

MULTIDIMENSIONAL COMMON FIXED POINT THEOREMS IN \mathcal{V} - FUZZY METRIC SPACES

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ABSTRACT. In this article, we extend the concept of the \mathcal{CLR}_{Ψ} -property in \mathcal{V} - Fuzzy Metric Space, in the setting of multidimensional fuzzy metric spaces. Further, we prove the existence and uniqueness of multidimensional common fixed point theorems for weakly compatible mappings with the \mathcal{CLR}_{Ψ} -property.

1. Introduction

Mustafa and Sims [11] brought the however of the thought of \mathcal{G} - metric spaces as a speculation of metric spaces. Besides, Sedghi et. al [13] presented the idea of S - metric spaces as one of the speculations of the metric spaces. Abbas et. al. [2] broadened the thought of S - metric spaces to A - metric space by stretching out the definition to n -tuple. In 1965, Zadeh [18] at first presented the idea of fuzzy sets. From that point forward, a few powerful mathematicians thought about the idea of fuzzy sets to present many energizing ideas in the field of science, like fuzzy differential equations, fuzzy logic and fuzzy metric spaces. A fuzzy metric space is notable to be a significant speculation of the metric space. In 1975, kramosil and Michalek [8] utilized the idea of fuzzy sets to present the thought of fuzzy metric spaces. George and Veeramani [3] madified the idea of fuzzy metric spaces in the feeling of Kramosil and Michalek [8]. Sun and Yang [15] begat the possibility of \mathcal{G} - fuzzy metric spaces. Aamri and D. El Moutawakil [1] generalized the idea of non similarity by characterizing E.A. property for self mappings. Sintunavarat and Kumam [14] gave the meaning of as far as possible in the range property on

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fuzzy metric spaces. After the comprehensive survey of the past writing, Gupta and Kanwar [16] presented the thought of \mathcal{V} -fuzzy metric spaces. In 1987, Guo and Lakshmikantham [4] presented a significant idea of coupled fixed point for some continuous and discontinuous operators. From there on, numerous authors presented coupled, tripled and higher dimensional fixed point brings about various spaces. Sedgi et. al. [12] presented coupled fixed point brings about fuzzy metric spaces. As of late, Roldan et. al. [9] set up multidimensional coincidence results in partially ordered fuzzy metric spaces for compatible mappings. In 2015, Roldan et. al. [7] has studied multidimensional coincidence point results using CLRg property in ordered fuzzy metric spaces for compatible mappings.

The purpose of the paper is to verify the existence of multidimensional common fixed point for weakly compatible mapping in \mathcal{V} -fuzzy metric spaces by using \mathcal{CLR}_Ψ -property and $E.A$ -property.

2. Preliminaries

Let p be a positive integer and let $\Gamma_p = \{1, 2, \dots, p\}$. Let $\Xi : \mathcal{X}^p \rightarrow \mathcal{X}$ and $\Psi : \mathcal{X} \rightarrow \mathcal{X}$ be two mappings. Henceforth, let $\mu_1, \mu_2, \dots, \mu_p : \Gamma_p \rightarrow \Gamma_p$ be p mappings from Γ_p into itself and let φ be the p -tuple $(\mu_1, \mu_2, \dots, \mu_p)$.

DEFINITION 2.1. [16] Let \mathcal{X} be a non-empty set. A triple $(\mathcal{X}, \mathcal{V}, *)$ is said to be \mathcal{V} -FMS where $*$ is a continuous t -norm and \mathcal{V} is a fuzzy set on $\mathcal{X}^n \times (0, \infty)$ satisfying the following conditions for all $t, s > 0$:

