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DECOMPOSITIONS OF $n^*\mu$-CONTINUITY IN NANO TOPOLOGICAL SPACES

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Abstract. The aim of this article is to give decomposition of a weaker form of nano continuity, namely $n^*\mu$-continuity, by providing the concepts of $n^*\mu_t$-sets, $n^*\mu_\alpha*$-sets, $n^*\mu_t$-continuity and $n^*\mu_\alpha*$-continuity.

1. Introduction

Various interesting problems arise when one considers nano continuity and nano generalized continuity. Recently some decompositions of nano continuity are obtained by various authors with the help of nano generalized continuous functions in nano topological spaces ([**4, 15, 16**]).

In this article, we obtained decomposition of $n^*\mu$-continuity in nano topological spaces using $n^*\mu_p$-continuity [**7**], $n^*\mu_\alpha$-continuity [**7**], $n^*\mu_t$-continuity and $n^*\mu_\alpha*$-continuity.

2. Preliminaries

Definition 2.1. ([**10**]) If $(K, \tau_R(X))$ is the nano topological space with respect to X where $X \subseteq K$ and if $S \subseteq K$, then:

(1) The nano interior of the set S is defined as the union of all nano open subsets contained in S and it is denoted by $ninte(S)$. That is, $ninte(S)$ is the largest nano open subset of S.

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(2) The nano closure of the set S is defined as the intersection of all nano closed sets containing S and it is denoted by $nclo(S)$. That is, $nclo(S)$ is the smallest nano closed set containing S.

Definition 2.2. ([**10**]) A subset S of a space $(K, \tau_R(X))$ is called:

(1) nano α-open set if $S \subseteq ninte(nclo(ninte(S)))$.
(2) nano semi-open set if $S \subseteq nclo(ninte(S))$.
(3) nano pre-open set if $S \subseteq ninte(nclo(S))$.

The complements of the above mentioned nano open sets are called their respective nano closed sets.

The nano α-closure [**14**] (resp. nano semi-closure [**2, 3**], nano pre-closure [**1**]) of a subset S of K, denoted by $n\alpha$clo(S) (resp. nsclo(S), npclo(S)) is defined to be the intersection of all nano α-closed (resp. nano semi-closed, nano pre-closed) sets of $(K, \tau_R(X))$ containing S.

The nano α-interior [**14**] (resp. nano semi-interior [**2, 3**], nano pre-interior[**1**]) of a subset S of K, denoted by $n\alpha$inte(S) (resp. nsinte(S), npinte(S)) is defined to be the union of all nano α-open (resp. nano semi-open, nano pre-open) sets of (K, $\tau_R(X)$) containing S.

Definition 2.3. ([**6**]) A subset M of a space (U, $\tau_R(X)$) is called:

(1) Nano *g-semi closed set (briefly n*gs-closed) if nsclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is $n\hat{g}$-open in (K, $\tau_R(X)$). The complement of n*gs-closed set is called n*gs-open set.
(2) Nano $^*\mu$-closed set (briefly $n^*\mu$-closed) if nclo(M) $\subseteq$ T whenever M $\subseteq$ T and T is n*gs-open in (K, $\tau_R(X)$). The complement of $n^*\mu$-closed set is called $n^*\mu$-open set.
(3) Nano $^*\mu_\alpha$-closed (briefly $n^*\mu_\alpha$-closed) set if $n\alpha clo(M) \subseteq T$ whenever $M \subseteq T$ and T is n*gs-open in $(K, \tau_R(X))$. The complement of $n^*\mu_\alpha$-closed set is called $n^*\mu_\alpha$-open set.
(4) Nano $^*\mu_p$-closed (briefly $n^*\mu_p$-closed) set if $npclo(M) \subseteq T$ whenever $M \subseteq T$ and T is n*gs-open in $(U, \tau_R(X))$. The complement of $n^*\mu_p$-closed set is called $n^*\mu_p$-open set.

Definition 2.4. A subset S of a space (K, $\tau_R(X)$) is called:

(1) nt-set [**9**] if $ninte(S) = ninte(nclo(S))$.
(2) $n\alpha$*-set [**13**] if $ninte(S) = ninte(nclo(inte(S)))$.
(3) an $n\eta$-set [**5**] if $S = M \cap P$ where M is nano open and P is a nano α-closed set.
(4) $n^\iota\eta$-set [**7**] if S = M $\cap$ P, where M is n*gs-open and P is $n\alpha$-closed in (K, $\tau_R(X)$).
(5) $n^{\iota\iota}\eta$-set [**7**] if S = M $\cap$ P, where M is $n^*\mu_\alpha$-open and P is nt-set in $(K, \tau_R(X))$.

