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APPROXIMATION OF SPHERICAL FUZZY SET

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Abstract. This paper deals with the rough approximations of spherical fuzzy sets. Also we study the applications of rough spherical fuzzy sets. Distance between rough spherical fuzzy set, similarity measure between rough spherical fuzzy sets. Finally, a numerical example is solved to show the feasibility, applicability and effectiveness of the proposed methods.

1. Introduction

The fundamental concept of fuzzy set was introduced by Zadeh [**12**] in 1965. Atanassov [**2, 3**] introduced the concept of intutionisitc fuzzy sets. Cuong [**4, 5, 6, 7**] initiated the concept of the picture fuzzy set as a direct extension of intuitonistic fuzzy sets, which may be adequate in cases when human opinions are of types: yes, abstain, no, and refusal. Picture fuzzy sets have many applications in fuzzy inference, clustering, decision making etc. The spherical fuzzy set, proposed by Gndogdu and Kahraman [**9**], is an extension of Picture fuzzy set, as it provides enlargement of the space of degrees of truthness, abstinence, and falseness in the interval $[0, 1]$ with a condition $0 \leqslant A^2 + B^2 + C^2 \leqslant 1$. Ashraf et al. [**1**] presented the notion of spherical fuzzy sets with applications in decision making problems. The famous rough set theory was studied by Pawlak [**11**]. Many researchers are interested in rough set theory.

2. Preliminaries

This section deals with basic concepts related to this work. For basic definitions let us see [**1**] - [**12**].

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3. Rough spherical set

In this section we introduce rough spherical set. Rough spherical set is the approximation of a spherical set with respect to crisp approximation space.

DEFINITION 3.1. The upper and lower approximations of a spherical set denoted by $U(S)$ and $L(S)$ w.r.t the approximation space $(\mathcal{U}, \Omega)$ are defined as follows:

$U(S) = \left\{ \left\langle y, A_{U(S)}, B_{U(S)}, C_{U(S)} \right\rangle | y \in \mathcal{U} \right\}$, $L(S) = \left\{ \left\langle y, A_{L(S)}, B_{L(S)}, C_{L(S)} \right\rangle | y \in \mathcal{U} \right\}$

where

$$A_{U(S)}(z) = \bigvee_{v \in [z]_\Omega} A_S(v),\ B_{U(S)}(z) = \bigwedge_{v \in [z]_\Omega} B_S(v) \text{ and } C_{U(S)}(z) = \bigwedge_{v \in [z]_\Omega} C_S(v)$$

Also

$$A_{L(S)}(z) = \bigwedge_{v \in [z]_\Omega} A_S(v), B_{L(S)}(z) = \bigvee_{v \in [z]_\Omega} B_S(v) \text{ and } C_{L(S)}(z) = \bigwedge_{v \in [z]_\Omega} C_S(v).$$

The pair $(L(S), U(S))$ is called the rough spherical set of S w.r.t the approximation space $(\mathcal{U}, \Omega)$.

EXAMPLE 3.1. Let $\mathcal{U} = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}$ be the universe in the approximation space Ω. Let $\mathcal{U}/\Omega = \{\{q_1, q_3, q_9\}, \{q_2, q_7, q_{10}\}, \{q_4\}, \{q_5, q_8\}, \{q_6\}\}$ be the set of equivalence classes of $\mathcal{U}$. Let S be an spherical fuzzy set defined by

$$S = \begin{cases} \langle q_1, .4, .3, .5 \rangle \\ \langle q_2, .6, .3, .5 \rangle \\ \langle q_3, .7, .3, .5 \rangle \\ \langle q_4, .4, .6, .3 \rangle \\ \langle q_5, .6, .4, .5 \rangle \\ \langle q_6, .4, .6, .3 \rangle \\ \langle q_7, .3, .3, .5 \rangle \\ \langle q_8, .5, .4, .6 \rangle \\ \langle q_9, .4, .3, .4 \rangle \\ \langle q_{10}, .5, .2, .6 \rangle \end{cases}$$

