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SOME CURVATURE RESULTS ON KENMOTSU METRIC SPACES

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Abstract. In this paper we present the curvature tensors of Kenmotsu manifold satisfying the conditions $R(X,Y)\cdot W_0=0$, $R(X,Y)\cdot W_1^{\star}=0$, $R(X,Y)\cdot W_1=0$, $R(X,Y)\cdot W_3=0$ and $R(X,Y)\cdot W_4=0$. According these cases, Kenmotsu manifolds have been characterized. I think that some interesting results on a Kenmotsu metric manifold are obtained.

1. Introduction

K.Kobayashi and K. Nomizu shown that any two simply connected complete Riemannian manifolds of constant curvature k are isometric to each other in 1963 [**9**]. After that Kenmotsu manifolds have been studied by many authors in several ways to a different extent such as [**17**].

K. Kenmotsu studied a class of contact Riemannian manifolds an call them Kenmotsu manifold [**8**]. He denote that if a Kenmotsu manifold satisfies the condition $R(X,Y)\cdot R=0$, where R is the Riemanniann curvature tensor and $R(X,Y)$ denotes the derivation of the tensor algebra at each point of the tangent space.

Subsequent to, K. De and U.C. De obtained conharmonically flat and $\phi-$conhar monically flat Kenmotsu manifold and they proved that the manifold is an Einstein manifold and a $\eta-$Einstein manifold. They researched a $3-$ dimensional Kenmotsu manifold admitting a non-null concircular vector field [**4**].

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The object of this paper is to study properties of the some certain curvature tensor in a Kenmotsu metric manifold. In the present paper we survey $R(X,Y)\cdot W_0 = 0$, $R(X,Y)\cdot W_1^\star = 0$, $R(X,Y)\cdot W_1 = 0$, $R(X,Y)\cdot W_3 = 0$ and $R(X,Y)\cdot W_4 = 0$, where W_0, W_1, $W_1^\star$, W_3, and W_4 denote the curvature tensors of a manifold, respectively.

2. Preliminaries

Let M be a $(2n+1)-$dimensional connected almost contact metric manifold with an almost contact metric structure (ϕ,ξ,η,g), that is, ϕ is an $(1,1)$ tensor field, ξ is a vector field, η is a 1-form and the Riemannian metric g satisfying

$$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\phi X) = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta\phi = 0 \tag{2.2}$$

for all $X, Y \in \chi(M)$ [**8**]. Let g be Riemannian metric compatible with (ϕ,ξ,η), that is

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.3}$$

or equivalently,

$$g(X,\phi Y) = -g(\phi X, Y) \quad \text{and} \quad g(X,\xi) = \eta(X) \tag{2.4}$$

for all $X, Y \in \chi(M)$ [**2**]. If moreover,

$$(\nabla_X \phi)Y = -\eta(Y)\phi X - g(X,\phi Y)\xi, \tag{2.5}$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.6}$$

where ∇ denotes the Riemannian connection of g hold, then (M,ϕ,ξ,η,g) is called an almost Kenmotsu manifold. An almost Kenmotsu manifold becomes a Kenmotsu manifold if

$$g(X,\phi Y) = d\eta(X,Y). \tag{2.7}$$

In a Kenmotsu manifold M, the following relation holds [**8, 5**]:

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y), \tag{2.8}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.9}$$

$$R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{2.10}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{2.11}$$

$$Q\xi = -(n-1)\xi, \tag{2.12}$$

where R is the Riemannian curvature tensor and S is Ricci tensor defined by $S(X,Y) = g(QX,Y)$, where Q is Ricci operator. It yields to

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y). \tag{2.13}$$

A Kenmotsu manifold M is said to be an $\eta-$Einstein manifold if its Ricci tensor S of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \tag{2.14}$$

for arbitrary vector fields X, Y; where a and b are functions on (M^{2n+1}, g). If $b = 0$, then $\eta-$ Einstein manifold becomes Einstein manifold [**13, 8**].

Let M be an $(2n+1)-$dimensional Kenmotsu manifold. The curvature tensor $\widetilde{R}$ of M with respect to the connection $\widetilde{\nabla}$ is defined by

$$\widetilde{R}(X,Y)Z = \widetilde{\nabla}_X\widetilde{\nabla}_Y Z - \widetilde{\nabla}_Y\widetilde{\nabla}_X Z - \widetilde{\nabla}_{[X,Y]}Z. \tag{2.15}$$

Then, in a Kenmotsu manifold, we have

$$\widetilde{R}(X,Y)Z = R(X,Y)Z + g(Y,Z)X - g(X,Z)Y, \tag{2.16}$$

where $R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$, is the curvature tensor of M with respect to the connection ∇.

