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ON b-+-OPEN SETS IN TOPOLOGICAL SPACES

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Abstract. In this paper, we used the notion of operator A^+ for defining a new class of set which will be called b-+-open set besides we define the concepts of b-+-continuous, b-+-irresolute. We define the concepts of generalized b-+-closed sets, regular generalized b-+-closed sets and gb-+-continuous and rgb-+-continuous. Moreover, some of their properties are shown.

1. Introduction and Preliminaries

The concept of operator A^+ was introduced by Elez and Papaz [**4**], they defined an operator A^+ by $A^+ = \mathrm{Cl(A)} - \mathrm{A}$. Carlos Granados [**7**] introduced and studied semi-+-open sets in topological spaces. S. Ganesan [**6**] introduced and studied α-+-open set, pre-+-open set and β-+-open set in topological spaces. In this paper, we used the operator A^+ for defining a new class of open sets which will be called b-+-open set.

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), Cl(A), Int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

Definition 1.1. A subset A of a space (X, τ) is called:

(1) ([**1**]) b-open set if $\mathrm{A} \subseteq \mathrm{Int(Cl(A))} \cap \mathrm{Cl(Int(A))}$.

(2) ([**8**]) regular open set [**8**] if $\mathrm{A} = \mathrm{Int(Cl(A))}$.

The complements of the above mentioned open sets are called their respective closed sets.

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Definition 1.2. Let (X, τ) be a topological space and A $\subseteq$ X. Then A is called:

([**7**]) semi-+-open if $A^+ \subseteq Cl(Int(A^+))$.

([**6**]) pre-+-open if $A^+ \subseteq Int(Cl(A^+))$.

([**6**]) α-+-open if $A^+ \subseteq Int(Cl(Int(A^+)))$.

The complements of the above mentioned open sets are called their respective closed sets.

Remark 1.1. ([**7**]) A^+ does not induce a topological space, because

$$X^+ = Cl(X) - X = X - X = \emptyset \text{ and } \emptyset^+ = Cl(\emptyset) - \emptyset = \emptyset \cap X = \emptyset.$$

We can see that X will never be in the topology.

Remark 1.2. ([**7**]) The operator $A^+ = Cl(A) - A = Cl(A) \cap A^c$, where A^c means the complement of the set A.

Definition 1.3. A function $f : (K, \tau) \to (L, \sigma)$ is said to be continuous if $f^{-1}(A)$ is closed set in (K, τ) for every closed set A of (L, σ).

2. b-+-open sets and generalized b-+-closed sets

Definition 2.1. A subset A of a space (X, τ) is said to be b-+-open if

$$A^+ \subseteq Int(Cl(A^+)) \cup Cl(Int(A^+)).$$

The complement of b-+-open sets is called b-+-closed sets.

The collection of all b-+-open sets and b-+-closed sets are denoted by $b + O(X, \tau)$ and $b + C(X, \tau)$ respectively.

We denote the power set of X by $P(X)$.

Theorem 2.1. *Every closed set is b-+-open.*

Proof. Let A be a closed set of (X, τ). Since A is a closed set, then $Cl(A) = A$ and so $A^+ = Cl(A) - A = A \cap A^c = \phi$. Now,

$$\phi \subseteq Int(Cl(A^+)) \cup Cl(Int(A^+))) = Int(Cl(\phi)) \cup Cl(Int(\phi)) = \phi \cup \phi = \phi.$$

This shows that A is b-+-open. □

The following example shows that open sets is independent of b-+-open sets.

Example 2.1. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then b-+-open sets are $\emptyset, X, \{c\}, \{a, c\}, \{b, c\}$. Here $\{a\}$ is a open set but it is not a b-+-open set. Also it is clear that $\{c\}$ is a b-+-open set but it is not a open set.

The following example shows that the notion of b-open set and b-+-open set are independent.

