**BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE**
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
*Bull. Int. Math. Virtual Inst.*, **12**(2)(2022), 387-395
DOI: 10.7251/BIMVI2202387V

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

# ON THE $(s,t)$-PADOVAN $n$-DIMENSIONAL RECURRENCES

**Renata Passos Machado Vieira, Francisco Regis Vieira Alves, and Paula Maria Machado Cruz Catarino**

Abstract. Based on the generalized sequence $(s,t)$-Padovan, a study is carried out around this one-dimensional sequence. In this way, its dimension is expanded, resulting in two-dimensional and three-dimensional relationships, until obtaining its complex generalized form, called $n$-dimensional.

## 1. Introduction

Numerical and recurring sequences are being classified as areas of mathematics and mathematics, more specifically the sequence of mathematics sequence. Investigations of these numbers being worked on in articles carried out and their generalization process comes [**5, 6**]. Thus, there is the study of the Padovan sequence, which is a third order sequence. The Padovan sequence presents the plastic number as a solution of its characteristic polynomial. The Fibonacci sequence presents the golden number as a solution of its characteristic polynomial. Thus, the Padovan sequence is considered to be similar to the Fibonacci sequence [**9, 10**].

The Padovan sequence is a numerical sequence and created by Richard Padovan, being similar to the Fibonacci sequence. Thus, we have the definition of the

2010 *Mathematics Subject Classification.* Primary 11B37; Secondary 11B39.

*Key words and phrases.* Generalization, Padovan sequence, recurrences.

The research development part in Brazil had the financial support of the National Council for Scientific and Technological Development - CNPq and the Ceará Foundation for Support to Scientific and Technological Development (Funcap). The research development aspect in Portugal is financed by National Funds through FCT - Fundação para a Ciência e Tecnologia. I.P, within the scope of the UID / CED / 00194/2020 project.

Communicated by Dusko Bogdanic.

Padovan sequence $Pa_n$, given by:

$$Pa_n = Pa_{n-2} + Pa_{n-3}, n \geqslant 3,$$

with $Pa_0 = Pa_1 = Pa_2 = 1$.

Some studies around this sequence are carried out, investigating the process of complexification and generalization of the one-dimensional Padovan model [**7, 8, 9, 10**].

Based on this, we have the generalization of the coefficients of the recurrence formula of the Padovan sequence, calling it $(s,t)$-Padovan $Pa_n(s,t)$, where [**1, 3, 4, 11**]:

$$Pa_n(s,t) = sPa_{n-2}(s,t) + tPa_{n-3}(s,t), n > 0,$$

assuming the valors initial $Pa_0(s,t) = 1, Pa_1(s,t) = 1, Pa_2(s,t) = s$ and $s \geqslant 0, t \neq 0, 27t^2 - 4s^3 \neq 0$.

Starting from Padovan's one-dimensional model, we have the insertion of imaginary units up to the imaginary unit of order $n$, according to the study carried out by Vieira, Alves and Catarino [**12**].

In view of this, and based on the work of Diskaya and Menken [**2**], in which they deal with the $n$-dimensional relations of the $(p,q)$-Fibonacci sequence, carrying out a study of the generalization of the coefficients of the Fibonacci recurrence and its complexification process with the insertion of imaginary units.

That said, there is an introduction and study of the $n$-dimensional of the sequence $(s,t)$-Padovan. In this way, the generalization of the coefficients of the recurrence formula of Padovan's primitive sequence is combined with the complexification of these numbers, in order to introduce imaginary units up to the order $n$.

In the following sections, these relations are introduced, in order to obtain their generalization and complexity.

## 2. Two-dimensional recurrences of the $(s,t)$-Padovan sequence

In this section, we introduce the two-dimensional recurrences of the $(s,t)$-Padovan sequence based on the one-dimensional recurrence.