The Ricci tensor $\widetilde{S}$ and the scalar curvature $\widetilde{r}$ of the Kenmotsu manifold M with respect to the connection $\widetilde{\nabla}$ is given by

$$\widetilde{S}(X,Y) = \sum_{i=1}^{n} g(\widetilde{R}(e_i, X)Y, ei) = S(X,Y) + (n-1)g(X,Y) \tag{2.17}$$

and

$$\widetilde{r} = \sum_{i=1}^{n} \widetilde{S}(e_i, e_i) = r + n(n-1), \tag{2.18}$$

where $\widetilde{r}$ and r are the scalar curvatures of the connection $\widetilde{\nabla}$ and ∇, respectively [**18, 19, 21**].

The concept of W_0-curvature tensor was defined by [**12**]. W_0-curvature tensor, W_1-curvature tensor, $W_1^{\star}-$curvature tensor, W_3-curvature tensor and W_4-curvature tensor of a $(2n+1)$-dimensional Riemannian manifold are, respectively, defined as

$$W_0(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - g(X,Z)QY], \tag{2.19}$$

$$W_1(X,Y)Z = R(X,Y)Z + \frac{1}{2n}[S(Y,Z)X - S(X,Z)Y], \tag{2.20}$$

$$W_1^{\star}(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - S(X,Z)Y], \tag{2.21}$$

$$W_3(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(X,Z)Y - g(Y,Z)QX], \tag{2.22}$$

$$W_4(X,Y)Z = R(X,Y)Z + \frac{1}{2n}[g(X,Z)QY - g(X,Y)QZ], \tag{2.23}$$

for all $X, Y, Z \in \chi(M)$ [**11, 12**].

3. Some Curvature Results On Kenmotsu Metric Spaces

In this section, we will give the main results for this paper.

Let M be $(2n+1)-$dimensional Kenmotsu metric manifold and we denote W_0 curvature tensor from (2.19), we have for later

$$W_0(X,Y)\xi = \eta(X)Y - \frac{n+1}{2n}\eta(Y)X + \frac{1}{2n}\eta(X)QY. \tag{3.1}$$

Putting $X = \xi$, in (3.1)

$$W_0(\xi,Y)\xi = Y - \frac{n+1}{2n}\eta(Y)\xi + \frac{1}{2n}QY. \tag{3.2}$$

In (2.20) choosing $Z = \xi$ and using (2.9), we obtain

$$W_1(X,Y)\xi = \frac{3n-1}{2n}(\eta(X)Y - \eta(Y)X). \tag{3.3}$$

In (3.3), it follows

$$W_1(\xi,Y)\xi = \frac{3n-1}{2n}(Y - \eta(Y)\xi). \tag{3.4}$$

From (2.21) and (2.9), we arrive

$$W_1^{\star}(X,Y)\xi = \frac{n+1}{2n}(\eta(X)Y - \eta(Y)X), \tag{3.5}$$

and

$$W_1^{\star}(\xi,Y)\xi = \frac{n+1}{2n}(Y - \eta(Y)\xi). \tag{3.6}$$

Choosing $Z = \xi$, in (2.22), we obtain

$$W_3(X,Y)\xi = \frac{3n-1}{2n}\eta(X)Y - \eta(Y)X + \frac{1}{2n}\eta(Y)QX. \tag{3.7}$$

In (3.7) it follows

$$W_3(\xi,Y)\xi = \frac{3n-1}{2n}(Y - \eta(Y)\xi). \tag{3.8}$$

In (2.23), choosing $Z = \xi$ and using (2.9), we get

$$W_4(X,Y)\xi = \eta(X)Y - \eta(Y)X + \frac{1}{2n}\{\eta(X)QY + (n-1)g(X,Y)\xi\}. \tag{3.9}$$

Setting $X = \xi$, in (3.9), we arrive

$$W_4(\xi,Y)\xi = Y - \frac{n+1}{2n}\eta(Y)\xi + \frac{1}{2n}QY. \tag{3.10}$$

THEOREM 1. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Kenmotsu manifold. Then M is a W_0 semi-symmetric if and only if M is an Einstein manifold.*