(6) $n^*\mu$lc*-set [**8**] if S = M ∩ P, where M is n*gs-open and P is nano closed $(K, \tau_R(X))$.

The family of all $n\eta$-sets (resp. $n^\iota\eta$-sets, $n^{\iota\iota}\eta$-sets) in a space $(K, \tau_R(X))$ is denoted by $n\eta(K, \tau_R(X))$ (resp. $n^\iota\eta(K, \tau_R(X))$, $n^{\iota\iota}\eta(K, \tau_R(X))$).

Remark 2.1. (1) Every nano closed set is $n^*\mu$-closed but not conversely [**6**].

(2) Every $n^*\mu$-closed set is $n^*\mu_\alpha$-closed but not conversely [**7**].

(3) Every $n\alpha$-closed set is $n^*\mu_\alpha$-closed but not conversely [**7**].

(4) Every $n^*\mu_\alpha$-closed set is $n^*\mu_p$-closed but not conversely [**7**].

Remark 2.2. The concepts of $n\alpha$-closed sets and $n^*\mu$-closed sets are independent.

Example 2.1. (1) Let $K = \{11, 12, 13\}$ with $K/R = \{\{13\}, \{11, 12\}\, \{12, 11\}\}$ and $X = \{11, 12\}$. The nano topology $\tau_R(X) = \{\phi, \{11, 12\}, K\}$. Then $\{11, 13\}$ is $n^*\mu$-closed set but it is not an $n\alpha$-closed set.

(2) Let $K = \{11, 12, 13\}$ with $K/R = \{\{11\}, \{12, 13\}\}$ and $X = \{11\}$. The nano topology $\tau_R(X) = \{\phi, \{11\}, K\}$. Then $\{13\}$ is $n\alpha$-closed set but it is not an $n^*\mu$-closed set.

Remark 2.3. ([**13**])

(1) Every nt-set is an $n\alpha$*-set but not conversely.
(2) The union of two $n\alpha$*-sets need not be an $n\alpha$*-set.
(3) The intersection of two $n\alpha$*-sets is an $n\alpha$*-set.

Definition 2.5. A function $f : (K, \tau_R(X)) \to (L, \sigma_R(Y))$ is called:

(1) nano continuous [**11**] if for each nano open set V of $(L, \sigma_R(Y))$, $f^{-1}(V)$ is a nano open in $(K, \tau_R(X))$.
(2) $n\alpha$-continuous [**12**] if for each nano open set V of $(L, \sigma_R(Y))$, $f^{-1}(V)$ is $n\alpha$-open in $(K, \tau_R(X))$.
(3) $n^*\mu$-continuous [**8**] if for each nano open set V of $(L, \sigma_R(Y))$, $f^{-1}(V)$ is $n^*\mu$-open in $(K, \tau_R(X))$.
(4) $n^*\mu_\alpha$-continuous [**8**](resp. $n^*\mu_p$-continuous [**8**]) if for each nano open set V of $(L, \sigma_R(Y))$, $f^{-1}(V)$ is $n^*\mu_\alpha$-open (resp. $n^*\mu_p$-open) set in $(K, \tau_R(X))$.
(5) $n^*\mu$lc*-continuous [**8**]if for each nano open set V of $(L, \sigma_R(Y))$, $f^{-1}(V) \in n^*\mu lc*(K, \tau_R(X))$.
(6) $n^\iota\eta$-continuous [**7**] if for each nano open set V of $(L, \sigma_R(Y))$, $f^{-1}(V) \in n^\iota\eta(K, \tau_R(X))$.
(7) $n^{\iota\iota}\eta$-continuous [**7**] if for each nano open set V of $(L, \sigma_R(Y))$, $f^{-1}(V) \in n^{\iota\iota}\eta(K, \tau_R(X))$.

Recently, the following decompositions have been established.