Definition 2.1. For $n, m \in \mathbb{N}$ and $s \geqslant 0, t \neq 0, 27t^2 - 4s^3 \neq 0$, the two-dimensaional of $(s,t)$-Padovan sequence $Pa_{n,m}(s,t)$ is defined by the recurrences:

$$Pa_{n+3,m}(s,t) = sPa_{n+1,m}(s,t) + tPa_{n,m}(s,t)$$
$$Pa_{n,m+3}(s,t) = sPa_{n,m+1}(s,t) + tPa_{n,m}(s,t)$$

with the initial values:

$$Pa_{0,0}(s,t) = 1, Pa_{1,0}(s,t) = 1, Pa_{0,1}(s,t) = 1+i, Pa_{1,1}(s,t) = 1+i,$$
$$Pa_{2,0}(s,t) = s, Pa_{0,2}(s,t) = s+i, Pa_{1,2}(s,t) = s+i, Pa_{2,1}(s,t) = s+si,$$
$$Pa_{2,2}(s,t) = s+si, i^2 = -1.$$

PROPOSITION 2.1. *The following properties are validy:*
(a) $Pa_{n,0}(s,t) = Pa_n(s,t)$;
(b) $Pa_{0,m}(s,t) = Pa_m(s,t) + iPa_{m-1}(s,t)$;
(c) $Pa_{n,1}(s,t) = Pa_n(s,t) + iPn(s,t)$;
(d) $Pa_{1,m}(s,t) = Pa_m(s,t) + iPa_{m-1}(s,t)$;
(e) $Pa_{n,m}(s,t) = Pa_n(s,t)Pa_m(s,t) + iPa_n(s,t)Pa_{m-1}(s,t)$.

PROOF. (a) The Mathematical Principle of Induction and by recurrence:
$Pa_{n+3,m}(s,t) = Pa_{n+1,m}(s,t) + Pa_{n,m}(s,t), m = 0$, we can prove the first proposition.

For $n = 0$:

$$Pa_{0,0}(s,t) = Pa_0(s,t) = 1$$

Suppose that the desired equality is true for any $n \geqslant k, k \in \mathbb{N}$, we have to:

$$Pa_{k,0}(s,t) = Pa_k(s,t)$$

Let's show that it is true for $k + 1$.

$$\begin{aligned} Pa_{k+4,0}(s,t) &= sPa_{k+2,0}(s,t) + tPa_{k+1,0}(s,t) \\ &= s(Pa_{k+2}(s,t)) + t(Pa_{k+1}(s,t)) \\ &= Pa_{k+4}(s,t) \end{aligned}$$

Validating proposition (a).

(b) The Mathematical Principle of Induction and by recurrence:
$Pa_{n,m+3}(s,t) = Pa_{n,m+1}(s,t) + Pa_{n,m}(s,t), n = 0$, we can prove the proposition.

For $m = 0$:

$$Pa_{0,0}(s,t) = Pa_0(s,t) + iPa_{-1}(s,t) = 1$$

Suppose that the desired equality is true for any $m \geqslant k, k \in \mathbb{N}$, we have to:

$$Pa_{0,k}(s,t) = Pa_k(s,t) + iPa_{k-1}(s,t)$$

Let's show that it is true for $k + 1$.

$$\begin{aligned} Pa_{0,k+4}(s,t) &= sPa_{0,k+2}(s,t) + tPa_{0,k+1}(s,t) \\ &= s(Pa_{k+2}(s,t) + iPa_{k+1}(s,t)) + t(Pa_{k+1}(s,t) + iPa_k(s,t)) \\ &= Pa_{k+4}(s,t) + iPa_{k+3}(s,t) \end{aligned}$$

Validating proposition (b).

(c) The Mathematical Principle of Induction and by recurrence:
$Pa_{n+3,m}(s,t) = Pa_{n+1,m}(s,t) + Pa_{n,m}(s,t), m = 1$, we can prove the proposition.

For $n = 0$:

$$Pa_{0,1}(s,t) = Pa_0(s,t) + iPa_0(s,t) = 1 + i$$

Suppose that the desired equality is true for any $n \geqslant k, k \in \mathbb{N}$, we have to:

$$Pa_{k,1}(s,t) = Pa_k(s,t) + iPa_k(s,t)$$

Let's show that it is true for $k+1$.

$$\begin{aligned} Pa_{k+4,1}(s,t) &= sPa_{k+2,1}(s,t) + tPa_{k+1,1}(s,t) \\ &= s(Pa_{k+2}(s,t) + iPa_{k+2}(s,t)) + t(Pa_{k+1}(s,t) + iPa_{k+1}(s,t)) \\ &= Pa_{k+4}(s,t) + iPa_{k+4}(s,t) \end{aligned}$$

Validating proposition (c).

(d) The Mathematical Principle of Induction and by recurrence:

$Pa_{n,m+3}(s,t) = Pa_{n,m+1}(s,t) + Pa_{n,m}(s,t), n = 1$, we can prove the proposition.