PROOF. Suppose M is a W_0 semi-symmetric. This implies that

$$\begin{aligned}(R(X,Y)W_0)(U,W)Z &= R(X,Y)W_0(U,W)Z - W_0(R(X,Y)U,W)Z \\ &\quad -W_0(U,R(X,Y)W)Z \\ &\quad -W_0(U,W)R(X,Y)Z = 0, \qquad (3.11)\end{aligned}$$

for any $X,Y,U,W,Z \in \chi(M)$. Taking $X = Z = \xi$ in (3.11), making use of (3.1), (2.9) and (2.10), for $A = -\frac{n+1}{2n}$, $B = \frac{1}{2n}$, we have

$$\begin{aligned}(R(\xi,Y)W_0)(U,W)\xi &= R(\xi,Y)(\eta(U)W + A\eta(W)U + B\eta(U)QW) \\ &\quad -W_0(\eta(U)Y) - g(Y,U)\xi, W)\xi \\ &\quad -W_0(U,\eta(W)Y - g(Y,W)\xi)\xi \\ &\quad -W_0(U,W)(Y - \eta(Y)\xi) = 0. \qquad (3.12)\end{aligned}$$

Taking into account (3.1), (3.2), (2.9) in (3.12), we obtain

$$\begin{aligned}&W_0(U,W)Y + \eta(U)g(Y,W)\xi + B(n-1)\eta(U)\eta(W)Y \\ &+B\eta(U)S(Y,W)\xi - g(Y,U)W - Bg(Y,U)QW \\ &+B\eta(U)\eta(W)QY + g(Y,W)U + A\eta(U)g(Y,W)\xi \\ &+Bg(Y,W)QU = 0. \qquad (3.13)\end{aligned}$$

Putting (2.19), (2.4), choosing $W = \xi$ in (3.13), we arrive

$$\begin{aligned}&BS(U,Y)\xi - \eta(Y)U - B\eta(Y)QU + \eta(U)\eta(Y)\xi + B(n-1)\eta(U)Y \\ &-B(n-1)\eta(U)\eta(Y)\xi + B(n-1)g(Y,U)\xi + B\eta(U)QY \\ &+\eta(Y)U + A\eta(U)\eta(Y)\xi + B\eta(Y)QU = 0. \qquad (3.14)\end{aligned}$$

Inner product both sides of (3.14) by $\xi \in \chi(M)$ and using (2.11),we conclude

$$S(U,Y) = (1-n)g(U,Y).$$

So, M is an Einstein manifold. Conversely, let $M^{2n+1}(\phi,\xi,\eta,g)$ be an Einstein manifold i.e. $S(U,Y) = (1-n)g(U,Y)$, then from (3.14), (3.13), (3.12) and (3.11), we have $R(X,Y) \cdot W_0 = 0$. □

THEOREM 2. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Kenmotsu manifold. Then M is a W_1 semi-symmetric if and only if M is an Einstein manifold.*

PROOF. Suppose that M is a W_1 semi-symmetric. This yields to

$$\begin{aligned}(R(X,Y)W_1)(U,W)Z &= R(X,Y)W_1(U,W)Z - W_1(R(X,Y)U,W)Z \\ &\quad -W_1(U,R(X,Y)W)Z \\ &\quad -W_1(U,W)R(X,Y)Z = 0, \qquad (3.15)\end{aligned}$$

for any $X,Y,U,W,Z \in \chi(M)$. Taking $X = Z = \xi$ in (3.15) and using (3.3), (2.9), (2.10), for $A = \frac{3n-1}{2n}$, we obtain

$$\begin{aligned}(R(\xi,Y)W_1)(U,W)\xi &= R(\xi,Y)(A\eta(U)W - A\eta(W)U) \\ &\quad -W_1(\eta(U)Y) - g(Y,U)\xi, W)\xi \\ &\quad -W_1(U,\eta(W)Y - g(Y,W)\xi)\xi \\ &\quad -W_1(U,W)(Y - \eta(Y)\xi) = 0. \qquad (3.16)\end{aligned}$$

And we arrive

$$\begin{aligned}
&A\eta(U)R(\xi,Y)W - A\eta(W)R(\xi,Y)U - \eta(U)W_1(Y,W)\xi \\
&+g(Y,U)W_1(\xi,W)\xi - \eta(W)W_1(U,Y)\xi + g(Y,W)W_1(U,\xi)\xi \\
&-W_1(U,W)Y + \eta(Y)W_1(U,W)\xi = 0. \qquad (3.17)
\end{aligned}$$