then lower and upper-approximations of S are

$$L(S) = \begin{cases} \langle q_1, .4, .3, .4 \rangle \\ \langle q_2, .3, .3, .5 \rangle \\ \langle q_3, .4, .3, .4 \rangle \\ \langle q_4, .4, .6, .3 \rangle \\ \langle q_5, .5, .4, .5 \rangle \\ \langle q_6, .4, .6, .3 \rangle \\ \langle q_7, .3, .3, .5 \rangle \\ \langle q_8, .5, .4, .5 \rangle \\ \langle q_9, .4, .3, .4 \rangle \\ \langle q_{10}, .3, .3, .5 \rangle \end{cases}$$

also

$$U(S) = \begin{cases} \langle q_1, .7, .3, .5\rangle \\ \langle q_2, .6, .2, .5\rangle \\ \langle q_3, .7, .3, .4\rangle \\ \langle q_4, .4, .6, .3\rangle \\ \langle q_5, .6, .4, .5\rangle \\ \langle q_6, .4, .6, .3\rangle \\ \langle q_7, .6, .2, .5\rangle \\ \langle q_8, .6, .4, .5\rangle \\ \langle q_9, .7, .3, .4\rangle \\ \langle q_{10}, .3, .3, .5\rangle \end{cases}$$

4. Distance between two rough spherical fuzzy sets

In this section we define the distance between two rough spherical fuzzy sets $\mathcal{S}_1$ and $\mathcal{S}_2$ with respect to the approximation space $\mathcal{R}$ in the universe $\mathcal{U}$.

Definition 4.1. Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two rough spherical fuzzy sets with respect to the approximation space $\mathcal{R}$ in the universe $\mathcal{U}$. Also let $\mathcal{LA}$ and $\mathcal{UA}$ denotes the lower approximation and upper approximation of spherical fuzzy set.

(i) The Hamming distance of $\mathcal{S}_1$ and $\mathcal{S}_2$:

$HD_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) =$

$$\sum_{l=1}^{n} \left\{ \left|\mu_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \mu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right| + \left|\eta_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \eta_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right| + \left|\nu_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \nu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right| \right\}$$

$HD_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_2) =$

$$\sum_{l=1}^{n} \left\{ \left|\mu_{\mathcal{U}(\mathcal{S}_1)}(a_i) - \mu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right| + \left|\eta_{\mathcal{U}(\mathcal{S}_1)}(a_i) - \eta_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right| + \left|\nu_{\mathcal{U}(\mathcal{S}_1)}(a_i) - \nu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right| \right\}$$

(ii) The Normalized Hamming distance of $\mathcal{S}_1$ and $\mathcal{S}_2$:

$HD_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) =$

$$\frac{1}{3n}\sum_{l=1}^{n} \left\{ \left|\mu_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \mu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right| + \left|\eta_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \eta_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right| + \left|\nu_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \nu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right| \right\}$$

$HD_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_2) =$

$$\frac{1}{3n}\sum_{l=1}^{n} \left\{ \left|\mu_{\mathcal{U}(\mathcal{S}_1)}(a_i) - \mu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right| + \left|\eta_{\mathcal{U}(\mathcal{S}_1)}(a_i) - \eta_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right| + \left|\nu_{\mathcal{U}(\mathcal{S}_1)}(a_i) - \nu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right| \right\}$$

(iii) The Euclidean distance of $\mathcal{S}_1$ and $\mathcal{S}_2$:

$ED_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) =$

$$\sqrt{\sum_{l=1}^{n} \left(\mu_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \mu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right)^2 + \left(\eta_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \eta_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right)^2 + \left(\nu_{\mathcal{L}(\mathcal{S}_1)}(a_i) - \nu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right)^2}$$

