**BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE**
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
*Bull. Int. Math. Virtual Inst.*, **12**(2)(2022), 273-285
DOI: 10.7251/BIMVI2201273R

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

# WEAK-INTERIOR IDEALS

**M. Murali Krishna Rao**

ABSTRACT. In this paper, we introduce the notion of weak-interior ideal as a generalization of quasi ideal, interior ideal, left(right) ideal, ideal of semiring. Then, we study the properties of weak-interior ideals of semiring and characterize the weak-interior simple semiring, regular semiring using weak-interior ideal ideals of semiring.

## 1. Introduction

Ideals play an important role in advance studies and uses of algebraic structures. Generalization of ideals in algebraic structures is necessary for further study of algebraic structures. Many mathematicians introduced various generalizations of concept of ideals in algebraic structures, proved important results and charecterizations of algebraic structures. The notion of a semiring was introduced by Vandiver [**25**] in 1934, but semirings had appeared in earlier studies on the theory of ideals of rings. Semiring is a generalization of ring but also of a generalization of distributive lattice. Semirings are structually similar to semigroups than to rings. Semiring theory has many applications in other branches of mathematics.

In 1995, M.Murali Krishna Rao [**15–18**] introduced the notion of semiring as a generalization of ring, ternary semiring and semiring. As a generalization of ring, the notion of a ring was introduced by Nobusawa [**22**] in 1964. In 1981, Sen introduced the notion of a semigroup as a generalization of semigroup. The notion of a ternary algebraic system was introduced by Lehmer. Murali Krishna Rao and Venkateswarlu [**20,21**] studied regular incline, field semiring and derivations. Dutta

---

2010 *Mathematics Subject Classification.* Primary 16Y60; Secondary 16Y99.

*Key words and phrases.* Bi-interior ideal, bi-quasi ideal, bi-ideal, quasi ideal, interior ideal, regular semiring, weak-interior ideal, weak-interior simple semiring.

Communicated by Daniel A. Romano.

& Sardar [**2**] introduced the notion of operator semirings of semiring.

Henriksen [**4**] and Shabir et al. [**23**] studied ideals in semirings. We know that the notion of an one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left ideal and right ideal whereas the bi-ideals are generalization of quasi ideals.
In 1952, the concept of bi-ideals was introduced by Good and Hughes [**3**] for semigroups. The notion of bi-ideals in rings and semirings were introduced by Lajos and Szasz [**10, 11**]. Bi-ideal is a special case of ( m-n) ideal. Steinfeld [**24**] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [**5**–**7**] introduced the concept of quasi ideal for a semiring. Quasi ideals, bi-ideals in $\Gamma-$semirings studied by Jagtap and Pawar [**8, 9**]. Murali Krishna Rao introduced the notion of left (right) bi-quasi ideal, the notion of bi-interior ideal and the notion of bi weak-interior ideal of semiring as a generalization of ideal of semiring, studied their properties and characterized the left bi-quasi simple semiring and regular semiring.

In this paper, we introduce the notion of weak-interior ideals as a generalization of quasi ideal, interior ideal,left(right) ideal and ideal of semiring and study the properties of weak-interior ideals of semiring.

## 2. Preliminaries

In this section, we recall some of the fundamental concepts and definitions which are necessary for this paper.

Definition 2.1. A set $S$ together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called semiring provided

(i) addition is a commutative operation.
(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

Example 2.1. Let $M$ be the set of all natural numbers. Then $(M, max, min)$ is a semiring.

Definition 2.2. Let $M$ be a semiring. If there exists $1 \in M$ such that $a \cdot 1 = 1 \cdot a = a$, for all $a \in M$, is called an unity element of $M$ then $M$ is said to be semiring with unity.

Definition 2.3. An element $a$ of a semiring $S$ is called a regular element if there exists an element $b$ of $S$ such that $a = aba$.

Definition 2.4. A semiring $S$ is called a regular semiring if every element of $S$ is a regular element.

Definition 2.5. An element $a$ of a semiring $S$ is called a multiplicatively idempotent (an additively idempotent) element if $aa = a(a + a = a)$.