Theorem 2.1 ([**8**]). *A function $f : (K, \tau_R(X)) \to (L, \sigma_R(Y))$ is nano continuous if and only if it is both $n^*\mu$-continuous and $n^*\mu lc^*$-continuous.*

Theorem 2.2 ([7]). *A function* $f : (K, \tau_R(X)) \to (L, \sigma_R(Y))$ *is* $n\alpha$*-continuous if and only if it is both* $n^*\mu_\alpha$*-continuous and* $n^\iota\eta$*-continuous.*

Theorem 2.3 ([7]). *A function* $f : (K, \tau_R(X)) \to (L, \sigma_R(Y))$ *is* $n^*\mu_\alpha$*-continuous if and only if it is both* $n^*\mu_p$*-continuous and* $n^{\iota\iota}\eta$*-continuous.*

3. On $n^*\mu_t$-sets and $n^*\mu_\alpha*$-sets

Definition 3.1. A subset S of a space (K, $\tau_R(X)$) is called:

(1) an $n^*\mu_t$-set if S = M ∩ O, where M is $n^*\mu$-open in K and O is a nt-set in K.
(2) an $n^*\mu_\alpha*$-set if S = M ∩ O, where M is $n^*\mu$-open in K and O is a $n\alpha^*$-set in K.

The family of all $n^*\mu_t$-sets (resp. $n^*\mu_\alpha*$-sets) in a space $(K, \tau_R(X))$ is denoted by $n^*\mu_t(K, \tau_R(X))$ (resp. $n^*\mu_\alpha * (K, \tau_R(X))$).

Proposition 3.1. *Let S be a subset of K. Then:*

(1) *if S is a nt-set, then* $S \in n^*\mu_t(K, \tau_R(X))$.
(2) *if S is an* $n\alpha^*$*-set, then* $S \in n^*\mu_\alpha * (K, \tau_R(X))$.
(3) *if S is an* $n^*\mu$*-open set in X, then* $S \in n^*\mu_t(K, \tau_R(X))$ *and* $S \in n^*\mu_\alpha * (K, \tau_R(X))$.

Proof. The proof is straightforward from the definitions. □

Proposition 3.2. *In a space K, every* $n^*\mu_t$*-set is an* $n^*\mu_\alpha*$*-set but not conversely.*

Proof. The proof is straightforward from the definitions. □

Example 3.1. Let K and $\tau_R(X)$ as in the Example 2.1(1). Then the set $\{12, 13\}$ is $n^*\mu_\alpha*$-set but it is not an $n^*\mu_t$-set.

Remark 3.1. The following examples show that:

(1) the converse of Proposition 3.2 need not be true.
(2) the concepts of $n^*\mu_t$-sets and $n^*\mu_p$-open sets are independent.
(3) the concepts of $n^*\mu_\alpha*$-sets and $n^*\mu_\alpha$-open sets are independent.

Example 3.2. Let K and $\tau_R(X)$ as in the Example 2.1(1). Then the set $\{11\}$ is $n^*\mu_t$-set but not a nt-set and the set $\{11, 12\}$ is an $n^*\mu_\alpha^*$-set but it is not an $n\alpha^*$-set.

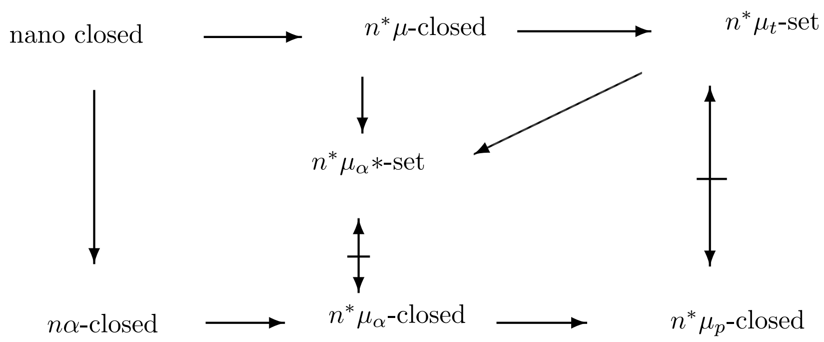
Example 3.3. Let K and $\tau_R(X)$ as in the Example 2.1(1). Then the set $\{13\}$ is both $n^*\mu_t$-set and $n^*\mu_\alpha^*$-set but it is not an$n^*\mu$-open set.