For $m = 0$:

$$Pa_{1,0}(s,t) = Pa_0(s,t) + iPa_{-1}(s,t) = 1$$

Suppose that the desired equality is true for any $m \geqslant k, k \in \mathbb{N}$, we have to:

$$Pa_{1,k}(s,t) = Pa_k(s,t) + iPa_{k-1}(s,t)$$

Let's show that it is true for $k+1$.

$$\begin{aligned} Pa_{1,k+4}(s,t) &= sPa_{1,k+2}(s,t) + tPa_{1,k+1}(s,t) \\ &= s(Pa_{k+2}(s,t) + iPa_{k+1}(s,t)) + t(Pa_{k+1}(s,t) + iPa_k(s,t)) \\ &= Pa_{k+4}(s,t) + iPa_{k+3}(s,t) \end{aligned}$$

Validating proposition (d).

(e) The Mathematical Principle of Induction and by recurrences:

$$Pa_{n+3,m}(s,t) = Pa_{n+1,m}(s,t) + Pa_{n,m}(s,t)$$

and

$$Pa_{n,m+3}(s,t) = Pa_{n,m+1}(s,t) + Pa_{n,m}(s,t),$$

we can prove the proposition.

For $n = 0$:

$$\begin{aligned} Pa_{0,m}(s,t) &= Pa_0(s,t)Pa_m(s,t) + iPa_0(s,t)Pa_{m-1}(s,t) \\ &= Pa_m(s,t) + iPa_{m-1}(s,t) \end{aligned}$$

For $m = 0$:

$$\begin{aligned} Pa_{n,0}(s,t) &= Pa_n(s,t)Pa_0(s,t) + iPa_n(s,t)Pa_{-1}(s,t) \\ &= Pa_n(s,t) \end{aligned}$$

Suppose that the desired equality is true for any $n \geqslant k, m \geqslant k, k \in \mathbb{N}$, we have to:

$$Pa_{k,m}(s,t) = Pa_k(s,t)Pa_m(s,t) + iPa_k(s,t)Pa_{m-1}(s,t)$$

and

$$Pa_{n,k}(s,t) = Pa_n(s,t)Pa_k(s,t) + iPa_n(s,t)Pa_{k-1}(s,t)$$

Let's show that it is true for $n = k+1$.

$$\begin{aligned} Pa_{k+4,m}(s,t) &= sPa_{k+2,m}(s,t) + tPa_{k+1,m}(s,t) \\ &= s(Pa_{k+2}(s,t) + Pa_m(s,t) + iPa_{k+1}(s,t)Pa_{m-1}(s,t)) \\ &+ t(Pa_{k+1}(s,t) + Pa_m(s,t) + iPa_{k+1}(s,t)Pa_{m-1}(s,t)) \\ &= Pa_{k+4}(s,t)Pa_m(s,t) + iPa_{k+4}(s,t)Pa_{m-1}(s,t) \end{aligned}$$

and For $m = k+1$.

$$\begin{aligned} Pa_{n,k+4}(s,t) &= sPa_{n,k+2}(s,t) + tPa_{n,k+1}(s,t) \\ &= s(Pa_n(s,t)Pa_{k+2}(s,t) + iPa_n(s,t)Pa_{k+1}(s,t)) \\ &+ t(Pa_n(s,t)Pa_{k+1}(s,t) + iPa_n(s,t)Pa_k(s,t)) \\ &= Pa_n(s,t)Pa_{k+4}(s,t) + iPa_n(s,t)Pa_{k+3}(s,t) \end{aligned}$$

Validating proposition (e). □

## 3. Three-dimensional recurrences of the $(s,t)$-Padovan sequence

In this section, we introduce the three-dimensional recurrences of the $(s,t)$-Padovan sequence based on the one-dimensional recurrence.

DEFINITION 3.1. For $n,m,p \in \mathbb{N}$ and $s \geqslant 0, t \neq 0, 27t^2 - 4s^3 \neq 0$, the three-dimensaional of $(s,t)$-Padovan sequence $Pa_{n,m,p}(s,t)$ is defined by the recurrences:

$$\begin{aligned} Pa_{n+3,m,p}(s,t) &= sPa_{n+1,m,p}(s,t) + tPa_{n,m,p}(s,t) \\ Pa_{n,m+3,p}(s,t) &= sPa_{n,m+1,p}(s,t) + tPa_{n,m,p}(s,t) \\ Pa_{n,m,p+3}(s,t) &= sPa_{n,m,p+1}(s,t) + tPa_{n,m,p}(s,t) \end{aligned}$$

