BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., Vol. **12**(1)(2022), 83-89
DOI: 10.7251/BIMVI2201083G

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

ON $n\mathcal{I}_g$-HOMEOMORPHISM IN NANO IDEAL TOPOLOGICAL SPACES

Selvaraj Ganesan

Abstract. In this paper, we introduce and study two new homeomorphisms namely n$\mathcal{I}_g$-homeomorphism and $^*n\mathcal{I}_g$-Homeomorphism in nano ideal topological spaces.

1. Introduction

Let $(U,\mathcal{N},\mathcal{I})$ be an nano ideal topological space with an ideal $\mathcal{I}$ on U, where $\mathcal{N} = \tau_R(X)$ and $(.)^*_n : \wp(\mathrm{U}) \to \wp(\mathrm{U})$ ($\wp$(U) is the set of all subsets of U) ([**6, 7**]). For a subset $A \subseteq U$, let $\mathrm{A}^*_n(\mathcal{I}, \mathcal{N}) = \{x \in U : G_n \cap A \notin \mathcal{I}$, for every $G_n \in G_n(x)\}$, where $\mathrm{G}_n = \{\mathrm{G}_n \mid \mathrm{x} \in \mathrm{G}_n,\ \mathrm{G}_n \in \mathcal{N}\}$ is called the nano local map (briefly n-local map) of A with repect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write A^*_n for A^*_n $(\mathcal{I}, \mathcal{N})$. Parimala et al. [**7**] introduced the concept of nano ideal generalized closed sets in nano ideal topological spaces and investigated some of its basic properties. Recently, Ganesan [**2**] introduced and studied n$\mathcal{I}_g$-continuous map and n$\mathcal{I}_g$-irresolute map in nano ideal topological spaces. In this paper, we introduced the concept of n$\mathcal{I}_g$-closed maps, n$\mathcal{I}_g$-open maps, n$\mathcal{I}_g$-Homeomorphism in nano ideal topological spaces and study its relationship with excisting homeomorphisms. A new class of maps $^*n\mathcal{I}_g$-Homeomorphism is introduced which from a subclass of n$\mathcal{I}_g$-Homeomorphisms. we establish that the set of all $^*n\mathcal{I}_g$-Homeomorphism (O, $\mathcal{N}$, $\mathcal{I}$) onto itself is a group under the composition of maps.

2010 *Mathematics Subject Classification.* Primary 20M99, 13F10; Secondary 13A15, 13M05.
Key words and phrases. n$\mathcal{I}_g$-closed sets, n$\mathcal{I}_g$-continuous map, n$\mathcal{I}_g$-irresolute map, n$\mathcal{I}_g$-closed map, n$\mathcal{I}_g$-open map and n$\mathcal{I}_g$-homeomorphism.
Communicated by Daniel A. Romano.

2. Preliminaries

DEFINITION 2.1. [**6, 7**] A subset A of a nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be n⋆-closed if $A_n^* \subseteq$ A.

LEMMA 2.1. [**6, 7**] *Let (U, $\mathcal{N}$, $\mathcal{I}$) be an nano topological space with an ideal $\mathcal{I}$ and A⊆A_n^*, then A_n^*=n-cl(A_n^*)=n-cl(A)*

DEFINITION 2.2. [**7**] A subset A of a nano ideal topological space (U, $\mathcal{N}$, $\mathcal{I}$) is said to be

(1) nano-$\mathcal{I}$-generalized closed (briefly, n$\mathcal{I}_g$-closed if $A_n^* \subseteq$ V whenever A⊆ V and V is n-open.
(2) n$\mathcal{I}_g$-open if its complement is n$\mathcal{I}_g$-closed.

DEFINITION 2.3. [**5**] A map f : (K, $\mathcal{N}$, $\mathcal{I}$) → (L, $\mathcal{N}'$, $\mathcal{J}$) is said to be n⋆-continuous if f^{-1}(G) is n⋆-closed in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set G of (L, $\mathcal{N}'$).

DEFINITION 2.4. [**2, 4**] A map f : (K, $\mathcal{N}$, $\mathcal{I}$) → (L, $\mathcal{N}'$, $\mathcal{J}$) is said to be n$\mathcal{I}_g$-continuous if f^{-1}(G) is n$\mathcal{I}_g$-closed in (K, $\mathcal{N}$, $\mathcal{I}$) for every n-closed set G of (L, $\mathcal{N}'$).