$ED_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_2) =$

$$\sqrt{\sum_{l=1}^{n}\left(\mu_{\mathcal{U}(\mathcal{S}_1)}(a_i)-\mu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right)^2+\left(\eta_{\mathcal{U}(\mathcal{S}_1)}(a_i)-\eta_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right)^2+\left(\nu_{\mathcal{U}(\mathcal{S}_1)}(a_i)-\nu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right)^2}$$

(iv) The Normalized Euclidean distance of $\mathcal{S}_1$ and $\mathcal{S}_2$:

$NED_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_2)=$

$$\sqrt{\frac{1}{3n}\sum_{l=1}^{n}\left(\mu_{\mathcal{L}(\mathcal{S}_1)}(a_i)-\mu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right)^2+\left(\eta_{\mathcal{L}(\mathcal{S}_1)}(a_i)-\eta_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right)^2+\left(\nu_{\mathcal{L}(\mathcal{S}_1)}(a_i)-\nu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right)^2}$$

$NED_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_2)=$

$$\sqrt{\frac{1}{3n}\sum_{l=1}^{n}\left(\mu_{\mathcal{U}(\mathcal{S}_1)}(a_i)-\mu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right)^2+\left(\eta_{\mathcal{U}(\mathcal{S}_1)}(a_i)-\eta_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right)^2+\left(\nu_{\mathcal{U}(\mathcal{S}_1)}(a_i)-\nu_{\mathcal{U}(\mathcal{S}_2)}(a_i)\right)^2}.$$

5. Simiarity Measure between Rough Spherical Fuzzy sets

This section deals with similarity measures of two rough spherical fuzzy set. There are

(i) Distance similarity measure.

(ii) Similarity measure based on membership degrees.

5.1. Distance based similarity measure. Consider the Euclidean distance of two rough spherical fuzzy sets $\mathcal{S}_1$ and $\mathcal{S}_2$ then its similarity measure is defined as follows,

$$\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_2)=\frac{1}{1+ED_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_2)} \text{ and } \mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_2)=\frac{1}{1+ED_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_2)}.$$

Proposition 5.1. *The defined distance based similarity measure of lower and upper approximation of two rough spherical fuzzy sets $\mathcal{S}_1$ and $\mathcal{S}_2$ satisfies the following properties,*

PPT(1) $0\leqslant \mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_2)\leqslant 1$ *and* $0\leqslant \mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_2)\leqslant 1$.

PPT(2) $\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_2)=1\Leftrightarrow \mathcal{S}_1=\mathcal{S}_2$ *and* $\mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_2)=1\Leftrightarrow \mathcal{S}_1=\mathcal{S}_2$

PPT(3) $\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_2)=\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_2,\mathcal{S}_1)$ *and* $\mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_2)=\mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_2,\mathcal{S}_1)$

PPT(4) $\mathcal{S}_1\subset\mathcal{S}_2\subset\mathcal{S}_3\Rightarrow \mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_3)\leqslant min\left\{\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1,\mathcal{S}_2),\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_2,\mathcal{S}_3)\right\}$ *and*
$\mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_3)\leqslant min\left\{\mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_1,\mathcal{S}_2),\mathcal{S}^a_{\mathcal{U}}(\mathcal{S}_2,\mathcal{S}_3)\right\}$

Proof. PPT(1), PPT(2) and PPT(3) are obvious from definition. Let $\mathcal{S}_1,\mathcal{S}_2$ and $\mathcal{S}_3$ be three rough spherical fuzzy sets in the universe $\mathcal{U}=\{a_1,a_2,...,a_n\}$. Let $\mathcal{S}_1\subset\mathcal{S}_2\subset\mathcal{S}_3$ then for any $a\in\mathcal{U}$ we have,

$$\mu_{\mathcal{L}(\mathcal{S}_1)}(a)\leqslant\mu_{\mathcal{L}(\mathcal{S}_2)}(a)\leqslant\mu_{\mathcal{L}(\mathcal{S}_3)}(a),\;\; \eta_{\mathcal{L}(\mathcal{S}_1)}(a)\geqslant\eta_{\mathcal{L}(\mathcal{S}_2)}(a)\geqslant\eta_{\mathcal{L}(\mathcal{S}_3)}(a) \text{ and}$$
$$\nu_{\mathcal{L}(\mathcal{S}_1)}(a)\geqslant\nu_{\mathcal{L}(\mathcal{S}_2)}(a)\geqslant\nu_{\mathcal{L}(\mathcal{S}_3)}(a).$$