- (V-1) $\mathcal{V}(\aleph, \aleph, \dots, \aleph, \eta, t) > 0$ for all $\aleph, \eta \in \mathcal{X}$ with $\aleph \neq \eta$,
- (V-2) $\mathcal{V}(\aleph_1, \aleph_1, \dots, \aleph_1, \aleph_2, t) \geq \mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, t)$ for all $\aleph_1, \aleph_2, \dots, \aleph_n \in \mathcal{X}$ with $\aleph_2 \neq \aleph_3 \neq \dots \neq \aleph_n$,
- (V-3) $\mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, t) = 1 \Leftrightarrow \aleph_1 = \aleph_2 = \aleph_3 = \dots = \aleph_n$,
- (V-4) $\mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, t) = \mathcal{V}(p(\aleph_1, \aleph_2, \dots, \aleph_n), t)$, where p is a permutation function,
- (V-5) $\mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, t + s) \geq \mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_{n-1}, l, t) * \mathcal{V}(l, l, \dots, l, \aleph_n, s)$,
- (V-6) $\lim_{t \rightarrow \infty} \mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, t) = 1$,
- (V-7) $\mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

EXAMPLE 2.1. Let $\mathcal{X} = R$ and (\mathcal{X}, A) be a A metric space. Define $\mathcal{V} : \mathcal{X}^n \times (0, \infty) \rightarrow [0, 1]$ such that

$$\mathcal{V}(\aleph_r, \aleph_r, \dots, \aleph_n, t) = \exp\left(-\frac{A(\aleph_r, \aleph_r, \dots, \aleph_n)}{t}\right)$$

for all $\aleph_r, \aleph_r, \dots, \aleph_n \in \mathcal{X}$ and $t > 0$. Then $(\mathcal{X}, \mathcal{V}, *)$ is a \mathcal{V} -fuzzy metric space.

LEMMA 2.1. [16] Let $(\mathcal{X}, \mathcal{V}, *)$ be a \mathcal{V} -FMS such that $\mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, kt) \geq \mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, t)$ with $k \in (0, 1)$. Then $\aleph_1 = \aleph_2 = \aleph_3 = \dots = \aleph_n$.

DEFINITION 2.2. [16] Let $(\mathcal{X}, \mathcal{V}, *)$ be a \mathcal{V} -FMS. A sequence $\{\aleph_r\}$ is said to be convergent to \aleph if $\lim_{r \rightarrow \infty} \mathcal{V}(\aleph_r, \aleph_r, \dots, \aleph_r, \aleph, t) = 1$

DEFINITION 2.3. [16] Let $(\mathcal{X}, \mathcal{V}, *)$ be a \mathcal{V} -FMS. A sequence $\{\aleph_r\}$ is said to be a Cauchy sequence if $\mathcal{V}(\aleph_r, \aleph_r, \dots, \aleph_r, \aleph_q, t) \rightarrow 1$ as $r, q, \rightarrow \infty$ for all $t > 0$.

DEFINITION 2.4. [16] The \mathcal{V} – FMS $(\mathcal{X}, \mathcal{V}, *)$ is said to be complete if every Cauchy sequence in \mathcal{X} is convergent.

DEFINITION 2.5. The mappings $\Xi : \mathcal{X}^p \rightarrow \mathcal{X}$ and $\Psi : \mathcal{X} \rightarrow \mathcal{X}$ are said to be φ – weakly compatible on \mathcal{V} – FMS if

$$\mathcal{V}\left(\Psi\Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)}), \dots, \Psi\Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)}), \right. \\ \left. \Xi(\Psi(\aleph_{\mu_i(1)}), \Psi(\aleph_{\mu_i(2)}) \dots, \Psi(\aleph_{\mu_i(p)}), t)\right) = 1$$

whenever $\Psi\aleph_i = \Psi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)})$, for all i and some $(\aleph_1, \aleph_2, \dots, \aleph_p) \in \mathcal{X}^p$.