DEFINITION 2.5. [**2**] A map f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is called n$\mathcal{I}_g$-irresolute if f^{-1}(G) is a n$\mathcal{I}_g$-closed set of (O, $\mathcal{N}$, $\mathcal{I}$) for every n$\mathcal{I}_g$-closed set G of (P, $\mathcal{N}'$, $\mathcal{J}$).

DEFINITION 2.6. [**1**] A map f : (O, $\mathcal{N}$) → (P, $\mathcal{N}'$) is said to be n-closed map if for every n-closed subset G of (O, $\mathcal{N}$), f(G) is n-closed in (P, $\mathcal{N}'$).

THEOREM 2.1. (1) *Every n-closed is n⋆-closed set but not conversely* [**3**].
(2) *Every n⋆-closed set is n$\mathcal{I}_g$-closed but not conversely* [**7**]

PROPOSITION 2.1. [**2**] *Every n⋆-continuous is n$\mathcal{I}$g-continuous but not conversely.*

3. n$\mathcal{I}_g$-Closed and n$\mathcal{I}_g$-Open maps

In this section, we introduce the concepts of n⋆-closed map, n$\mathcal{I}_g$-closed map and n$\mathcal{I}_g$-open map.

DEFINITION 3.1. A map f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is called

(1) n⋆-closed if for every n-closed subset G of (O, $\mathcal{N}$, $\mathcal{I}$), f(G) is n⋆closed in (P, $\mathcal{N}'$, $\mathcal{J}$).
(2) n$\mathcal{I}_g$-closed (resp. n$\mathcal{I}_g$-open) map if for every n-closed (resp. n-open) subset G of (O, $\mathcal{N}$, $\mathcal{I}$). f(G) is n$\mathcal{I}_g$-closed (resp. n$\mathcal{I}_g$-open) in (P, $\mathcal{N}'$, $\mathcal{J}$).

THEOREM 3.1. *A map f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is n$\mathcal{I}_g$-closed if and only if for each n-open set U containing f^{-1}(S), there is a n$\mathcal{I}_g$-open set V of (P, $\mathcal{N}'$, $\mathcal{J}$) such that S ⊆ V and f^{-1}(V) ⊆ U.*

PROOF. Necessity: Let U be n-open in (O, $\mathcal{N}$, $\mathcal{I}$). Then U^c is n-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Since f is an n$\mathcal{I}_g$-closed map. $f(U^c)$ is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Thus (P, $\mathcal{N}'$, $\mathcal{J}$) − f(U^c) is n$\mathcal{I}_g$-open, say V containing S such that

$$f^{-1}(V) \subseteq f^{-1}((P, \mathcal{N}', \mathcal{J}) - f(U^c)) = U.$$

Sufficiency: Let F be a n-closed set in $(O,\mathcal{N},\mathcal{I})$. Then F^c is n-open in $(O,\mathcal{N},\mathcal{I})$. By hypothesis, there exits an $n\mathcal{I}_g$-open set V of $(P,\mathcal{N}',\mathcal{J})$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c = f^{-1}(V^c)$. Therefore, $f(F) = V^c$. Since V^c is $n\mathcal{I}_g$-closed, then $f(F)$ is $n\mathcal{I}_g$-closed in $(P,\mathcal{N}',\mathcal{J})$. Hence f is $n\mathcal{I}_g$-closed. □

THEOREM 3.2. *If $f : (O, \mathcal{N}, \mathcal{I}) \to (P, \mathcal{N}', \mathcal{J})$ is a n-closed map, then it is n⋆-closed map.*

PROOF. Let G be a n-closed subset of (O, $\mathcal{N}$, $\mathcal{I}$). Since f is a n-closed map, f(G) is n-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Every n-closed is a n⋆-closed, f(G) is n⋆-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Hence f is a n⋆-closed map. □