with the initial values:

$$\begin{aligned} &Pa_{0,0,0}(s,t) = 1, Pa_{1,0,0}(s,t) = 1, Pa_{2,0,0}(s,t) = s, Pa_{0,1,0}(s,t) = 1+i, \\ &Pa_{0,0,1}(s,t) = 1+j, Pa_{1,1,1}(s,t) = 1+i+j, Pa_{0,1,1}(s,t) = 1+i+j, \\ &Pa_{1,0,1}(s,t) = 1+j, Pa_{1,1,0}(s,t) = 1+i, Pa_{2,1,1}(s,t) = s+si+sj, \\ &Pa_{2,1,0}(s,t) = s+si, Pa_{0,2,0}(s,t) = s+i, Pa_{0,2,1}(s,t) = s+i+sj, \\ &Pa_{1,2,0}(s,t) = s+i, Pa_{0,0,2}(s,t) = s+j, Pa_{0,1,2}(s,t) = s+si+j, \\ &Pa_{1,0,2}(s,t) = s+j, Pa_{1,1,2}(s,t) = s+si+j, \\ &Pa_{1,2,1}(s,t) = s+i+sj, Pa_{0,2,2}(s,t) = s^2+si+sj, \\ &Pa_{2,0,1}(s,t) = s+sj, Pa_{2,2,1}(s,t) = s^2+si+s^2j, Pa_{2,1,2}(s,t) = s^2+s^2i+sj, \\ &Pa_{1,2,2}(s,t) = s^2+si+sj, Pa_{2,2,2}(s,t) = s^3+s^2i+s^2j, Pa_{2,0,2}(s,t) = s^2+sj, \\ &Pa_{2,2,0}(s,t) = s^2+si, i^2 = j^2 = -1. \end{aligned}$$

PROPOSITION 3.1. *The following properties are validy:*

*(a)* $Pa_{n,0,0}(s,t) = Pa_n(s,t)$*;*
*(b)* $Pa_{n,0,1}(s,t) = Pa_n(s,t) + jPa_n(s,t)$*;*
*(c)* $Pa_{n,1,0}(s,t) = Pa_n(s,t) + iPa_n(s,t)$*;*
*(d)* $Pa_{n,1,1}(s,t) = Pa_n(s,t) + iPa_n(s,t) + jPa_n(s,t)$*;*

*(e)* $Pa_{0,m,0}(s,t) = Pa_m(s,t) + iPa_{m-1}(s,t)$;
*(f)* $Pa_{0,m,1}(s,t) = Pa_m(s,t) + iPa_{m-1}(s,t) + jPa_m(s,t)$;
*(g)* $Pa_{1,m,0}(s,t) = Pa_m(s,t) + iPa_{m-1}(s,t)$;
*(h)* $Pa_{1,m,1}(s,t) = Pa_m(s,t) + iPa_{m-1}(s,t) + jPa_m(s,t)$;
*(i)* $Pa_{0,0,p}(s,t) = Pa_p(s,t) + jPa_{p-1}(s,t)$;
*(j)* $Pa_{0,1,p}(s,t) = Pa_p(s,t) + iPa_p(s,t) + jPa_{p-1}(s,t)$;
*(k)* $Pa_{1,0,p}(s,t) = Pa_p(s,t) + jPa_{p-1}(s,t)$;
*(l)* $Pa_{1,1,p}(s,t) = Pa_p(s,t) + iPa_p(s,t) + jPa_{p-1}(s,t)$;
*(m)* $Pa_{n,m,p}(s,t) = Pa_n(s,t)Pa_m(s,t)Pa_p(s,t) + iPa_n(s,t)Pa_{m-1}(s,t)Pa_p(s,t) + jPa_n(s,t)Pa_m(s,t)Pa_{p-1}(s,t)$.

PROOF. The demonstrations are carried out in accordance with the Proposition 2.1. □

## 4. $n$-dimensional recurrences of the $(s,t)$-Padovan sequence

In this section, we introduce the $n$-dimensional recurrences of the $(s,t)$-Padovan sequence based on the one-dimensional, two-dimensional and three-dimensional recurrences.