DEFINITION 2.6. Let $(\mathcal{X}, \mathcal{V}, *)$ be a \mathcal{V} – FMS. The mapping $\Xi : \mathcal{X}^p \rightarrow \mathcal{X}$ and $\Psi : \mathcal{X} \rightarrow \mathcal{X}$ satisfy \mathcal{CLR}_Ψ - property if there exists $\{\aleph_r^1\}, \{\aleph_r^2\}, \dots, \{\aleph_r^p\} \in \mathcal{X}$ such that

$$\lim_{r \rightarrow \infty} \mathcal{V}\left(\Psi(\aleph_r^i), \dots, \Psi(\aleph_r^i), \Psi(\aleph_i), t\right) = 1 = \\ \lim_{r \rightarrow \infty} \mathcal{V}\left(\Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \dots, \Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \Psi(\aleph_i), t\right)$$

for some $\aleph_1, \aleph_2, \dots, \aleph_p \in \mathcal{X}$.

3. Main results

In the rest of this paper, we say that $(\eta_1, \eta_2, \dots, \eta_p) \in \mathcal{X}^p$ is a φ – common fixed point for the mappings $\Xi : \mathcal{X}^p \rightarrow \mathcal{X}$ and $\Psi : \mathcal{X} \rightarrow \mathcal{X}$ if $\Psi\eta_i = \Xi(\eta_{\mu_i(1)}, \eta_{\mu_i(2)}, \dots, \eta_{\mu_i(p)}) = \eta_i$.

THEOREM 3.1. Let $(\mathcal{X}, \mathcal{V}, *)$ be a \mathcal{V} – FMS and $\varphi = (\mu_1, \mu_2, \dots, \mu_p)$ be an p -tuple of mappings from Γ_p to itself. Consider the functions $\Xi : \mathcal{X}^p \rightarrow \mathcal{X}$ and $\Psi : \mathcal{X} \rightarrow \mathcal{X}$ such that

- (3.1.1) Ξ and Ψ are φ – weakly compatible,
- (3.1.2) The pair (Ξ, Ψ) holds \mathcal{CLR}_Ψ - property,
- (3.1.3) Assume that there exists $k \in (0, 1)$ such that

$$\mathcal{V}\left(\Xi(\aleph_1, \aleph_2, \dots, \aleph_p), \dots, \Xi(\aleph_1, \aleph_2, \dots, \aleph_p), \Xi(\eta_1, \eta_2, \dots, \eta_p), kt\right) \\ \geq \theta\left(*_{i=1}^p \mathcal{V}(\Psi\aleph_i, \dots, \Psi\aleph_i, \Psi\eta_i, t)\right)$$

for all $t > 0$, $\aleph_1, \aleph_2, \dots, \aleph_p, \eta_1, \eta_2, \dots, \eta_p \in \mathcal{X}$, where $\theta : [0, 1] \rightarrow [0, 1]$ is a continuous mapping such that $*^p\theta(a) \geq a$ for each $a \in [0, 1]$. Suppose that

$$\theta\left(*_{i=1}^p \mathcal{V}(\Psi\aleph_{\mu_j(i)}, \dots, \Psi\aleph_{\mu_j(i)}, \Psi\eta_{\mu_j(i)}, t)\right) \geq \theta\left(*_{i=1}^p \mathcal{V}(\Psi\aleph_i, \dots, \Psi\aleph_i, \Psi\eta_i, t)\right)$$

for all $i, j \in \{1, 2, \dots, p\}$ and $\aleph_1, \aleph_2, \dots, \aleph_p, \eta_1, \eta_2, \dots, \eta_p \in \mathcal{X}$.

Then Ξ and Ψ have a unique φ - common fixed point.