DEFINITION 2.6. An element $b$ of a semiring $M$ is called an inverse element of $a$ of $M$ if $ab = ba = 1$.

DEFINITION 2.7. A non-empty subset $A$ of semiring $M$ is called

(i) a subsemiring of $M$ if $A$ is an additive subsemigroup of $M$ and $AA \subseteq A$.
(ii) a left(right) ideal of $M$ if $A$ is an additive subsemigroup of $M$ and $MA \subseteq A(AM \subseteq A)$.
(iii) an ideal if $A$ is an additive subsemigroup of $M$, $MA \subseteq A$ and $AM \subseteq A$.
(iv) a $k-$ideal if $A$ is a subsemiring of $M, AM \subseteq A, MA \subseteq A$ and $x \in M,\ x+y \in A, y \in A$ then $x \in A$.

DEFINITION 2.8. A semiring $M$ is called a division semiring if for each non-zero element of $M$ has multiplication inverse.

DEFINITION 2.9. A non-empty subset $A$ of semiring $M$ is called

(i) an interior ideal of $M$ if $A$ is a subsemiring of $M$ and $MAM \subseteq A$.
(ii) a quasi ideal of $M$ if $A$ is a subsemiring of $M$ and $AM \cap MA \subseteq A$.
(iii) a bi-ideal of $M$ if $A$ is a subsemiring of $M$ and $AMA \subseteq A$.

DEFINITION 2.10. A non-empty subset $B$ of semiring $M$ is said to be bi-interior ideal of $M$ if $B$ is a subsemiring of $M$ and $MBM \cap BMB \subseteq B$.

DEFINITION 2.11. Let $M$ be a semiring. A non-empty subset $L$ of $M$ is said to be left bi-quasi ideal (right bi-quasi ideal) of $M$ if $L$ is a subsemigroup of $(M, +)$ and $ML \cap LML \subseteq L$ ($LM \cap LML \subseteq L$).

DEFINITION 2.12. Let $M$ be a semiring. $L$ is said to be bi-quasi ideal of $M$ if it is both a left bi-quasi and a right bi-quasi ideal of $M$.

EXAMPLE 2.2.

(i) Let $Q$ be the set of all rational numbers, $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in Q \right\}$ be the additive semigroup of $M$ matrices.With respect to usual matrix multiplication, M is a semiring

(a) If $R = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$ then $R$ is a quasi ideal of semiring $M$ and $R$ is neither a left ideal nor a right ideal.
(b) If $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid 0 \neq a \in Q \right\}$ then $S$ is a bi-ideal of semiring $M$.

(ii) If $M = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid a, b, c \in Q \right\}$ and $\Gamma = M$ then $M$ is a semiring with respect to usual addition of matrices and ternary operation is defined as usual matrix multiplication and $A = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$. Then $A$ is not a bi-ideal of semiring $M$.

DEFINITION 2.13. A semiring $M$ is called a left bi-quasi simple semiring if $M$ has no left bi-quasi ideal other than $M$ itself.

## 3. Weak-interior ideals of semirings

In this section we introduce the notion of weak-interior ideal as a generalization of quasi-ideal and interior ideal of semiring and study the properties of weak-interior ideal of semiring.Throughout tmis paper$M$ is a semiring with unity element.

Definition 3.1. A non-empty subset $B$ of a semiring $M$ is said to be left weak-interior ideal of $M$ if $B$ is a subsemiring of $M$ and $MBB \subseteq B$.

Definition 3.2. A non-empty subset $B$ of a semiring $M$ is said to be right weak-interior ideal of $M$ if $B$ is a subsemiring of $M$ and $BBM \subseteq B$.

Definition 3.3. A non-empty subset $B$ of a semiring $M$ is said to be weak-interior ideal of $M$ if $B$ is a subsemiring of $M$ and $B$is left and right weak-interior ideal of $M$.

Remark: A weak-interior ideal of a semiring $M$ need not be quasi-ideal, interior ideal, bi-interior ideal. and bi-quasi ideal of semiring $M$.