EXAMPLE 3.1. Let $O = \{5, 6, 7\}$, with $O/R = \{\{5\}, \{6, 7\}\}$ and $X = \{5, 6\}\}$. Then the nano topology $\mathcal{N} = \{\phi, \{5\}, \{6, 7\}, O\}$ and $\mathcal{I}$=$\{\emptyset\}$. Let $P = \{5, 6, 7\}$, with $P/R = \{\{5\}, \{6, 7\}\}$ and $X = \{5\}\}$. Then the nano topology $\mathcal{N}' = \{\phi, \{5\}, P\}$ and $\mathcal{J} = \{\emptyset, \{5\}\}$. Thus n⋆-closed sets are ϕ, P, $\{5\}$, $\{6, 7\}$. Define $f : (O,\mathcal{N},\mathcal{I}) \to (P,\mathcal{N}',\mathcal{J})$ be the identity map. Therefore f is a n⋆-closed map but not a n-closed map because the subset $\{5\}$ is n-closed in $(O,\mathcal{N},\mathcal{I})$, $f(\{5\}) = \{5\}$ is not n-closed in $(P,\mathcal{N}',\mathcal{J})$.

THEOREM 3.3. *If $f : (O, \mathcal{N}, \mathcal{I}) \to (P, \mathcal{N}', \mathcal{J})$ is a n-closed map, then it is $n\mathcal{I}_g$-closed map.*

PROOF. Let G be a n-closed subset of (O, $\mathcal{N}$, $\mathcal{I}$). Since f is a n-closed map, f(G) is n-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Every n-closed is n$\mathcal{I}_g$-closed, f(G) is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Hence f is a n$\mathcal{I}_g$-closed map. □

EXAMPLE 3.2. Let O, $\mathcal{N}$, $\mathcal{I}$ and f be defined as in Example 3.1. Let $P = \{5, 6, 7\}$, with $P/R = \{\{7\}, \{5, 6\}, \{6, 5\}\}$ and $X = \{5, 6\}$. Then the nano topology $\mathcal{N}' = \{\phi$, {5, 6}, P} and $\mathcal{J} = \{\emptyset$, {5}}. The n$\mathcal{I}_g$-closed sets are ϕ, P, {5}, {7}, {5, 7}, {6, 7}. Then f is a n$\mathcal{I}_g$-closed map but not a n-closed map because the subset {6, 7} is n-closed in (O, $\mathcal{N}$, $\mathcal{I}$), f({6, 7}) = {6, 7} is not n-closed in (P, $\mathcal{N}'$, $\mathcal{J}$).

THEOREM 3.4. *If $f : (O, \mathcal{N}, \mathcal{I}) \to (P, \mathcal{N}', \mathcal{J})$ is a n⋆-closed map, then it is $n\mathcal{I}_g$-closed map.*

PROOF. Let G be a n-closed subset of (O, $\mathcal{N}$, $\mathcal{I}$). since f is a n⋆-closed map, f(G) is n⋆-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Every n⋆-closed is n$\mathcal{I}_g$-closed, f(G) is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Hence f is a n$\mathcal{I}_g$-closed map. □

EXAMPLE 3.3. Let O, $\mathcal{N}$, $\mathcal{I}$, P, $\mathcal{N}'$, $\mathcal{J}$ and f be defined as in Example 3.2. Then n⋆-closed sets are ϕ, P, {5}, {7}, {5, 7}. Then f is a n$\mathcal{I}_g$-closed map but not a n⋆-closed map because the subset {6, 7} is n-closed in (O, $\mathcal{N}$, $\mathcal{I}$), f({6, 7}) = {6, 7} is not n⋆-closed in (P, $\mathcal{N}'$, $\mathcal{J}$).

4. n$\mathcal{I}_g$-Homeomorphisms

DEFINITION 4.1. A bijection f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is called a n$\star$-Homeomorphism, if both f and f^{-1} are n$\star$-continuous.

DEFINITION 4.2. A bijection f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is called a n$\mathcal{I}_g$-Homeomorphism, if both f and f^{-1} are n$\mathcal{I}_g$-continuous.

We say that the space (O, $\mathcal{N}$, $\mathcal{I}$) and (P, $\mathcal{N}'$, $\mathcal{J}$) are n$\mathcal{I}_g$-Homeomorphism if there exists an n$\mathcal{I}_g$-Homeomorphic from (O, $\mathcal{N}$, $\mathcal{I}$) onto (P, $\mathcal{N}'$, $\mathcal{J}$) .