Example 3.4. Let K and $\tau_R(X)$ as in the Example 2.1(2). Then the set $\{13\}$ is an $n^*\mu_t$-set but not a $n^*\mu_p$-open set whereas the set {11, 12} is a $n^*\mu_p$-open set but not an $n^*\mu_t$-set.

Example 3.5. Let K and $\tau_R(X)$ as in the Example 2.1(2). Then the set $\{12\}$ is an $n^*\mu_\alpha$*-set but not an $n^*\mu_\alpha$-open set whereas the set $\{11, 13\}$ is an $n^*\mu_\alpha$-open set but not an $n^*\mu_\alpha$*-set.

Example 3.6. Let K and $\tau_R(X)$ as in the Example 2.1(1). Then the set $\{12\}$ is $n^*\mu_\alpha$*-set and $n^*\mu_t$-set but it is not an $n^*\mu$-closed.

Remark 3.2. From the above discussions, we have the following diagram of implications where A $\longrightarrow$ B (resp. A $\nleftrightarrow$ B) represents A implies B but not conversely (resp. A and B are independent of each other).



Remark 3.3. (1) The union of two $n^*\mu_t$-sets need not be an $n^*\mu_t$-set.

(2) The union of two $n^*\mu_\alpha*$-sets need not be an $n^*\mu_\alpha*$-set.

Example 3.7. (1) Let K and $\tau_R(X)$ as in the Example 2.1(1). The the sets are $\{12\}$ and $\{13\}$ are $n^*\mu_t$-sets but $\{11\} \cup \{13\} = \{11, 13\}$ is not an $n^*\mu_t$-set.

(2) Let K and $\tau_R(X)$ as in the Example 2.1(2). Then the sets are $\{11\}$ and $\{12\}$ are $n^*\mu_\alpha*$-sets but $\{11\} \cup \{12\} = \{11, 12\}$ is not an $n^*\mu_\alpha*$-set.

Remark 3.4. (1) The intersection of any numbers of $n^*\mu_t$-sets belongs to $n^*\mu_t$ (K, $\tau_R(X)$).

(2) The intersection of any numbers of $n^*\mu_\alpha*$-sets belongs to $n^*\mu_\alpha*$ (K, $\tau_R(X)$).

Lemma 3.1. *The following holds:*

(1) *A subset S of (K, $\tau_R(X)$) is n*μ-open* [**14**] *if and only if F $\subseteq$ ninte(S) whenever F $\subseteq$ S and F is n*gs-closed in K.*

(2) *A subset S of (K, $\tau_R(X)$) is n*μ_α-open* [**16**] *if and only if F $\subseteq$ nαinte(S) whenever F $\subseteq$ S and F is n*gs-closed in K.*

(3) *A subset S of (K, $\tau_R(X)$) is n*μ_p-open* [**16**] *if and only if F $\subseteq$ npinte(S) whenever F $\subseteq$ S and F is n*gs-closed in K.*

Theorem 3.1. *A subset S is n*μ-open in $(K, \tau_R(X))$ if and only if it is both $n^*\mu_\alpha$-open and $n^*\mu_\alpha$*-set in $(K, \tau_R(X))$.*

Proof. Necessity. The proof is obvious.

Sufficiency. Let S be both $n^*\mu_\alpha$-open set and $n^*\mu_\alpha$*-set. Since S is an $n^*\mu_\alpha$*-set, $S = M \cap O$, where M is $n^*\mu$-open and O is an $n\alpha$*-set. Assume that $F \subseteq S$, where F is n*gs-closed in K. Since M is $n^*\mu$-open, by Lemma 3.1 (1), $F \subseteq ninte(A)$. Since S is $n^*\mu_\alpha$-open in K, by Lemma 3.1 (2),

$$\begin{aligned} F &\subseteq n\alpha inte(S) = S \cap ninte(nclo(ninte(S))) \\ &= (M \cap O) \cap ninte(nclo(ninte(M \cap O))) \\ &\subseteq M \cap O \cap ninte(nclo(ninte(M))) \cap ninte(nclo(ninte(O))) \\ &= M \cap O \cap ninte(nclo(ninte(M))) \cap ninte(O) \\ &\subseteq ninte(O). \end{aligned}$$

Therefore, we obtained $F \subseteq ninte(O)$ and hence

$$F \subseteq ninte(M) \cap ninte(O) = ninte(S).$$

Hence S is n*gs-open, by Lemma 3.1 (1). □

THEOREM 3.2. *A subset S is $n^*\mu$-open in $(K, \tau_R(X))$ if and only if it is both $n^*\mu_p$-open and $n^*\mu_t$-set in $(K, \tau_R(X))$.*