PROOF. The \mathcal{CLR}_Ψ - property for the pair (Ξ, Ψ) implies that

$$(3.1.4) \quad \lim_{r \rightarrow \infty} \mathcal{V} \left(\Psi(\aleph_r^i), \dots, \Psi(\aleph_r^i), \Psi(\aleph_i), t \right) = 1 = \\ \lim_{r \rightarrow \infty} \mathcal{V} \left(\Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \dots, \Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \Psi(\aleph_i), t \right)$$

for sequences $\{\aleph_r^1\}, \{\aleph_r^2\}, \dots, \{\aleph_r^p\}$ in \mathcal{X} and for some $\aleph_1, \aleph_2, \dots, \aleph_p \in \mathcal{X}$.
By using (3.1.1) and (3.1.4), we get

$$\mathcal{V} \left(\Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \dots, \Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \right. \\ \left. \Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)}, kt) \right) \\ \geq \theta \left(*_{j=1}^p \mathcal{V}(\Psi \aleph_r^{\mu_i(j)}, \dots, \Psi \aleph_r^{\mu_i(j)}, \Psi \aleph_{\mu_i(j)}, t) \right)$$

Let $r \rightarrow \infty$, we get

$$\mathcal{V} \left(\Psi \aleph_i, \dots, \Psi \aleph_i, \Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)}), kt \right) \\ \geq \theta \left(*_{j=1}^p \mathcal{V}(\Psi \aleph_{\mu_i(j)}, \dots, \Psi \aleph_{\mu_i(j)}, \Psi \aleph_{\mu_i(j)}, t) \right) \\ \geq \theta(1).$$

Since, θ verifies $1 \leq *^p \theta(1) \leq \min(\theta(1), \theta(1), \dots, \theta(1)) = \theta(1)$, then we have $\theta(1) = 1$.

Therefore,

$$(3.1.5) \quad \Psi \aleph_i = \Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)})$$

Suppose that

$$(3.1.6) \quad \Psi \aleph_i = \Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)}) = \eta_i$$

Since Ξ and Ψ are φ - weakly compatible mappings, we have

$$\mathcal{V} \left(\Psi \eta_i, \dots, \Psi \eta_i, \Xi(\eta_{\mu_i(1)}, \eta_{\mu_i(2)}, \dots, \eta_{\mu_i(p)}), t \right) \\ = \mathcal{V} \left(\Psi \Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)}), \dots, \Psi \Xi(\aleph_{\mu_i(1)}, \aleph_{\mu_i(2)}, \dots, \aleph_{\mu_i(p)}), \right. \\ \left. \Xi(\Psi \aleph_{\mu_i(1)}, \Psi \aleph_{\mu_i(2)}, \dots, \Psi \aleph_{\mu_i(p)}), t \right) = 1$$

This gives,

$$(3.1.7) \quad \Psi \eta_i = \Xi(\eta_{\mu_i(1)}, \eta_{\mu_i(2)}, \dots, \eta_{\mu_i(p)})$$

From (3.1.5) and (3.1.7) we get

$(\aleph_1, \aleph_2, \dots, \aleph_p), (\eta_1, \eta_2, \dots, \eta_p) \in \mathcal{X}^p$ are two φ - coincidence points of Ξ and Ψ .
For all t and q , define

$$\Delta_q(t) = *_{j=1}^q \mathcal{V}(\Psi\eta_q^j, \dots, \Psi\eta_q^j, \Psi\aleph_j, t).$$

We Claim that $\Delta_{q+1}(kt) \geq \Delta_q(t)$ for all $q, t > 0$.

For all q, t and j ,

$$\begin{aligned} \mathcal{V}(\Psi\eta_{q+1}^j, \dots, \Psi\eta_{q+1}^j, \Psi\aleph_j, kt) &= \mathcal{V}\left(\Xi(\eta_q^{\mu_j(1)}, \eta_q^{\mu_j(2)}, \dots, \eta_q^{\mu_j(p)}), \dots, \right. \\ &\quad \Xi(\eta_q^{\mu_j(1)}, \eta_q^{\mu_j(2)}, \dots, \eta_q^{\mu_j(p)}), \\ &\quad \left. \Xi(\aleph_{\mu_j(1)}, \aleph_{\mu_j(2)}, \dots, \aleph_{\mu_j(p)}), kt\right) \\ &\geq \theta\left(*_{i=1}^p \mathcal{V}(\Psi\eta_q^{\mu_j(i)}, \dots, \Psi\eta_q^{\mu_j(i)}, \Psi\aleph_{\mu_j(i)}, t)\right) \\ &\geq \theta\left(*_{i=1}^p \mathcal{V}(\Psi\eta_q^i, \dots, \Psi\eta_q^i, \Psi\aleph_i, t)\right) \\ &= \theta(\Delta_q(t)) \end{aligned}$$