Example 3.1. Let $Q$ be the set of all rational numbers, $M = \left\{ \begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix} \mid b, d \in Q \right\}$ be the additive semigroup of $M$ matrices and $=$ $M$ is a multiplicative semigroup with respect to usual addition and multiplication of matrices. $A, \alpha, B$, for all $A, \alpha, B \in M$.Then M is a semiring. If $R = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \mid 0 \neq b \in Q \right\}$ then $R$ is a left weak interior ideal of the semiring $M$ and $R$ is neither a left ideal nor a right ideal , not a weak interior ideal and not a interior ideal of the semiring $M$ .

Theorem 3.1. *If $B$ is an interior ideal of a semiring $M$, then $B$ is a left weak-interior ideal of $M$.*

Proof. Suppose $B$ is an interior ideal of the semiring $M$.Then $MBB \subseteq MBM \subseteq B$. Hence $B$ is a left weak-interior ideal of $M$. □

Corollary 3.1. *If $B$ is an interior ideal of a semiring $M$, then $B$ is a right weak-interior ideal of $M$.*

Corollary 3.2. *If $B$ is an interior ideal of a semiring $M$, then $B$ is a weak-interior ideal of $M$.*

Theorem 3.2. *Let $M$ be a semiring and $B$ be a subsemiring of $M$. $B$ is a weak-interior ideal of $M$ if and only if there exists left ideal $L$ such that $LB \subseteq B \subseteq L$ .*

Proof. Suppose $B$ is a weak-interior ideal of the semiring$M$.
Then $MBB \subseteq B$. Let $L = MB$. Then $L$ is a left ideal of $M$.
Therefore $LB \subseteq B \subseteq L$ .
Conversely, suppose that there exists a left ideal $L$ of $M$ such that $LB \subseteq B \subseteq L$ . Then $MBB \subseteq MLB \subseteq LB \subseteq B$ .
Hence $B$ is a left weak-interior ideal of the semiring $M$. □

COROLLARY 3.3. *Let $M$ be a semiring and $B$ be a subsemiring of $M$. $B$ is a right weak-interior ideal of $M$ if and only if there exists a right ideal $R$ such that $RR \subseteq B \subseteq R$. .*

COROLLARY 3.4. *Let $M$ be a semiring and $B$ be a subsemiring of $M$. $B$ is a weak-interior ideal of $M$ if and only if there exist ideal $R$ such that $BR \subseteq B \subseteq R$. .*

THEOREM 3.3. *The intersection of a left weak-interior ideal $B$ of a semiring $M$ and a left ideal $A$ of $M$ is always a left weak-interior ideal of $M$.*

PROOF. Suppose $C = B \cap A$.

$$MCC \subseteq MBB \subseteq B$$

$$MCC \subseteq MAA \subseteq A \text{ Since } A \text{ is a left ideal of } M$$

Therefore $MCC \subseteq B \cap A = C$.

Hence the intersection of a right weak-interior ideal $B$ of a semiring $M$ and a right ideal $A$ of $M$ is always a right weak-interior ideal of $M$. □

COROLLARY 3.5. *The intersection of a right weak-interior ideal $B$ of a semiring $M$ and a right ideal $A$ of $M$ is always a right weak-interior ideal of $M$.*

COROLLARY 3.6. *The intersection of a weak-interior ideal $B$ of a semiring $M$ and an ideal $A$ of $M$ is always a weak-interior ideal of $M$.*

THEOREM 3.4. *Let $A$ and $C$ be left weak-interior ideals of a semiring $M$, $B = AC$ and $B$ is additively subsemigroup of $M$. If $AA = C$ then $B$ is a left left weak-interior ideal of $M$.*

PROOF. Let $A$ and $C$ be left weak-interior ideals of the semiring $M$ and $B = AC$. Then $BB = AMAAC = AAC \subseteq AC = B$. Therefore $B = AC$ is a subsemiring of $M$

$$\begin{aligned} MBB &= MACAC \\ &\subseteq MAAC \subseteq AC = B. \end{aligned}$$

Hence $B$ is a left weak-interior ideal of $M$. □

COROLLARY 3.7. *Let $A$ and $C$ be weak-interior ideals of a semiring $M$, $B = CA$ and $B$ is additively subsemigroup of $M$. If $CC = C$ then $B$ is a left weak-interior ideal of $M$.*