PROOF. Similar to Theorem 3.1. □

DEFINITION 3.2. A function f : (K, $\tau_R(X)$) → (L, $\sigma_R(Y)$) is said to be

(1) $n^*\mu_t$-continuous if for each open set V of L, f^{-1}(V) $\in n^*\mu_t$(K, $\tau_R(X)$).
(2) $n^*\mu_\alpha*$-continuous if for each open set V of L, f^{-1}(V) $\in n^*\mu_\alpha*$(K, $\tau_R(X)$).

THEOREM 3.3. *For a function f : (K, $\tau_R(X)$) → (L, $\sigma_R(Y)$), the following implications hold:*

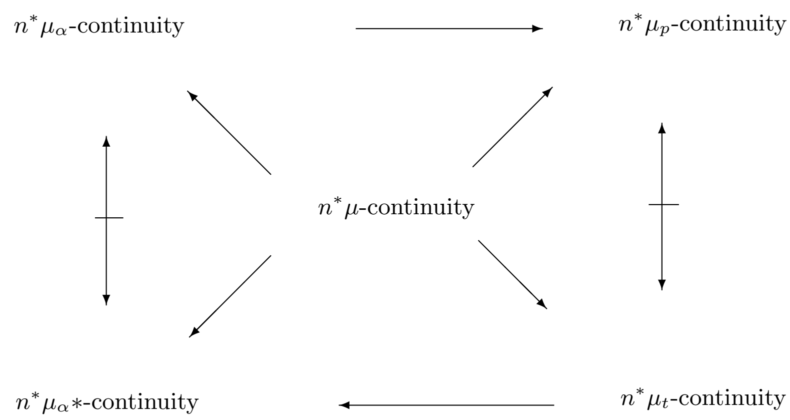
(1) *$n^*\mu$-continuity $\Rightarrow n^*\mu_t$-continuity.*
(2) *$n^*\mu$-continuity $\Rightarrow n^*\mu_\alpha*$-continuity.*
(3) *$n^*\mu_t$-continuity is an $n^*\mu_\alpha*$-continuity.*
(4) *$n^*\mu$-continuity $\Rightarrow n^*\mu_\alpha$-continuity $\Rightarrow n^*\mu_p$-continuity.* [**16**]

PROOF. (1) and (2). The proof follows from Proposition 3.1.
(3). The proof follows from Proposition 3.2. □

REMARK 3.5. (1) The concepts of $n^*\mu_t$-continuity and $n^*\mu_p$-continuity are independent.

(2) The concepts of $n^*\mu_\alpha*$-continuity and $n^*\mu_\alpha$-continuity are independent.

REMARK 3.6. From the above discussions, we have the following diagram of implications where A → B (resp. A ↮ B) represents A implies B but not conversely (resp. A and B are independent of each other).



We obtained some dcompositions of $n^*\mu$-continuity

THEOREM 3.4. *A function $f : (K, \tau_R(X)) \to (L, \sigma_R(Y))$ is $n^*\mu$-continuous if and only if it is both $n^*\mu_\alpha$-continuous and $n^*\mu_\alpha$*-continuous.*

PROOF. The proof follows immediately from Theorem 3.1. □

THEOREM 3.5. *A function $f : (K, \tau_R(X)) \to (L, \sigma_R(Y))$ is $n^*\mu$-continuous if and only if it is both $n^*\mu_p$-continuous, $n^{\iota\iota}\eta$-continuous and $n^*\mu_\alpha$*-continuous.*

PROOF. It follows from Theorem 2.3 and Theorem 3.4. □

THEOREM 3.6. *A function $f : (K, \tau_R(X)) \to (L, \sigma_R(Y))$ is $n^*\mu$-continuous if and only if it is both $n^*\mu_p$-continuous and $n^*\mu_t$-continuous.*

PROOF. The proof follows immediately from Theorem 3.2. □

Conclusion: We obtained decompositions of $n^*\mu$-continuity in nano topological spaces using $n^*\mu_p$-continuity, $n^*\mu_\alpha$-continuity, $n^*\mu_t$-continuity and $n^*\mu_\alpha$*-continuity. The results of this study may be help to many researches.

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