via ideal nano space

Definition 4.1. A function $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ is said to be $n\mathcal{Z}_I$-continuous (resp. $n\mathcal{SB}_I$-continuous and $\mathcal{R}$-nI-continuous) if for every $H_2 \in \mathcal{N}'$, $f^{-1}(H_2)$ is a $n\mathcal{Z}_I$-set (resp. $n\mathcal{SB}_I$-set and $\mathcal{R}$-nI-set) in $(U, \mathcal{N}, I)$.

Proposition 4.1. *For a function $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$, the following properties hold: If*

(1) *f is $n\mathcal{Z}_I$-continuous, then f is $n\mathcal{SB}_I$-continuous.*
(2) *f is $n\mathcal{SB}_I$-continuous, then f is $\mathcal{R}$-nI-continuous.*
(3) *f is n-continuous, then f is $n\mathcal{SB}_I$-continuous.*

Proof.
The proof is obvious from Proposition 3.1. □

Remark 4.1. The converse of Proposition 4.1 need not be true as the following Example shows.

Example 4.1. (1) Let $U = \{10, 20, 30\}$ with $U/R = \{\{10\}, \{20, 30\}\}$ and $X = \{20, 30\}$. Then $\mathcal{N} = \{\phi, U, \{20, 30\}\}$ if the ideal $I = \{\phi\}$ and Let $V = \{5, 6, 7\}$ with $U/R = \{\{5\}, \{6, 7\}\}$ and $Y = \{6, 7\}$.
Then $\mathcal{N}' = \{V, \phi, \{6, 7\}\}$
Let $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ be defined by $f(10) = 5, f(20) = 6$ and $f(30) = 7$. Then f is $n\mathcal{SB}_I$-continuous but not $n\mathcal{Z}_I$-continuous.

(2) $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ be a function defined as follows; $f(10) = f(30) = 20$ and $f(20) = f(40) = 10$. Then f is $\mathcal{R}$-nI-continuous but not $n\mathcal{SB}_I$-continuous by Example 3.1(2).

(3) Let $U = \{10, 20, 30\}$ with $U/R = \{\{10\}, \{20, 30\}\}$ and $X = \{20, 30\}$. Then $\mathcal{N} = \{\phi, U, \{20, 30\}\}$ if the ideal $I = \{\phi\}$ and Let $V = \{5, 6, 7\}$ with $U/R = \{\{5\}, \{6, 7\}\}$ and $Y = \{5\}$.
Then $\mathcal{N}' = \{V, \phi, \{5\}\}$

Let $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ be defined by $f(10) = 5, f(20) = 6$ and $f(30) = 7$.
Then f is $n\mathcal{SB}_I$-continuous but not n-continuous.

(4) Let $U = \{10, 20, 30\}$ with $U/R = \{\{10\}, \{20, 30\}\}$ and $X = \{20, 30\}$. Then $\mathcal{N} = \{\phi, U, \{20, 30\}\}$ if the ideal $I = \{\phi\}$ and Let $V = \{5, 6, 7\}$ with $U/R = \{\{5\}, \{6, 7\}\}$ and $Y = \{5\}$.
Then $\mathcal{N}' = \{V, \phi, \{5\}\}$
Let $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ be defined by $f(10) = 5, f(20) = 6$ and $f(30) = 7$.
Then f is $n\mathcal{SB}_I$-continuous but not β-nI-continuous.

(5) Let $U = \{10, 20, 30\}$ with $U/R = \{\{10\}, \{20, 30\}\}$ and $X = \{20, 30\}$. Then $\mathcal{N} = \{\phi, U, \{20, 30\}\}$ if the ideal $I = \{\phi\}$ and Let $V = \{5, 6, 7\}$ with $U/R = \{\{5\}, \{6, 7\}\}$ and $Y = \{6, 7\}$.
Then $\mathcal{N}' = \{V, \phi, \{6\}, \{5, 7\}\}$
Let $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ be defined by $f(10) = 5, f(20) = 6$ and $f(30) = 7$.
Then f is β-nI-continuous but not $\mathcal{R}$-nI-continuous and hence $n\mathcal{SB}_I$-continuous.

Remark 4.2. In a ideal nano topological space, $n\mathcal{SB}_I$-continuous and β-nI-continuous are independent.

Example 4.2. (1) Let $U = \{10, 20, 30\}$ with $U/R = \{\{10\}, \{20, 30\}\}$ and $X = \{20, 30\}$. Then $\mathcal{N} = \{\phi, U, \{20, 30\}\}$ if the ideal $I = \{\phi\}$ and Let $V = \{5, 6, 7\}$ with $U/R = \{\{5\}, \{6, 7\}\}$ and $Y = \{5\}$.
Then $\mathcal{N}' = \{V, \phi, \{5\}\}$
Let $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ be defined by $f(10) = 5, f(20) = 6$ and $f(30) = 7$. Then f is $n\mathcal{SB}_I$-continuous but not β-nI-continuous.