THEOREM 3.5. *Let $A$ and $C$ be subsemirings of a semiring $M$ and $B = AC$ and $B$ is additively subsemigroup of $M$. If $A$ is the left ideal of $M$ then $B$ is a left weak-interior ideal of $M$.*

PROOF. Let $A$ and $C$ be subsemirings of $M$ and $B = AC$. Suppose $A$ is the left ideal of $M$. Then $BB = ACAC \subseteq AC = B$.

$$\begin{aligned} MBB &= MACAC \\ &\subseteq AC = B. \end{aligned}$$

Hence $B$ is a left weak-interior ideal of $M$. □

Corollary 3.8. *Let $A$ and $C$ be subsemirings of a semiring $M$ and $B = AC$and $B$ is additively subsemigroup of $M$. If $C$ is a right idealof $M$ then $B$ is a right weak-interior ideal of $M$.*

Theorem 3.6. *Let $M$ be a semiring and $T$ be a non-empty subset of $M$. If a subsemiring $B$ of $M$ containing $MTT$ and $B \subseteq T$ then $B$ is a left weak-interior ideal of a semiring $M$.*

Proof. Let $B$ be a subsemiring of $M$ containing $MTT$. Then

$$\begin{aligned} MBB &\subseteq MTT \\ &\subseteq B. \end{aligned}$$

Therefore $MBB \subseteq B$.
Hence $B$ is a left weak-interior ideal of $M$. □

Theorem 3.7. *If $B$ is a left weak-interior ideal of semiring $M$, $T \subseteq B$ and $BT$ is an additively subsemigroup of $M$ then $BT$ is a left weak-interior ideal of $M$.*

Proof. Suppose $B$ is a left weak-interior ideal of the semiring $M$, $T \subseteq B$ and $BT$ is an additively subsemigroup of $M$ .Then $BTBT \subseteq BT$. Hence $BT$ is a subsemiring of $M$.

$$\text{We have } MBTBT \subseteq MBBT \subseteq BT.$$

Hence $BT$ is a left weak-interior ideal of the semiring $M$. □

Theorem 3.8. *Let $B$ and $I$ be left weak-interior ideals of a semiring $M$. Then $B \cap I$ is a left weak-interior ideal of $M$.*

Proof. Suppose $B$ and $I$ are left weak interior ideals of $M$. Obviously $B \cap I$ is a subsemiring of $M$. Then

$$M(B \cap I)(B \cap I) \subseteq MBB \subseteq B$$
$$M(B \cap I)(B \cap I) \subseteq MII \subseteq I$$

Therefore $M(B \cap I)M(B \cap I) \subseteq B \cap I$.
Hence $B \cap I$ is a left weak-interior ideal of $M$. □

Theorem 3.9. *Let $M$ be a semiring and $T$ be a subsemiring of $M$. Then every subsemiring of $T$ containing $MTT$ is a left weak-interior ideal of $M$.*

Proof. Let $C$ be a subsemiring of $T$ containing $MTMT$. Then

$$MCC \subseteq MTT \subseteq C.$$

Hence $C$ is a left weak-interior ideal of $M$ . □

Theorem 3.10. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ left weak-interior ideals of a semiring $M$ is a left weak-interior ideal of $M$.*

Proof. Let $B = \bigcap_{\lambda \in A} B_\lambda$. Then $B$ is a subsemiring of $M$.
Since $B_\lambda$ is a left weak-interior ideal of $M$, we have

$$\begin{aligned} &MB_\lambda B_\lambda \subseteq B_\lambda, \text{ for all } \lambda \in A \\ \Rightarrow &M(\cap B_\lambda)(\cap B_\lambda) \subseteq \cap B_\lambda \\ \Rightarrow &MBB \subseteq B. \end{aligned}$$

Hence $B$ is a left weak-interior ideal of $M$. □

Corollary 3.9. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ right weak-interior ideals of a semiring $M$ is a right weak-interior ideal of $M$.*

Corollary 3.10. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ weak-interior ideals of a semiring $M$ is a weak-interior ideal of $M$.*