Therefore,

$$\begin{aligned} \Delta_{q+1}(kt) &= *_{i=1}^p \mathcal{V}(\Psi\eta_{q+1}^i, \dots, \Psi\eta_{q+1}^i, \Psi\aleph_i, kt) \\ &\geq *_{i=1}^p \theta(\Delta_q(t)) \\ &\geq \Delta_q(t) \end{aligned}$$

That is, $\Delta_{q+1}(kt) \geq \Delta_q(t) \geq \dots \geq \Delta_0(\frac{t}{k^q})$ and as a consequence $\lim_{q \rightarrow \infty} \Delta_q(t) = 1$ for all $t > 0$. For all i and t

$$\begin{aligned} \mathcal{V}(\Psi\eta_q^i, \dots, \Psi\eta_q^i, \Psi\aleph_i, t) &\geq *_{j=1}^p \mathcal{V}(\Psi\eta_q^j, \dots, \Psi\eta_q^j, \Psi\aleph_j, t) \\ &\geq \Delta_q(t). \end{aligned}$$

Which means that $\lim_{q \rightarrow \infty} \mathcal{V}(\Psi\eta_q^i, \dots, \Psi\eta_q^i, \Psi\aleph_i, t) = 1$ for all $t > 0$. That is, $\lim_{q \rightarrow \infty} \Psi\eta_q^i = \Psi\aleph_i$. Thus, $\Psi\aleph_i = \Psi\eta_i$.

From (3.1.6) and (3.1.7), we get

$$\Psi\eta_i = \Xi(\eta_{\mu_i(1)}, \eta_{\mu_i(2)}, \dots, \eta_{\mu_i(p)}) = \eta_i$$

Hence, $(\eta_1, \eta_2, \dots, \eta_p) \in \mathcal{X}^p$ be a φ - common fixed point of Ξ and Ψ .

Suppose that $(\aleph_1, \aleph_2, \dots, \aleph_p), (\eta_1, \eta_2, \dots, \eta_p) \in \mathcal{X}^p$ are two φ - common fixed points of Ξ and Ψ with $\aleph_i \neq \eta_i$.

Clearly, $\aleph_i = \Psi\aleph_i = \Psi\eta_i = \eta_i$, which proves uniqueness. \square

THEOREM 3.2. *Let $(\mathcal{X}, \mathcal{V}, *)$ be a \mathcal{V} -FMS and $\varphi = (\mu_1, \mu_2, \dots, \mu_p)$ be an p -tuple of mappings from Γ_p to itself. Consider the functions $\Xi : \mathcal{X}^p \rightarrow \mathcal{X}$ and $\Psi : \mathcal{X} \rightarrow \mathcal{X}$ such that*

(3.2.1) Ξ and Ψ are φ - weakly compatible,

(3.2.2) The pair (Ξ, Ψ) holds E.A - property,

(3.2.3) Assume that there exists $k \in (0, 1)$ such that

$$\mathcal{V}\left(\Xi(\aleph_1, \aleph_2, \dots, \aleph_p), \dots, \Xi(\aleph_1, \aleph_2, \dots, \aleph_p), \Xi(\eta_1, \eta_2, \dots, \eta_p), kt\right) \geq \theta \left(\ast_{i=1}^p \mathcal{V}(\Psi \aleph_i, \dots, \Psi \aleph_i, \Psi \eta_i, t) \right)$$

for all $t > 0$, $\aleph_1, \aleph_2, \dots, \aleph_p, \eta_1, \eta_2, \dots, \eta_p \in \mathcal{X}$, where $\theta : [0, 1] \rightarrow [0, 1]$ is a continuous mapping such that $\ast^p \theta(a) \geq a$ for each $a \in [0, 1]$. Suppose that