Taking into account that (2.9), (2.10) and (3.3) in (3.17), we get

$$W_1(U,W)Y - Ag(Y,U)W + Ag(Y,W)U = 0. \qquad (3.18)$$

Putting $U = \xi$, using (2.20) in (3.18) and inner product both sides of (3.18) by $\xi \in \chi(M)$,we conclude

$$S(Y,W) = (1-n)g(Y,W).$$

Thus, M is an Einstein manifold. Conversely, let $M^{2n+1}(\phi,\xi,\eta,g)$ be an Einstein manifold i.e. $S(Y,W) = (1-n)g(Y,W)$, then from (3.18), (3.17), (3.16) and (3.15), we have $R(X,Y)\cdot W_1 = 0$. □

Theorem 3. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Kenmotsu manifold. Then M is a $W_1^\star$ semi-symmetric if and only if M is an Einstein manifold.*

Proof. Suppose that M is a $W_1^\star$ semi-symmetric. This yields to

$$\begin{aligned}
(R(X,Y)W_1^\star)(U,W)Z &= R(X,Y)W_1^\star(U,W)Z - W_1^\star(R(X,Y)U,W)Z \\
&\quad -W_1^\star(U,R(X,Y)W)Z \\
&\quad -W_1^\star(U,W)R(X,Y)Z = 0, \qquad (3.19)
\end{aligned}$$

for any $X,Y,U,W,Z \in \chi(M)$. Taking $X = Z = \xi$ in (3.19) and using (3.5), (2.9), (2.10), for $A = \frac{n+1}{2n}$, we obtain

$$\begin{aligned}
(R(\xi,Y)W_1^\star)(U,W)\xi &= R(\xi,Y)(A\eta(U)W - A\eta(W)U) \\
&\quad -W_1^\star(\eta(U)Y) - g(Y,U)\xi,W)\xi \\
&\quad -W_1^\star(U,\eta(W)Y - g(Y,W)\xi)\xi \\
&\quad -W_1^\star(U,W)(Y - \eta(Y)\xi) = 0. \qquad (3.20)
\end{aligned}$$

and from (3.20), we arrive

$$\begin{aligned}
&A\eta(U)R(\xi,Y)W - A\eta(W)R(\xi,Y)U - \eta(U)W_1^\star(Y,W)\xi \\
&+g(Y,U)W_1^\star(\xi,W)\xi - \eta(W)W_1^\star(U,Y)\xi + g(Y,W)W_1^\star(U,\xi)\xi \\
&-W_1^\star(U,W)Y + \eta(Y)W_1^\star(U,W)\xi = 0. \qquad (3.21)
\end{aligned}$$

Taking into account that (2.9), (2.10) and (3.5) in (3.21), we get

$$W_1^\star(U,W)Y - Ag(Y,U)W + Ag(Y,W)U = 0. \qquad (3.22)$$

Setting $U = \xi$ and using (2.11), inner product both sides of (3.22) by $\xi \in \chi(M)$, we have

$$S(Y,W) = (1-n)g(Y,W).$$

Thus, M is an Einstein manifold. Conversely, let $M^{2n+1}(\phi,\xi,\eta,g)$ be an Einstein manifold i.e. $S(Y,W) = (1-n)g(Y,W)$, then from (3.22), (3.21), (3.20) and (3.19), we have $R(X,Y)\cdot W_1^\star = 0$. □

THEOREM 4. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Kenmotsu manifold. Then M is a W_3 semi-symmetric if and only if M is an Einstein manifold.*

PROOF. Suppose that M is a W_3 semi-symmetric. This means that

$$\begin{aligned}(R(X,Y)W_3)(U,W,Z) &= R(X,Y)W_3(U,W)Z - W_3(R(X,Y)U,W)Z\\ &\quad -W_3(U,R(X,Y)W)Z\\ &\quad -W_3(U,W)R(X,Y)Z = 0,\end{aligned} \tag{3.23}$$

for any $X,Y,U,W,Z \in \chi(M)$. Setting $X = Z = \xi$ in (3.23) and making use of (3.7), (2.9), for $A = \frac{3n-1}{2n}$, $B = \frac{1}{2n}$, we obtain

$$\begin{aligned}(R(\xi,Y)W_3)(U,W)\xi &= R(\xi,Y)(A\eta(U)W - \eta(W)U + B\eta(W)QU)\\ &\quad -W_3(\eta(U)Y - g(Y,U)\xi,W)\xi\\ &\quad -W_3(U,\eta(W)Y - g(Y,W)\xi)\xi\\ &\quad -W_3(U,W)(Y - \eta(Y)\xi) = 0.\end{aligned} \tag{3.24}$$