Example 2.2. Let X and $\tau(X)$ as in the Example 2.1. Then b-open sets are $\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}$. Here $\{a, b\}$ is a b-open set but it is not a b-+-open set. Also it is clear that $\{c\}$ is a b-+-open set but it is not a b-open set.

THEOREM 2.2. *Every semi-+-open set is b-+-open set but not conversely.*

PROOF. Let A be a semi-+-open set in $(X, \tau(X))$. Then $A^+ \subseteq Cl(Int(A^+))$. Hence $A^+ \subseteq Cl(Int(A^+)) \cup Int(Cl(A^+))$ and A is b-+-open in $(X, \tau(X))$. □

EXAMPLE 2.3. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{b, c\}\}$. Then semi-+-open sets are $\emptyset, X, \{a, d\}$; b-+-open sets are

$$\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}.$$

Here, $\{a, b\}$ is b-+-open set but it is not a semi-+-open set.

THEOREM 2.3. *Every α-+-open set is b-+-open set but not conversely.*

PROOF. Let A be a α-+-open set in $(X, \tau(X))$. Then $A^+ \subseteq Int(Cl(Int(A^+)))$. Hence

$$A^+ \subseteq Int(Cl(Int(A^+))) \subseteq Cl(Int(A^+)) \subseteq Cl(Int(A^+)) \cup Int(Cl(A^+))$$

and A is b-+-open in $(X, \tau(X))$. □

EXAMPLE 2.4. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a, b\}\}$. Then α-+-open sets are $\emptyset, X, \{c, d\}$; b-+-open sets are

$$\emptyset, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}.$$

Here, $\{b, c\}$ is b-+-open set but it is not a α-+-open set.

THEOREM 2.4. *Every pre-+-open set is b-+-open set but not conversely.*

PROOF. Let A be a pre-+-open set in $(X, \tau(X))$. Then $A^+ \subseteq Int(Cl(A^+))$. Hence $Cl(A^+) \subseteq Cl(Int(A^+)) \cup Int(Cl(A^+))$ and A is b-+-open in $(X, \tau(X))$. □

REMARK 2.1. The union (intersection) of any two b-+-open sets in not b-+-open.

EXAMPLE 2.5. Let X and τ as in the Example 2.4.

(i) Let $M = \{a\}$ and $N = \{b\}$ are b-+-open sets, but $A \cup B = \{a, b\}$ is not b-+-open set.

(ii) Let $P = \{b, d\}$ and $Q = \{c, d\}$ are b-+-open sets, but $A \cap B = \{d\}$ is not b-+-open set.

DEFINITION 2.2. Let (X, τ) be a topological space and $A \subset X$. An element $x \in A$ is said to be b-+-interior point of A if there exits a b-+-open set U such that $x \in U \subseteq A$. The set of all b-+-interior points of A is said to be b+-interior of A an it is denoted by $Int_{b+(A)}$.

THEOREM 2.5. *Let (X, τ) be a topological space and $A \subset X$. Then, A is b-+-open if and only if $A = Int_{b+(A)}$.*

PROOF. Let A be a b-+-open set. Then, $A \subseteq A$ and this implies that $A \in \{U \mid U \text{ is b-+-open and } U \subset A\}$. Since union of this collection is in A. Therefore, $A = Int_{b+(A)}$.

Conversely, suppose that $A = Int_{b+(A)}$. Hence, A is b-+-open. □

Definition 2.3. Let $A \subset X$. Then $x \in X$ is b-+-adherent to A if $U \cap A \neq \emptyset$ for every b-+-open set U containing x. The set of all b-+-adherent points of A is said to be b-+-closure of A and it is denoted by $Cl_{b+(A)}$.