(2) Let $U = \{10, 20, 30\}$ with $U/R = \{\{10\}, \{20, 30\}\}$ and $X = \{20, 30\}$. Then $\mathcal{N} = \{\phi, U, \{20, 30\}\}$ if the ideal $I = \{\phi\}$ and Let $V = \{5, 6, 7\}$ with $U/R = \{\{6\}, \{5, 7\}\}$ and $Y = \{6, 7\}$.
Then $\mathcal{N}' = \{V, \phi, \{6\}, \{5, 7\}\}$
Let $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$ be defined by $f(10) = 5, f(20) = 6$ and $f(30) = 7$. Then f is β-nI-continuous but not $n\mathcal{SB}_I$-continuous.

Theorem 4.1. *For a function $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$, the following properties are equivalent:*

(1) *f is nano continuous.*
(2) *f is α-nI-continuous and $n\mathcal{SB}_I$-continuous.*
(3) *f is pre-nI-continuous (or semi-nI-continuous) and $n\mathcal{SB}_I$-continuous.*
(4) *f is β-nI-continuous and $n\mathcal{SB}_I$-continuous.*

Proof.
This is an immediate consequence of Proposition 3.3. □

Corollary 4.1. *Let $(U, \mathcal{N}, I)$ be an ideal nano topological space and $I = \{\phi\}$. For a function $f : (U, \mathcal{N}, I) \to (V, \mathcal{N}')$, the following properties are equivalent:*

(1) *f is nano continuous,*
(2) *f is $n\alpha$-continuous and $n\mathcal{SB}$-continuous,*
(3) *f is np-continuous (or ns-continuous) and $n\mathcal{SB}$-continuous,*
(4) *f is $n\beta$-continuous and $n\mathcal{SB}$-continuous.*

Proof.
Since $I = \phi$, we have $H_n^\star = C_n(H)$ and $C_n^\star(H) = H \cup H_n^\star = C_n(H)$ for any subset H of U. Therefore, we obtain

(1) H is α-nI-open (pre-nI-open, semi-nI-open, β-nI-open) $\iff$ it is $n\alpha$-open (np-open, ns-open, β-open) and
(2) H is a $n\mathcal{SB}_I$-set $\iff$ it is a $n\mathcal{SB}$-set.

The proof follows from Theorem 4.1 immediately.

□

References

1. V. Inthumathi, R. Abinprakash, and M. Parveen Banu, *Some weak forms of continuous and irresolute mappings in nano ideal topological spaces*, Journal of new results in Science, 8(1)(2019), 14-25.
2. A. Jayalakshmi and C. Janaki, *A new form of nano locally closed sets in nano topological spaces*, Global Journal of Pure and Applied Mathematics, 13(9)2017, 5997-6006.
3. K. Kuratowski, *Topology*, Vol I, Academic Press (New York), 1966.
4. M. L. Thivagar and C. Richard, *On nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention,1(1)(2013), 31-37.
5. M. Parimala, T. Noiri, and S. Jafari, *New types of nano topological spaces via nano ideals* (to appear).
6. M. Parimala and S. Jafari, *On some new notions in nano ideal topological spaces*, International Balkan Journal of Mathematics(IBJM), 1(3)(2018), 85-92.
7. Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11(5)(1982), 341-356.
8. I. Rajasekaran and O. Nethaji, *Simple forms of nano open sets in an ideal nano topological spaces*, Journal of New Theory, 24(2018), 35-43.
9. I. Rajasekaran and O. Nethaji, *A new form of some nano sets*, Bull. Int. Math. Virtual Inst., 9(2)(2019), 355-360.
10. I. Rajasekaran, O. Nethaji, and R. Premkumar, *Perception of several an ideal nano topological spaces*, Journal of New Theory, 23(2018), 78-84.
11. R. Vaidyanathaswamy, *The localization theory in set topology*, Proc. Indian Acad. Sci., 20(1945), 51-61.
12. R. Vaidyanathaswamy, *Set topology*, Chelsea Publishing Company, New York, 1946.

Received by editors 6.8.2021; Revised version 20.10.2022; Available online 1.12.2022.

Ilangovan Rajasekaran, Department of Mathematics, Tirunelveli Dakshina Mara Nadar Sangam College, T. Kallikulam - 627 113, Tirunelveli District, Tamil Nadu, India.

Email address: sekarmelakkal@gmail.com.