Also we can prove,

$$\left|\mu_{\mathcal{L}(\mathcal{S}_1)}(a)-\mu_{\mathcal{L}(\mathcal{S}_2)}(a)\right|\leqslant\left|\mu_{\mathcal{L}(\mathcal{S}_1)}(a)-\mu_{\mathcal{L}(\mathcal{S}_3)}(a)\right| \text{ and}$$
$$\left|\mu_{\mathcal{L}(\mathcal{S}_2)}(a)-\mu_{\mathcal{L}(\mathcal{S}_3)}(a)\right|\leqslant\left|\mu_{\mathcal{L}(\mathcal{S}_1)}(a)-\mu_{\mathcal{L}(\mathcal{S}_3)}(a)\right|.$$

Similarly,

$$\left|\eta_{\mathcal{L}(\mathcal{S}_1)}(a)-\eta_{\mathcal{L}(\mathcal{S}_2)}(a)\right|\geqslant\left|\eta_{\mathcal{L}(\mathcal{S}_1)}(a)-\eta_{\mathcal{L}(\mathcal{S}_3)}(a)\right| \text{ and}$$

$$\left|\eta_{\mathcal{L}(\mathcal{S}_2)}(a) - \eta_{\mathcal{L}(\mathcal{S}_3)}(a)\right| \geqslant \left|\eta_{\mathcal{L}(\mathcal{S}_1)}(a) - \eta_{\mathcal{L}(\mathcal{S}_3)}(a)\right|.$$

Also, holds

$$\left|\nu_{\mathcal{L}(\mathcal{S}_1)}(a) - \nu_{\mathcal{L}(\mathcal{S}_2)}(a)\right| \geqslant \left|\nu_{\mathcal{L}(\mathcal{S}_1)}(a) - \nu_{\mathcal{L}(\mathcal{S}_3)}(a)\right| \quad \text{and}$$

$$\left|\nu_{\mathcal{L}(\mathcal{S}_2)}(a) - \nu_{\mathcal{L}(\mathcal{S}_3)}(a)\right| \geqslant \left|\nu_{\mathcal{L}(\mathcal{S}_1)}(a) - \nu_{\mathcal{L}(\mathcal{S}_3)}(a)\right|.$$

Hence

$$\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_3) \leqslant min\left\{\mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2), \mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_2, \mathcal{S}_3)\right\}.$$

Similarly we prove for upper approximation. Hence the result. □

5.2. Similarity measure based on membership degrees.

Definition 5.1. The similarity measure based on membership degree between two rough spherical fuzzy sets $\mathcal{S}_1$ and $\mathcal{S}_2$ is defined as follows:

$$\mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) = \frac{\sum\limits_{l=1}^{n}\left\{min\left\{\mu_{\mathcal{L}(\mathcal{S}_1)}(a_i), \mu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right\} + min\left\{\eta_{\mathcal{L}(\mathcal{S}_1)}(a_i), \eta_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right\} + min\left\{\nu_{\mathcal{L}(\mathcal{S}_1)}(a_i), \nu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right\}\right\}}{\sum\limits_{l=1}^{n}\left\{max\left\{\mu_{\mathcal{L}(\mathcal{S}_1)}(a_i), \mu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right\} + max\left\{\eta_{\mathcal{L}(\mathcal{S}_1)}(a_i), \eta_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right\} + max\left\{\nu_{\mathcal{L}(\mathcal{S}_1)}(a_i), \nu_{\mathcal{L}(\mathcal{S}_2)}(a_i)\right\}\right\}}.$$