DEFINITION 4.1. For $n_0, n_1, \ldots, n_{n-1} \in \mathbb{N}$ and and $s^2 + 4t > 0$, the $n$-dimensional of $(s,t)$-Padovan sequence $Pa_{n_0,n_1,\ldots,n_{n-1}}(s,t)$ is defined by the recurrences:

$$\begin{aligned} Pa_{n_0+3,n_1,\ldots,n_{n-1}}(s,t) &= sPa_{s,t}^{n_0+1,n_1,\ldots,n_{n-1}} + tPa_{s,t}^{n_0,n_1,\ldots,n_{n-1}} \\ Pa_{n_0,n_1+3,\ldots,n_{n-1}}(s,t) &= sPa_{s,t}^{n_0,n_1+1,\ldots,n_{n-1}} + tPa_{s,t}^{n_0,n_1,\ldots,n_{n-1}} \\ &\vdots \\ Pa_{n_0,n_1,\ldots,n_{n-1}+3}(s,t) &= sPa_{s,t}^{n_0,n_1,\ldots,n_{n-1}+1} + tPa_{s,t}^{n_0,n_1,\ldots,n_{n-1}}, \end{aligned}$$

with the initial values:

$$\begin{aligned} Pa_{(0,0,0,\ldots,0)}(s,t) &= 0 \\ Pa_{(1,0,0,\ldots,0)}(s,t) &= 1 \\ Pa_{(0,1,0,\ldots,0)}(s,t) &= \mu_1 \\ Pa_{(0,0,1,\ldots,0)}(s,t) &= \mu_2 \end{aligned}$$

$$\begin{aligned}
Pa_{(0,0,0,1,\ldots,0)}(s,t) &= \mu_3\\
Pa_{(0,0,0,0,1,0,\ldots,0)}(s,t) &= \mu_4\\
&\vdots\\
Pa_{(0,0,0,\ldots,1)}(s,t) &= \mu_{n-1}\\
Pa_{(1,1,1,\ldots,1)}(s,t) &= 1+\mu_1+\ldots+\mu_n\\
Pa_{(0,1,1,\ldots,1)}(s,t) &= \mu_1+\ldots+\mu_n\\
Pa_{(1,0,1,\ldots,1)}(s,t) &= 1+\mu_2+\ldots+\mu_n\\
Pa_{(1,1,0,\ldots,1)}(s,t) &= 1+\mu_1+\mu_2+\ldots+\mu_n\\
&\vdots\\
Pa_{(1,1,1,\ldots,1,0)}(s,t) &= 1+\mu_1+\mu_2+\ldots+\mu_{n-1}.
\end{aligned}$$

and $\mu_0 = 1, \mu_1 = i, \mu_2 = j, \ldots, \mu_{n-1}$.

THEOREM 4.1. *Numbers of the form* $Pa_{n_0,n_1,\ldots,n_{n-1}}(s,t)$*, such that* $n_1, n_2, n_3, \ldots, n_n \in \mathbb{N}$*, are determined by:*

$$\begin{aligned}
Pa_{(n_1,n_2,n_3,\ldots,n_n)}(s,t) &= (Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3}(s,t)\ldots Pa_{n_n}(s,t))\\
&+(Pa_{n_1}(s,t)Pa_{n_2-1}(s,t)Pa_{n_3}(s,t)\ldots Pa_{n_n}(s,t))\mu_1\\
&+(Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3-1}(s,t)\ldots Pa_{n_n}(s,t))\mu_2\\
&+\ldots+\\
&+(Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3}(s,t)\ldots Pa_{n_n-1}(s,t))\mu_n.
\end{aligned}$$

PROOF. Starting from the demonstrations carried out in the previous subsections, in which the theorems are valid:

$$\begin{aligned}
Pa_{n,m}(s,t) &= Pa_n(s,t)Pa_m(s,t)+iPa_n(s,t)Pa_{m-1}(s,t),\\
Pa_{n,m,p}(s,t) &= Pa_n(s,t)Pa_m(s,t)Pa_p(s,t)+iPa_n(s,t)Pa_{m-1}(s,t)Pa_p(s,t)\\
&+jPa_n(s,t)Pa_m(s,t)Pa_{p-1}(s,t).
\end{aligned}$$