Using (3.7), (3.8), (2.9) in (3.24), we get

$$\begin{aligned}&W_3(U,W)Y - \eta(W)g(Y,U)\xi + B(n-1)\eta(W)\eta(U)Y\\ &+B\eta(W)S(Y,U)\xi + B\eta(W)\eta(U)QY - Ag(Y,U)W\\ &+A\eta(W)g(U,Y)\xi + Ag(Y,W)U = 0.\end{aligned} \tag{3.25}$$

Making use of (2.22), choosing $W = \xi$, and inner product both sides of (3.25) by $\xi \in \chi(M)$, we have

$$\begin{aligned}&BS(Y,U) - g(Y,U)\xi + B(n-1)\eta(U)Y\\ &+B\eta(U)QY + Ag(Y,U)\xi = 0.\end{aligned} \tag{3.26}$$

From (3.26) and by using (2.11), we conclude

$$S(Y,U) = (1-n)g(Y,U).$$

This tell us, M is an Einstein manifold. Conversely, let $M^{2n+1}(\phi,\xi,\eta,g)$ be an Einstein manifold i.e. $S(Y,U) = (1-n)g(Y,U)$, then from (3.26), (3.25), (3.24) and (3.23), we have $R(X,Y)\cdot W_3 = 0$. □

THEOREM 5. *Let $M^{2n+1}(\phi,\xi,\eta,g)$ be a Kenmotsu manifold. Then M is a W_4 semi-symmetric if and only if M is an $\eta-$Einstein manifold.*

PROOF. Suppose that M is a W_4 semi-symmetric. This means that

$$\begin{aligned}(R(X,Y)W_4)(U,W,Z) &= R(X,Y)W_4(U,W)Z - W_4(R(X,Y)U,W)Z\\ &\quad -W_4(U,R(X,Y)W)Z\\ &\quad -W_4(U,W)R(X,Y)Z = 0,\end{aligned} \tag{3.27}$$

for any $X, Y, U, W, Z \in \chi(M)$. Setting $X = Z = \xi$ in (3.27) and making use of (3.9), (2.9), (2.10), for $A = \frac{1}{2n}$, $B = \frac{n-1}{2n}$, we obtain

$$\begin{aligned}(R(\xi, Y)W_4)(U, W)\xi &= R(\xi, Y)(\eta(U)W - \eta(W)U + A\eta(U)QW \\ &\quad +Bg(U, W)\xi) - W_4(\eta(U)Y - g(Y, U)\xi, W)\xi \\ &\quad -W_4(U, \eta(W)Y - g(Y, W)\xi)\xi \\ &\quad -W_4(U, W)(Y - \eta(Y)\xi) = 0. \end{aligned} \tag{3.28}$$

Using (3.9) and (3.10) in (3.28), we get

$$\begin{aligned}&W_4(U, W)Y + \eta(U)g(Y, W)\xi - \eta(W)g(Y, U)\xi \\ &+A(n-1)\eta(U)\eta(W)Y + A\eta(U)S(Y, W)\xi \\ &+Bg(U, W)Y + g(Y, U)W + Ag(Y, U)QW \\ &+A(U)\eta(W)QY + g(Y, W)U + Ag(Y, W)QU = 0. \end{aligned} \tag{3.29}$$

Making use of (2.23) and choosing $U = \xi$ and inner product both sides of in (3.29) by $\xi \in \chi(M)$, we have

$$\begin{aligned}&\eta(Y)\eta(W) + g(Y, W) + AS(Y, W) + B\eta(Y)\eta(W) \\ &-A(n-1)\eta(Y)\eta(W) - A(n-1)g(Y, W) = 0. \end{aligned} \tag{3.30}$$

From (3.30) and (2.11), we obtain

$$S(Y, W) = -(n+1)g(Y, W) - 2n\eta(Y)\eta(W).$$

Thus, M is an $\eta-$Einstein manifold. Conversely, let $M^{2n+1}(\phi, \xi, \eta, g)$ be an $\eta-$Einstein manifold i.e. $S(Y, W) = -(n+1)g(Y, W) - 2n\eta(Y)\eta(W)$, then from (3.30), (3.29), (3.28) and (3.27), we have $R(X, Y) \cdot W_4 = 0$. □

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