THEOREM 4.1. *Every n$\star$-Homeomorphism is a n$\mathcal{I}_g$-Homeomorphism*

PROOF. Let f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) is be a n$\star$-homeomorphism. Then f and f^{-1} are n$\star$-continuous and f is a bijection. Since every n$\star$-continuous map is n$\mathcal{I}_g$-continuous, it follows that f is n$\mathcal{I}_g$-Homeomorphism. $\square$

EXAMPLE 4.1. Let O, $\mathcal{N}$, $\mathcal{I}$, P, $\mathcal{N}'$, $\mathcal{J}$ and f be defined as in Example 3.3. Then n$\mathcal{I}_g$-closed sets are ϕ, O, {5}, {6}, {7}, {5, 6}, {5, 7}, {6, 7}. Then f is n$\mathcal{I}_g$-homeomorphism but not n$\star$-homeomorphism because the subset f^{-1} ({7}) = {7} is n-closed in (P, $\mathcal{N}'$, $\mathcal{J}$) but not n$\star$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

REMARK 4.1. The composition of two n$\mathcal{I}_g$-homeomorphisms need not be n$\mathcal{I}_g$-homeomorphism.

EXAMPLE 4.2. Let O, $\mathcal{N}$, $\mathcal{I}$, P, $\mathcal{N}'$, $\mathcal{J}$ and f be defined as an Example 3.3. Let $Q = \{5, 6, 7\}$ with $Q/R = \{\{5\}, \{6, 7\}\}$ and $X = \{5\}$. Then the nano topology $\mathcal{N}'_* = \{\phi, \{5\}, Q\}$ and $\mathcal{K} = \{\emptyset\}$. Then n$\mathcal{I}_g$-open sets are ϕ, O, {5}, {6}, {7}, {5, 6}, {5, 7}, {6, 7}, n$\mathcal{I}_g$-open sets are ϕ, P, {5}, {6}, {5, 6}, {6, 7} and n$\mathcal{I}_g$-open sets are ϕ, Q, {5}, {6}, {7}, {5, 6}, {5, 7}. Define $g : (P, \mathcal{N}', \mathcal{J}) \rightarrow (Q, \mathcal{N}'_*, \mathcal{K})$ be the identity map. Then both f and g are n$\mathcal{I}_g$-homeomorphisms but their composition $g \circ f : (O, \mathcal{N}, \mathcal{I}) \rightarrow (Q, \mathcal{N}'_*, \mathcal{K})$ is not a n$\mathcal{I}_g$-homeomorphism, because for the open set $\{6, 7\}$ of $(O, \mathcal{N}, \mathcal{I}), g \circ f\{6, 7\} = g(f(\{6, 7\})) = g(\{6, 7\}) = \{6, 7\}$ which is not n$\mathcal{I}_g$-open in $(Q, \mathcal{N}'_*, \mathcal{K})$. Therefore, $g \circ f$ is not n$\mathcal{I}_g$-open and not an n$\mathcal{I}_g$-homeomorphism.

The following theorem gives a characterzation of n$\mathcal{I}_g$-homeomorphism.

THEOREM 4.2. *For any bijection f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) the following statements are equivalent.*

(1) *f^{-1} : (P, $\mathcal{N}'$, $\mathcal{J}$) $\rightarrow$ (O, $\mathcal{N}$, $\mathcal{I}$) is n$\mathcal{I}_g$-continuous map.*
(2) *f is a n$\mathcal{I}_g$-open map.*
(3) *f is a n$\mathcal{I}_g$-closed map.*

PROOF. (1) $\Rightarrow$ (2): Let G be an n-open set (O, $\mathcal{N}$, $\mathcal{I}$). By assumption $(f^{-1})^{-1}$(G) = f(G) is n$\mathcal{I}_g$-open in (P, $\mathcal{N}'$, $\mathcal{J}$). so f is a n$\mathcal{I}_g$-open map.

(2) $\Rightarrow$ (3): Let G be n-closed set of (O, $\mathcal{N}$, $\mathcal{I}$). Then G^c is n-open in (O, $\mathcal{N}$, $\mathcal{I}$). Since f is n$\mathcal{I}_g$-open, f (G^c) is n$\mathcal{I}_g$-open in (P, $\mathcal{N}'$, $\mathcal{J}$). This implies [f(G)]^c is n$\mathcal{I}_g$-open in (P, $\mathcal{N}'$, $\mathcal{J}$). This implies f(G) is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Therefore, f is a n$\mathcal{I}_g$-closed map.