Thus, through the inductive step, it can be verified that:

$$
\begin{aligned}
Pa_{n_1,n_2}(s,t) &= Pa_{n_1}(s,t)Pa_{n_2}(s,t) + Pa_{n_1}(s,t)Pa_{n_2-1}(s,t)\mu_1 \\
Pa_{n_1,n_2,n_3}(s,t) &= Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3}(s,t) \\
&\quad + Pa_{n_1}(s,t)Pa_{n_2-1}(s,t)Pa_{n_3}(s,t)\mu_1 \\
&\quad + Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3-1}(s,t)\mu_2 \\
Pa_{n_1,n_2,n_3,n_4}(s,t) &= Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3}(s,t)Pa_{n_4}(s,t) \\
&\quad + Pa_{n_1}(s,t)Pa_{n_2-1}(s,t)Pa_{n_3}(s,t)Pa_{n_4}(s,t)\mu_1 \\
&\quad + Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3-1}(s,t)Pa_{n_4}(s,t)\mu_2 \\
&\quad + Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3}(s,t)Pa_{n_4-1}(s,t)\mu_3 \\
&\vdots \\
Pa_{n_1,n_2,n_3,\ldots,n_n}(s,t) &= (Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3}(s,t)\ldots Pa_{n_n}(s,t)) \\
&\quad + (Pa_{n_1}(s,t)Pa_{n_2-1}(s,t)Pa_{n_3}(s,t)\ldots Pa_{n_n}(s,t))\mu_1 \\
&\quad + (Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3-1}(s,t)\ldots Pa_{n_n}(s,t))\mu_2 \\
&\quad + \ldots + \\
&\quad + (Pa_{n_1}(s,t)Pa_{n_2}(s,t)Pa_{n_3}(s,t)\ldots Pa_{n_n-1}(s,t))\mu_n.
\end{aligned}
$$

□

## 5. Conslusion

From the one-dimensional model, one can insert imaginary units, resulting in two-dimensional, three-dimensional recurrences until their generalization with the $n$-dimensional. Thus, it is possible to perceive the process of complexification of the primitive Padovan sequence, generalizing it through the terms of the recurrence formula $(s,t)$, as well as its generalization in the $n$-dimensional format.

## References


1. G. Cerda-Morales, *The (s, t)-Padovan and (s, t)-Perrin matrix sequences*, preprint in Researchgate. (2017).
2. O. Diskaya and H. Menken, *On the $(p,q)$-Fibonacci $N$-dimensional recurrences*, Bulletin of the International Mathematical Virtual Institute. **12**(2) (2022), 205–212.
3. O. Diskaya and H. Menken, *On the $(s,t)$-Padovan and $(s,t)$-Perrin quaternions*, Journal of Advanced Mathematical Studies. **12**(2) (2019), 186–192.
4. O. Diskaya and H. Menken, *(2019). On the Split (s, t)-Padovan and (s, t)-Perrin Quaternions*, International Journal of Applied Mathematics and Informatics. **13** (2019), 25–28.
5. A. F. Horadam, *Quaternion recurrence relations*, Fibonacci Quarterly. **2**(2) (1993), 23–33.
6. A. Ipek, *On (p; q)-Fibonacci quaternions and their Binet formulas, generating functions and certain binomial sums*, Advances in Applied Clifford Algebras. **27**(2) (2017), 1343–1351.
7. R. Padovan, *Dom hans van der laan and the plastic number*, Nexus Network Journal. **4**(1) (2002), 181–193.
8. S. Tas and E. Karaduman, *The Padovan sequences in finite groups*, Chiang Mai Journal of Science. **41**(9) (2014), 456–462.
9. R. Vieira and F. Alves, *Explorando a sequência de padovan através de investigação histórica e abordagem epistemológica*, Boletim Gepem. **74** (2019), 161–169.

10. R. Vieira, F. Alves, and P. Catarino, *A historical analysis of the padovan sequence*, International Journal of Trends in Mathematics Education Research. **3**(1) (2020), 8–12.
11. R. Vieira, F. Alves, and P. Catarino, *The (s, t)-Padovan Quaternions Matrix Sequence*, Punjab University Journal of Mathematics. **52**(11) (2020), 1–9.
12. R. Vieira, F. Alves, and P. Catarino, *Relations on higher dimensional Padovan sequences*, Notes on Number Theory and Discrete Mathematics. **27**(4) (2021), 167–179.

Received by editors 15.3.2022; Revised version 8.6.2022; Available online 1.8.2022.

Renata Passos Machado Vieira, Department of Mathematics, Federal University of Ceara, Ceara, Brazil
*Email address*: re.passosm@gmail.com

Francisco Regis Vieira Alves, Department of Mathematics, Federal Institute of Science and Technology Education of the State of Ceará, Ceara, Brazil
*Email address*: fregis@gmx.fr

Paula Maria Machado Cruz Catarino, Department of Mathematics, University of Trás-os-Montes and Alto Douro, Vila Real, Portugal
*Email address*: pcatarino23@gmail.com