Proposition 5.2. *The defined membership degree based similarity measure of lower and upper approximation of two rough spherical fuzzy sets $\mathcal{S}_1$ and $\mathcal{S}_2$ satisfies the following properties:*

PPT(1) $0 \leqslant \mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) \leqslant 1$ *and* $0 \leqslant \mathcal{S}^b_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_2) \leqslant 1$.

PPT(2) $\mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) = 1 \Leftrightarrow \mathcal{S}_1 = \mathcal{S}_2$ *and* $\mathcal{S}^b_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_2) = 1 \Leftrightarrow \mathcal{S}_1 = \mathcal{S}_2$

PPT(3) $\mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) = \mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_2, \mathcal{S}_1)$ *and* $\mathcal{S}^b_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_2) = \mathcal{S}^b_{\mathcal{U}}(\mathcal{S}_2, \mathcal{S}_1)$

PPT(4) $\mathcal{S}_1 \subset \mathcal{S}_2 \subset \mathcal{S}_3 \Rightarrow \mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_3) \leqslant min\left\{\mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2), \mathcal{S}^a_{\mathcal{L}}(\mathcal{S}_2, \mathcal{S}_3)\right\}$ *and* $\mathcal{S}^b_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_3) \leqslant min\left\{\mathcal{S}^b_{\mathcal{U}}(\mathcal{S}_1, \mathcal{S}_2), \mathcal{S}^b_{\mathcal{U}}(\mathcal{S}_2, \mathcal{S}_3)\right\}$

Proof. PPT(1), PPT(2) and PPT(3) are obvious from definition. Let us prove PPT(4) Let $\mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{S}_3$ be three rough spherical fuzzy sets in the universe $\mathcal{U} = \{a_1, a_2, ..., a_n\}$. Let $\mathcal{S}_1 \subset \mathcal{S}_2 \subset \mathcal{S}_3$ then for any $a \in \mathcal{U}$ we have

$$\mu_{\mathcal{L}(\mathcal{S}_1)}(a) \leqslant \mu_{\mathcal{L}(\mathcal{S}_2)}(a) \leqslant \mu_{\mathcal{L}(\mathcal{S}_3)}(a);\ \eta_{\mathcal{L}(\mathcal{S}_1)}(a) \geqslant \eta_{\mathcal{L}(\mathcal{S}_2)}(a) \geqslant \eta_{\mathcal{L}(\mathcal{S}_3)}(a) \text{ and}$$

$$\nu_{\mathcal{L}(\mathcal{S}_1)}(a) \geqslant \nu_{\mathcal{L}(\mathcal{S}_2)}(a) \geqslant \nu_{\mathcal{L}(\mathcal{S}_3)}(a).$$

Now,

$$\mu_{\mathcal{L}(\mathcal{S}_1)}(a) + \eta_{\mathcal{L}(\mathcal{S}_1)}(a) + \nu_{\mathcal{L}(\mathcal{S}_2)}(a) \geqslant \mu_{\mathcal{L}(\mathcal{S}_1)}(a) + \eta_{\mathcal{L}(\mathcal{S}_1)}(a) + \nu_{\mathcal{L}(\mathcal{S}_3)}(a) \quad \text{and}$$

$$\mu_{\mathcal{L}(\mathcal{S}_2)}(a) + \eta_{\mathcal{L}(\mathcal{S}_2)}(a) + \nu_{\mathcal{L}(\mathcal{S}_1)}(a) \leqslant \mu_{\mathcal{L}(\mathcal{S}_3)}(a) + \eta_{\mathcal{L}(\mathcal{S}_3)}(a) + \nu_{\mathcal{L}(\mathcal{S}_1)}(a).$$