(3) ⇒ (1): Suppose G is n-closed set in (O, $\mathcal{N}$, $\mathcal{I}$). Then by assumption $(f^{-1})^{-1}$(G) = f(G)is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). Hence f^{-1} is a n$\mathcal{I}_g$-continuous map. □

THEOREM 4.3. *Let f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) be a bijective and n$\mathcal{I}_g$-continuous map. Then the following statements are equivalent.*

(1) *f is a n$\mathcal{I}_g$-open map.*
(2) *f is a n$\mathcal{I}_g$-homeomorphism.*
(3) *f is a n$\mathcal{I}_g$-closed map.*

PROOF. Follows from Definitions 2.4, 3.1, 4.1 and Theorem 4.2. □

5. $^*n\mathcal{I}_g$-Homeomorphisms

We introduce a new class of maps called $^*n\mathcal{I}_g$-Homeomorphisms which forms a subclass of n$\mathcal{I}_g$-Homeomorphisms. This class of maps is closed under composition of maps.

DEFINITION 5.1. A map f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is called $^*n\mathcal{I}_g$-open if for every n$\mathcal{I}g$-open subset G of (O, $\mathcal{N}$, $\mathcal{I}$). f(G) is n$\mathcal{I}_g$-open in (P, $\mathcal{N}'$, $\mathcal{J}$).

THEOREM 5.1. *For any bijection f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) the following statements are equivalent.*

(1) *The inverse map $f^{-1} : (P, \mathcal{N}', \mathcal{J}) \to (O, \mathcal{N}, \mathcal{I})$ is n$\mathcal{I}_g$-irresolute.*
(2) *f is a $^*n\mathcal{I}_g$-open map.*
(3) *f is a $^*n\mathcal{I}_g$-closed map.*

PROOF. (1) ⇒ (2): Let G be n$\mathcal{I}_g$-open in (O, $\mathcal{N}$, $\mathcal{I}$). By (1), $(f^{-1})^{-1}$(G) = f(G) is n$\mathcal{I}_g$-open in (P, $\mathcal{N}'$, $\mathcal{J}$). Hence (2) holds.

(2) ⇒ (3): Let G be n$\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). Then (O, $\mathcal{N}$, $\mathcal{I}$) − G is n$\mathcal{I}_g$-open and by (2) f((O, $\mathcal{N}$, $\mathcal{I}$) − G) = (P, $\mathcal{N}'$, $\mathcal{J}$) − f(G) is n$\mathcal{I}_g$-open in (P, $\mathcal{N}'$, $\mathcal{J}$). That is f(G) is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$) and so f is $^*n\mathcal{I}_g$-closed map.

(3) ⇒ (1): Let G be n$\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$). By (3), f(G) is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$). But f(G)= $(f^{-1})^{-1}$(G) . Thus (1) holds. □

Next we introduce a new class of maps as follows.

DEFINITION 5.2. A bijection f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) is called $^*n\mathcal{I}_g$-Homeomorphisms if both f and f^{-1} are n$\mathcal{I}g$-irresolute.

We say that the spaces (O, $\mathcal{N}$, $\mathcal{I}$) and (P, $\mathcal{N}'$, $\mathcal{J}$) are $^*n\mathcal{I}_g$-Homeomorphic if there exists an $^*n\mathcal{I}_g$-Homeomorphisms from (O, $\mathcal{N}$, $\mathcal{I}$) onto (P, $\mathcal{N}'$, $\mathcal{J}$). The family of all n$\mathcal{I}g$-Homeomorphisms (resp $^*n\mathcal{I}_g$-Homeomorphisms) from (O, $\mathcal{N}$, $\mathcal{I}$) onto (P, $\mathcal{N}'$, $\mathcal{J}$) is denoted by n$\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$) (resp $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$)).