$$\text{theta} \left(\ast_{i=1}^p \mathcal{V}(\Psi \aleph_{\mu_j(i)}, \dots, \Psi \aleph_{\mu_j(i)}, \Psi \eta_{\mu_j(i)}, t) \right) \geq \theta \left(\ast_{i=1}^p \mathcal{V}(\Psi \aleph_i, \dots, \Psi \aleph_i, \Psi \eta_i, t) \right)$$

for all $i, j \in \{1, 2, \dots, p\}$ and $\aleph_1, \aleph_2, \dots, \aleph_p, \eta_1, \eta_2, \dots, \eta_p \in \mathcal{X}$.

(3.2.4) The Range of Ψ is a closed subspace of \mathcal{X} .

Then Ξ and Ψ have a unique φ - common fixed point.

PROOF. The Pair of the functions (Ξ, Ψ) holds *E.A* - property, there exists p sequences $\{\aleph_r^1\}, \{\aleph_r^2\}, \dots, \{\aleph_r^p\} \in \mathcal{X}$ such that

$$\lim_{r \rightarrow \infty} \mathcal{V}\left(\Psi(\aleph_r^i), \dots, \Psi(\aleph_r^i), \aleph_i, t\right) = 1 = \lim_{r \rightarrow \infty} \mathcal{V}\left(\Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \dots, \Xi(\aleph_r^{\mu_i(1)}, \aleph_r^{\mu_i(2)}, \dots, \aleph_r^{\mu_i(p)}), \aleph_i, t\right)$$

for all i and some $\aleph_1, \aleph_2, \dots, \aleph_p \in \mathcal{X}$.

Since the range of Ψ is a closed subspace of \mathcal{X} . That is, $\aleph_i \in \Psi(\mathcal{X})$ which implies that $\aleph_i = \Psi(\eta_i)$ for some $\eta_i \in \mathcal{X}$ and for all i . Hence Ξ and Ψ are satisfy the \mathcal{CLR}_Ψ - property. The result follows from the previous theorem. \square

EXAMPLE 3.1. Let $(\mathcal{X}, \mathcal{V}, \ast)$ be a \mathcal{V} - FMS with $\mathcal{X} = [0, 1]$ where

$$\mathcal{V}(\aleph_1, \aleph_2, \dots, \aleph_n, t) = \frac{t}{t + \sum_{i=1}^n \sum_{i < j} |\aleph_i - \aleph_j|}.$$

Suppose $\Xi : \mathcal{X}^p \rightarrow \mathcal{X}$ and $\Psi : \mathcal{X} \rightarrow \mathcal{X}$ defined by

$$\Xi(\aleph_1, \aleph_2, \dots, \aleph_p) = \frac{\sum_{i=1}^p \aleph_i^2}{p}, \quad \Psi(\aleph) = \frac{\aleph}{p}.$$

Consider the sequences for $i = 1, 2, \dots, p$,

$$\{\aleph_q^i\} = \left\{ \frac{1}{p} + \frac{(-1)^{i+1}}{q} \right\}$$

Then we have,

$$\lim_{q \rightarrow \infty} \Xi(\aleph_q^1, \aleph_q^2, \dots, \aleph_q^p) = \lim_{q \rightarrow \infty} \Psi(\aleph_q^i) = \Psi\left(\frac{1}{p}\right) = \frac{1}{p^2}$$

This implies that (Ξ, Ψ) holds \mathcal{CLR}_Ψ - property and *E.A* - property.

And also, the pair (Ξ, Ψ) is φ - weakly compatible. All the conditions of Theorem 3.1 are satisfied. Hence, Ξ and Ψ have φ - common fixed point.

4. Conclusion

In this paper, we characterize weakly compatible and \mathcal{CLR}_Ψ -property in p dimensional setting. We likewise show the unique common fixed point result utilizing this kind of mapping and give a reasonable example.

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