$$\begin{aligned} \mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2) &= \frac{\mu_{\mathcal{L}(\mathcal{S}_1)}(a) + \eta_{\mathcal{L}(\mathcal{S}_1)}(a) + \nu_{\mathcal{L}(\mathcal{S}_2)}(a)}{\mu_{\mathcal{L}(\mathcal{S}_2)}(a) + \eta_{\mathcal{L}(\mathcal{S}_2)}(a) + \nu_{\mathcal{L}(\mathcal{S}_1)}(a)} \\ &\geqslant \frac{\mu_{\mathcal{L}(\mathcal{S}_1)}(a) + \eta_{\mathcal{L}(\mathcal{S}_1)}(a) + \nu_{\mathcal{L}(\mathcal{S}_3)}(a)}{\mu_{\mathcal{L}(\mathcal{S}_3)}(a) + \eta_{\mathcal{L}(\mathcal{S}_3)}(a) + \nu_{\mathcal{L}(\mathcal{S}_1)}(a)} = \mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_3). \end{aligned}$$

Similarly we prove, $\mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_2, \mathcal{S}_3) \geqslant \mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_3)$. Hence,

$$\mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_2, \mathcal{S}_3) \leqslant min\left\{\mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_1, \mathcal{S}_2), \mathcal{S}^b_{\mathcal{L}}(\mathcal{S}_2, \mathcal{S}_3)\right\}.$$

Consequently, we can prove for upper approximation. □

6. Numerical example

Let us consider four kinds of minerals which are represented by the rough spherical sets $\mathcal{S}_i(i = 1, 2, 3)$ each of which is featured by the content of five minerals in the feature space $\mathcal{Q} = \{q_1, q_2, q_3, q_4, q_5\}$. Now we consider the another kind of unknown material

TABLE 1

	q_1	q_2	q_3	q_4	q_5
$\mathcal{S}_1$	(.5,.5,.3),(.7,.3,.3)	(.3,.8,.1),(.6,.3,.1)	(.3,.8,.1),(.6,.3,.1)	(.7,.0,.1),(.7,.0,.1)	(.5,.5,.3),(.7,.3,.3)
$\mathcal{S}_2$	(.1,.9,.1),(.8,.6,.1)	(.4,.6,.4),(.6,.5,.4)	(.4,.6,.4),(.6,.5,.4)	(.3,.2,.9),(.3,.2,.9)	(.1,.9,.1),(.8,.6,.1)
$\mathcal{S}_3$	(.4,.5,.5),(.5,.3,.5)	(.6,.4,.2),(.8,.3,.1)	(.6,.4,.2),(.8,.3,.1)	(.7,.3,.8),(.7,.3,.8)	(.4,.5,.5),(.5,.3,.5)

with data as given as follows.

$$\mathcal{S}_4 = \begin{cases} \langle q_1, (.6, .6, .2), (.6, .5, .2)\rangle \\ \langle q_2, (.5, .4, .1), (.8, .3, .1)\rangle \\ \langle q_3, (.5, .4, .1), (.8, .3, .1)\rangle \\ \langle q_4, (.7, .1, .6), (.7, .1, .6)\rangle \\ \langle q_5, (.6, .6, .2), (.6, .5, .2)\rangle \end{cases}$$

We can use the above proposed methods to identify to which type the unknown material $\mathcal{S}_4$ belongs. From the above table our conclusion is that unknown pattern S_4

TABLE 2

Similarity Measures	L(S)	U(S)
Hamming Distance	1.2333	.9667
Normalized Hamming Distance	.2067	.1933
Euclidean Distance	.6298	.4725
Membership Degree	.5488	.6548

belongs to S_2.

7. Conclusion

In this paper, we have approximations of spherical fuzzy sets. Also distance between two rough spherical fuzzy sets are defined. More over some similarity measures of rough spherical fuzzy sets are introduced. Some examples are investigated.

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