THEOREM 5.2. *Every $^*n\mathcal{I}_g$-Homeomorphism is a n$\mathcal{I}_g$-Homeomorphism.*

PROOF. Let f : (O, $\mathcal{N}$, $\mathcal{I}$) → (P, $\mathcal{N}'$, $\mathcal{J}$) be a $^*n\mathcal{I}_g$-homeomorphism. Then f and f^{-1} are n$\mathcal{I}_g$ irrosolute and f is a bijection. Thus f and f^{-1} are n$\mathcal{I}_g$-continuous. Hence f is n$\mathcal{I}_g$-Homeomorphism. □

Example 5.1. Let $O = \{5,6,7\}$, with $O/R = \{\{7\},\{5,6\}\{6,5\}\}$ and $X = \{5,6\}$. Then the Nano topology $\mathcal{N} = \{\phi$, {5, 6}, O} and $\mathcal{I}$={$\emptyset$, {5}}. Let $P = \{5,6,7\}$, with $P/R = \{\{5\},\{6,7\}\}$ and $X = \{5\}$. Then the Nano topology $\mathcal{N}' = \{\phi$, {5}, P} and $\mathcal{J}$={$\emptyset$}. Then n$\mathcal{I}_g$-closed sets are ϕ, O, {5}, {7}, {5, 7}, {6, 7} and n$\mathcal{I}_g$-closed sets are ϕ, P, {6}, {7}, {5, 6}, {5, 7}, {6, 7}. Define f : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (P, $\mathcal{N}'$, $\mathcal{J}$) be the identity map. Then f is a n$\mathcal{I}_g$-homeomorphism but not a $^*n\mathcal{I}_g$-homeomorphism because the subset {5, 6} is n$\mathcal{I}_g$-closed in (P, $\mathcal{N}'$, $\mathcal{J}$), but f^{-1} {5, 6} = {5, 6} is not n$\mathcal{I}_g$-closed in (O, $\mathcal{N}$, $\mathcal{I}$).

Theorem 5.3. *Let* $f : (O,\mathcal{N},\mathcal{I}) \rightarrow (P,\mathcal{N}',\mathcal{J})$ *and* $g : (P,\mathcal{N}',\mathcal{J}) \rightarrow (Q,\mathcal{N}'_*,\mathcal{K})$ *be a* $^*n\mathcal{I}_g$*-homeomoephism then their composition* $g \circ f : (O,\mathcal{N},\mathcal{I}) \rightarrow (Q,\mathcal{N}'_*,\mathcal{K})$ *is* $^*n\mathcal{I}_g$*-homeomoephism.*

Proof. Let G be a n$\mathcal{I}_g$-open set in $(Q,\mathcal{N}'_*,\mathcal{K})$ since g is n$\mathcal{I}_g$-irresolute, g^{-1}(G)is n$\mathcal{I}_g$-open set in (P, $\mathcal{N}'$, $\mathcal{J}$). Since f is n$\mathcal{I}_g$-irresolute, $f^{-1}(g^{-1}(G)) = (g\circ f)^{-1}(G)$ is an n$\mathcal{I}_g$-open set in $(O,\mathcal{N},\mathcal{I})$. Therefore $(O,\mathcal{N},\mathcal{I})$ is n$\mathcal{I}_g$-irrosolute. Also for a n$\mathcal{I}_g$-open set G in $(O,\mathcal{N},\mathcal{I})$. We have $(g \circ f)(G) = g(f(G)) = g(S)$ where $S = f(G)$. By the hypothesis $f(G)$ is n$\mathcal{I}_g$-open in $(P,\mathcal{N}',\mathcal{J})$ and also by hypothesis $g(f(G))$ is a n$\mathcal{I}_g$-open in $(Q,\mathcal{N}'_*,\mathcal{K})$. That is $(g \circ f)(G)$ is a n$\mathcal{I}_g$-open set in $(Q,\mathcal{N}'_*,\mathcal{K})$ and therefore $(g \circ f)^{-1}$ is n$\mathcal{I}_g$-irrosolute. Also $(g \circ f)$ is a bijection. Hence $(g \circ f)$ is $^*n\mathcal{I}_g$-Homeomorphism. □

Theorem 5.4. *The set* $^*n\mathcal{I}_g$*-h(O,* $\mathcal{N}$*,* $\mathcal{I}$*) is a group under the composition of maps.*