Theorem 3.11. *Let $B$ be a right weak-interior ideal of a semiring $M$ ,$e \in B$ and $e$ be an idempotent. Then $eB$ is a right weak-interior ideal of $M$.*

Proof. Let $B$ be a right weak-interior ideal of the semiring $M$. Suppose $x \in B \cap eM$. Then $x \in B$ and $x = ey, y \in M$.

$$\begin{aligned} x &= ey \\ &= eey \\ &= e(ey) \\ &= ex \in eB. \end{aligned}$$

$$\begin{aligned} &\text{Therefore } B \cap eM \subseteq eB \\ &\text{we have } eB \subseteq B \text{ and } eB \subseteq eM \\ &\Rightarrow eB \subseteq B \cap eM \\ &\Rightarrow eB = B \cap eM. \end{aligned}$$

Hence $eB$ is a right weak-interior ideal of $M$. □

Corollary 3.11. *Let $B$ be a left weak-interior ideal of a semiring $M$ ,$e \in B$ and $e$ be an idempotent. Then $Be$ is a left weak-interior ideal of $M$.*

Theorem 3.12. *Let $M$ be a semiring. If $M = aM$, for all $a \in M$. Then every right weak- interior ideal of $M$ is a right ideal of $M$.*

Proof. Let $B$ be a right weak-interior ideal of the semiring $M$ and $a \in B$. Then

$$\begin{aligned} &\Rightarrow aM \subseteq BM, \\ &\Rightarrow M \subseteq BM \subseteq M \\ &\Rightarrow BM = M \\ &\Rightarrow BM = BBM \subseteq B \\ &\Rightarrow BM \subseteq B. \end{aligned}$$

Therefore $B$ is a right ideal of $M$. Hence the theorem. □

THEOREM 3.13. *$B$ is a left weak-interior ideal of a semiring $M$ if and only if $B$ is a left ideal of some left ideal of a semiring $M$.*

PROOF. Suppose $B$ is a left weak-interior ideal of the semiring $M$ .Then $MBB \subseteq B$. Therefore $B$ is a left ideal of left ideal $MB$ of semiring $M$.
Conversely suppose that $B$ is a left ideal of left ideal $R$ of the semiring$M$.Then $RB \subseteq B, MR \subseteq R$ and $MBB \subseteq MRB \subseteq RB \subseteq B$. Therefore $B$ is a left weak-interior ideal of a semiring $M$ .

□

COROLLARY 3.12. *$B$ is a right weak-interior ideal of a semiring $M$ if and only if $B$ is a right ideal of some right ideal of a semiring$M$.*

## 4. Left weak-interior simple semiring

In this section,we introduce the notion of left weak-interior simple semiring and characterize the left weak-interior simple semiring using left weak-interior ideals of semiring and study the properties of minimal left weak-interior ideals of semirings.

DEFINITION 4.1. A semiring $M$ is a left (right) simple semiring if $M$ has no proper left (right) ideals of $M$.

DEFINITION 4.2. A semiring $M$ is said to be simple semiring if $M$ has no proper ideals of $M$.

DEFINITION 4.3. A semiring $M$ is said to be left(right) weak-interior simple semiring if $M$ has no proper left(right) weak-interior ideal other than $M$ itself.

DEFINITION 4.4. A semiring $M$ is said to be weak-interior simple semiring if $M$ has no weak-interior ideal other than $M$ itself.

THEOREM 4.1. *If $M$ is a division semiring then $M$ is a left weak-interior simple semiring.*

PROOF. Let $B$ be a proper left weak-interior ideal of the division semiring $M$, $x \in M$ and $0 \neq a \in B$. Since $M$ is a division semiring, there exists $b \in M$, such that $ab = 1$.Then a b x=1 x =x .Therefore $x \in BM$ and $M \subseteq BM$. We have $BM \subseteq M$. Hence $M = BM$. Similarly we can prove $MB = M$.

$$M = MB = MBB$$
$$\subseteq MBB \subseteq B$$
$$M \subseteq B$$
$$\text{Therefore} M = B.$$

Hence division semiring $M$ is a left weak-interior simple semiring. □

COROLLARY 4.1. *If $M$ is a division semiring then $M$ is a right weak-interior simple semiring.*

COROLLARY 4.2. *If $M$ is a division semiring then $M$ is a weak-interior simple semiring.*

THEOREM 4.2. *Let $M$ be a left simple semiring. Every left weak-interior ideal of $M$ is a left ideal of $M$.*

PROOF. Let $M$ be a left simple semiring and $B$ be a left weak-interior ideal of $M$.
Then $MBB \subseteq B$ and $MB$ is a left ideal of $M$.
Since $M$ is a left simple semiring, we have $MB = M$. Therefore

$$MBB \subseteq B$$
$$\Rightarrow MB \subseteq B.$$

Hence the theorem. □

COROLLARY 4.3. *Let $M$ be a right simple semiring. Every right weak-interior ideal is a right ideal of $M$.*

COROLLARY 4.4. *If semiring $M$ is right simple semiring then every right weak-interior ideal of $M$ is a left ideal of $M$.*

COROLLARY 4.5. *Every weak-interior ideal of left and right simple semiring $M$ is an ideal of $M$.*

THEOREM 4.3. *Let $M$ be a semiring. $M$ is a left weak-interior simple semiring if and only if $< a >= M$, for all $a \in M$ and where $< a >$ is the smallest left weak-interior ideal generated by $a$.*

PROOF. Let $M$ be a semiring. Suppose $M$ is the left weak-interior simple semiring, $a \in M$ and $B = Ma$.
Then $B$ is a left ideal of $M$.
Therefore, by Theorem[3.5 ], $B$ is a left weak-interior ideal of $M$.
Therefore $B = M$. Hence $Ma = M$, for all $a \in M$.

$$Ma \subseteq < a > \subseteq M$$
$$\Rightarrow M \subseteq < a > \subseteq M.$$
$$\text{Therefore } M = < a > .$$

Conversely suppose that $< a >$ is the smallest left weak-interior ideal of $M$ generated by $a$, $< a >= M$ , $A$ is the left weak-interior ideal and $a \in A$. Then

$$< a > \subseteq A \subseteq M$$
$$\Rightarrow M \subseteq A \subseteq M.$$

Therefore $A = M$. Hence $M$ is a left weak-interior ideal simple semiring. □

THEOREM 4.4. *Let $M$ be a semiring. Then $M$ is a left weak-interior simple semiring if and only if $Maa = M$, for all $a \in M$.*

PROOF. Suppose $M$ is the left weak- interior simple semiring and $a \in M$.
Then $Maa$ is a left weak-interior ideal of $M$.
Hence $Maa = M$, for all $a \in M$.

Conversely suppose that $Maa = M$, for all $a \in M$.
Let $B$ be a left weak-interior ideal of the semiring $M$ and $a \in B$.

$$\begin{aligned} M &= Maa \\ &\subseteq MBB \subseteq B \\ \text{Therefore } M &= B. \end{aligned}$$

Hence $M$ is a left weak-interior simple semiring. □

Corollary 4.6. *Let $M$ be a semiring. Then $M$ is a right weak-interior simple semrriring if and only if $aaM = M$, for all $a \in M$.*

Corollary 4.7. *Let $M$ be a semiring. Then $M$ is a weak-interior simple semrriring if and only if $aaM = M$ and $Maa = M$,for all $a \in M$.*

Theorem 4.5. *Let $M$ be a semiring and $B$ be a left weak-interior ideal of $M$. Then $B$ is a minimal left weak-interior ideal of $M$ if and only if $B$ is a left weak-interior simple subsemiring of $M$.*

Proof. Let $B$ be a minimal left weak-interior ideal of the semiring $M$ and $C$ be a left weak-interior ideal of $B$. Then $BCC \subseteq C$.
and $BCC$ is a left weak-interior ideal of $M$.
Since $B$ is a minimal weak-interior ideal of $M$,

$$\begin{aligned} &BCC = B \\ &\Rightarrow B = BCC \subseteq C \\ &\Rightarrow B = C. \end{aligned}$$

Conversely suppose that $B$ is the left weak-interior simple subsemiring of $M$. Let $C$ be a left weak-interior ideal of $M$ and $C \subseteq B$.