Proof. Define a binary operation $\star$: $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$) $\times$ $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$) by f $\star$ g = (g $\circ$ f) for all f, g $\in$ $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$) and $\circ$ is the usal operation of maps. Then by Theorem 5.3, (g $\circ$ f) $\in$ $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$). We know that composition of maps associative and the identity map I : (O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ (O, $\mathcal{N}$, $\mathcal{I}$) belonging to $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$) serves as the identity element. for any f $\in$ $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$), f $\circ$ f^{-1} = f^{-1} $\circ$ f = I. Hence inverse exists for each element of $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$). $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$) forms a group under the operation of composition of maps. □

Theorem 5.5. *Let* $f : (O,\mathcal{N},\mathcal{I}) \rightarrow (P,\mathcal{N}',\mathcal{J})$ *be an* $^*n\mathcal{I}_g$*-homeomorphism. Then* f *induces an isomorphism from the group* $^*n\mathcal{I}_g - h(O,\mathcal{N},\mathcal{I})$ *onto the group* $^*n\mathcal{I}_g - h(P,\mathcal{N}',\mathcal{J})$*.*

Proof. Let $f \in {}^*n\mathcal{I}_g - h(O,\mathcal{N},\mathcal{I})$. We define a map ϕ_f : $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$) $\rightarrow$ $^*n\mathcal{I}_g$-h(P, $\mathcal{N}'$, $\mathcal{J}$) by ϕ_f = f $\circ$ h $\circ$ f^{-1} for every h $\in$ $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$). Then f is a bijection. Now for all g, h $\in$ $^*n\mathcal{I}_g$-h(O, $\mathcal{N}$, $\mathcal{I}$), ϕ_f (g $\circ$ h) = f $\circ$ (g $\circ$ h) $\circ$ f^{-1} = (f $\circ$ g $\circ$ f^{-1}) $\circ$ (f $\circ$ h $\circ$ f^{-1}) = ϕ_f (g) $\circ$ ϕ_f (h). □

6. Conclusion

In this paper, we introduced the concepts of n$\mathcal{I}_g$-closed maps, n$\mathcal{I}_g$-open maps, n$\mathcal{I}_g$-Homeomorphism in nano ideal topological spaces and study its relationship with excisting homeomorphisms. A new class of maps $^*n\mathcal{I}_g$-Homeomorphism is introduced which from a subclass of n$\mathcal{I}_g$-Homeomorphisms. we establish that the set of all $^*n\mathcal{I}_g$-Homeomorphism (O, $\mathcal{N}$, $\mathcal{I}$) onto itself is a group under the composition of maps. In future, we have extended this work in various nano ideal topological spaces.

Acknowledgement I thank to referees for giving their useful suggestions and help to improve this manuscript.

References

[1] M. Bhuvaneswari and N. Nagaveni. On Nwg-normal and Nwg-regular spaces. *International Journal of Mathematics Trends and Technology (IJMTT)*, Special Issue *ICRMIT March* (2018), 1–5.

[2] S. Ganesan. Cn$\mathcal{I}$g-continuous maps in nano ideal topological spaces. *Math Lab Journal*, **4**(2019), 182-190.

[3] S. Ganesan, C. Alexander, S. Jeyashri and S. M. Sandhya. n$\mathcal{I}_{\ddot{g}}$-closed sets. *Math Lab Journal*, **5**(2020), 23-34.

[4] S. Ganesan. $n\mathcal{I}_{g\mu}$-closed sets in nano ideal topological spaces. *Palest. J. Math.*, **10**(1) (2021), 340-348.

[5] J. Jayasudha and T. Rekhapriyadharsini. On some decompositions of nano $\star$-continuity. *International Journal of Mathematics and Statistics Invention*, **7**(1)(2019), 01–06.

[6] M. Parimala, T. Noiri and S. Jafari. New types of nano topological spaces via nano ideals. *Q & A in General Topology*, **38**(1)(2020), 59–65.

[7] M. Parimala, S. Jafari and S. Murali. Nano ideal generalized closed sets in nano ideal topological spaces. *Ann. Univ. Sci. Budap. Rolando Etvs, Sect. Math.*, **60**(1)(2017), 3-11.

Received by editors 07.02.2021; Revised version 04.08.2021; Available online 09.08.2021.

PG & Research Department of Mathematics Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.

E-mail address: sgsgsgsgsg77@gmail.com