$\Rightarrow BCC \subseteq MCC \subseteq C,$ .Therefore $C$ is a left weak-interior of $B$.
$\Rightarrow B = C$.Since $B$ is a left weak-interior simple subsemiring of $M$.

Hence $B$ is a minimal left weak-interior ideal of $M$. □

Corollary 4.8. *Let $M$ be a semiring and $B$ be a right weak-interior ideal of $M$. Then $B$ is a minimal right weak-interior ideal of $M$ if and only if $B$ is a right weak-interior simple subsemiring of $M$.*

Theorem 4.6. *Let $M$ be a semiring and $B = LL$, where $L$ is a minimal left ideal of $M$ . Then $B$ is a minimal left weak-interior ideal of $M$.*

Proof. Obviously $B = LL$ is a left weak-interior ideal of $M$. Let $A$ be a left weak-interior ideal of $M$ such that $A \subseteq B$.

We have $MAA$ is a left ideal of $M$. Then

$$\begin{aligned} MAA &\subseteq MBB \\ &= MLLLL \\ &\subseteq L, \text{ since } L \text{ is a left ideal of } M. \\ \text{Therefore } MAA &= L, \text{ since } L \text{ is a minimal left ideal of } M. \\ \text{Hence } \ B &= MAA \\ &\subseteq A. \end{aligned}$$

Therefore $A = B$. Hence $B$ is a minimal left weak-interior ideal of $M$.

□

COROLLARY 4.9. *Let $M$ be a semiring and $B = RR$, where $R$ is a minimal right ideal of $M$ . Then $B$ is a minimal right weak-interior ideal of $M$.*

THEOREM 4.7. *$M$ is regular $-$semiring if and only if $AB = A \cap B$ for any right ideal $A$ and left ideal $B$ of $-$semiring $M$.*

THEOREM 4.8. *Let $M$ be semiring Then $B$ is a weak-interior ideal of an idempotent regular semiring $M$ if and only if $BBM = B$ and $MBB = B$ for all weak-interior ideals $B$ of $M$.*

PROOF. Suppose $M$ is the regular semiring, $B$ is the weak-interior ideal of $M$ and $x \in B$.Then $MBB \subseteq B$,there exists $y \in M$ such that $x = xx = xxyxx \in MBB$. Therefore $x \in MBB$.
Hence $MBB = B$. Similarly we can prove $BBM = B$.
Conversely suppose that $BBM = B$ and $MBB = B$ for all weak-interior ideals $B$ of $M$.
Let $B = R \cap L$ and $C = RL$, where $R$ is a right ideal and $L$ is a left ideal of $M$.
Then $B$ and $C$ are weak-interior ideals of $M$.
Therefore $(R \cap L)(R \cap L)M = R \cap L$

$$\begin{aligned} R \cap L &= (R \cap L)(R \cap L)M \\ &\subseteq RLM \\ R \cap L &= M(R \cap L)(R \cap L)M \\ &\subseteq MRLML \\ &\subseteq RL \\ &\subseteq R \cap L \text{ (since } RL \subseteq L \text{ and } RL \subseteq R). \end{aligned}$$

Therefore $R \cap L = RL$. Hence $M$ is a regular semiring. □

## 5. Conclusion

As a further generalization of ideals,we introduced the notion of weak-interior ideal of semiring as a generalization of ideal ,left ideal, right ideal,quasi ideal and interior ideal of semiring and studied some of their properties. We introduced the notion of weak-interior simple semiring and characterized the weak-interior simple semiring , regular semiring using weak-interior ideal ideals of semiring. In continuity of this paper, we study prime, maximal and minimal weak-interior ideals of semiring.

## References


[1] P. J. Allen, *A fundamental theorem of homomorphism for semirings*, Proc. Amer. Math. Sco., 21 (1969), 412–416.

[2] T. K. Dutta and S.K. Sardar, *On the operator semirings of a semiring*, Southeast Asian Bull. Math. 26 (2002), 203-213.

[3] R. A. Good and D. R. Hughes, *Associated groups for a semigroup*, Bull. Amer. Math. Soc., 58(1952), 624-625.

[4] M. Henriksen, *Ideals in semirings with commutative addition*, Amer. Math. Soc. Notices, 5 (1958), 321.

[5] K. Iseki, *Quasi-ideals in semirings without zero*, Proc. Japan Acad., 34 (1958), 79-84.

[6] K. Iseki, *Ideal theory of semiring*, Proc. Japan Acad., 32(1956), 554-559.

[7] K. Iseki, *Ideal in semirings*, Proc. Japan Acad., 34(1958), 29-31.

[8] R. D. Jagatap and Y. S. Pawar, *Quasi-ideals and minimal quasi-ideals in semirings*, Novi Sad J. Math., 39(2) (2009), 79-87.

[9] R.D. Jagatap and Pawar, Y.S. *Bi-ideals in semirings*, Bull. Inter. Math. Virtual Inst., 6(2)(2016), 169-179.

[10] S. Lajos, *On the bi-ideals in semigroups*, Proc. Japan Acad., 45(1969), 710-712.

[11] S. Lajos and F. A. Szasz, *On the bi-ideals in associative ring*, Proc. Japan Acad., 46 (1970), 505-507.

[12] M. Murali Krishna Rao, $\Gamma-$*semirings-I*, Southeast Asian Bull. Math. 19 (1)(1995), 49–54.

[13] M.Murali Krishna Rao, *The Jacobson radical of* $\Gamma-$*semiring*, South eastAsian Bulletin of Mathematics,23(1999)127-134

[14] M. Murali Krishna Rao, *semiring with identity*, Discussiones Mathematicae General Algebra and Applications., 37 (2017) 189-207, doi:10.7151/dmgaa.1276.

[15] M.Murali Krishna Rao, *Ideals in ordered* $\Gamma-$*semirings*, Discussiones Mathematicae General Algebra and Applications 38 (2018), 47-68 doi:10.7151/dmgaa.1284

[16] M. Murali Krishna Rao, *Bi-interior Ideals in semigroups*, Discussiones Mathematicae General Algebra and Applications, 38 (2018),69-78 doi:10.7151/dmgaa.1284

[17] M. Murali Krishna Rao, *Left bi-quasi ideals of semirings*, Bull. Int. Math. Virtual Inst ,45-53 DOI: Vol. 8(2018), 10.7251/BIMVI1801045R

[18] M.Murali Krishna Rao, *Bi-quasi-ideals and fuzzy bi-quasi ideals of* $\Gamma-$ *semigroups*, Bull. Int. Math. Virtual Inst., Vol. 7(2)(2017), 231-242.

[19] M. Murali Krishna Rao, B. Venkateswarlu and N.Rafi, *Left bi-quasi-ideals of* $\Gamma-$ *semirings*, Asia Pacific Journal of Mathematics, Vol. 4, No. 2 (2017), 144-153 ISSN 2357-2205.

[20] M. Murali Krishna Rao and B. Venkateswarlu, *On generalized right derivations of* $\Gamma-$*incline*,*Journal Of The International Mathematical Virtual Institute,* **6** (2016), 31-47.

[21] M. Murali Krishna Rao and B. Venkateswarlu, *Regular* $\Gamma-$ *incline and field semiring*, Novi Sad J. of Math., 45 (2), (2015), 155-171.

[22] N. Nobusawa, *On a generalization of the ring theory*, Osaka. J.Math., 1 (1964), 81 – 89.

[23] A. M. Shabir and A. Batod, *A note on quasi ideal in semirings*, Southeast Asian Bull. Math. 7(2004), 923-928.

[24] O. Steinfeld, *Uher die quasi ideals*, Von halbgruppend Publ. Math., Debrecen, 4 (1956), 262-275.

[25] H. S. Vandiver, *Note on a simple type of algebra in which cancellation law of addition does not hold*, Bull. Amer. Math. Soc.(N.S.), 40 (1934), 914–920.


Received by editors 29.3.2021; Revised version 30.3.2022; Available online 18.4.2022.

M. Murali Krishna Rao, Department of Mathematics GIT, GITAM(Deemed to be University),, Visakhapatnam- 530 045, Andhra Pradesh, India.
*Email address*: mmarapureddy@gmail.com