Theorem 2.6. *Let* (X, τ) *be a topological space and* $A, B \subseteq X$. *Then, the following statements hold:*

(1) $A \subseteq Cl_{b+(A)}$.
(2) $Cl_{b+(A)}$ *is the smallest b-+-closed set containing A, that is* $Cl_{b+(A)} = \bigcap\{W \mid W \text{ is b-+-closed and} A \subseteq W\}$.
(3) A *is b-+-closed if and only if* $A = Cl_{b+(A)}$.
(4) *If* $A \subseteq B$, *then* $Cl_{b+(A)} \subseteq Cl_{b+(B)}$.
(5) $Cl_{b+(A)} \cup Cl_{b+(A)} \subseteq Cl_{b+(A \cup B)}$.
(6) $Cl_{b+(A \cap B)} \subseteq Cl_{b+(A)} \cap Cl_{b+(B)}$.

Proof. (1) Let x ∈ A and suppose that x ∉ Cl$_{b+(A)}$. Then, there exits b-+-open set V containing x such that V ∩ A = ∅ and this is a contradiction. Therefore, x ∈ Cl$_{b+(A)}$.

(2) Let x ∈ Cl$_{b+(A)}$. Then, V ∩ A ≠ ∅ for every b-+-open set V containing x. Now, suppose the contrary, that x ∉ ⋂ { W | W is b-+-closed ans A ⊆ W}. Then, x ∉ W for some b-+-closed set W, so x ∈ X − W for some b-+-open set X − W. So, (X− W) ∩ A = ∅ for some b-+-open set X − W containing x and this is a contradiction. Therefore, x ∉ { W | W is b-+-closed ans A ⊆ W}. Conversely, let y ∈ x ∉ ⋂{ W | W is b-+-closed ans A ⊆ W}. Then, y ∈W for all b-+-closed set W containing A. Now, suppose that y ∉ Cl$_{b+(A)}$. Then, there exits b-+-open set V containing y such that V ∩ A = ∅. Therefore, X − V is b-+-closed set containing A and y ∉ X − V and this is a contradiction. Therefore, y ∈ Cl$_{b+(A)}$.

The proof of (3) and (4) are followed directly from the Definition 2.3.

The proof of (5) and (6) are followed by applying part (4) of this Theorem. □

Theorem 2.7. *Let (X, τ) be a topological space and A, B ⊆ X. Then, the following statements hold:*

(1) *If A ⊆ B, then* $Int_{b+(A)} \subseteq Int_{b+(B)}$.
(2) $Int_{b+(A)} \cup Int_{b+(A)} \subseteq Int_{b+(A \cup B)}$.
(3) $Int_{b+(A \cap B)} \subseteq Int_{b+(A)} \cap Int_{b+(B)}$.

Proof. The proof of (1) is followed directly from the Definition 2.2. (2) and (3) are followed by applying part (1) of this Theorem. □

Theorem 2.8. *Let (X, τ) be a topological space and A, B ⊆ X. Then, the following statements hold:*

(1) $Int_{b+(X \setminus A)} = X \setminus Cl_{b+(A)}$.
(2) $Cl_{b+(X \setminus A)} = X \setminus Int_{b+(A)}$.
(3) $X \setminus Cl_{b+(X \setminus A)} = Int_{b+(A)}$.
(4) $X \setminus Int_{b+(X \setminus A)} = Cl_{b+(A)}$.

(5) *$x \in Int_{b+(A)}$ if and only if there exists a b-+-open set M such that $x \in M \subseteq A$.*

Proof. (1) For $A \subseteq X$, holds

$$\begin{aligned} X \setminus Cl_{b+(A)} &= X \setminus \cap\{F : A \subseteq F, F \text{ is b-+-closed}\} \\ &= \cup\{X \setminus F : A \subseteq F, F \text{ is b-+-closed}\} \\ &= \cup\{X \setminus F : X \setminus F \subseteq X \setminus A, F \text{ is b-+-closed}\} = Int_{b+(X\setminus A)}. \end{aligned}$$

(2) For $A \subseteq X$, holds

$$\begin{aligned} X \setminus Int_{b+(A)} &= X \setminus \cup\{U : U \subseteq A, U \text{ is b-+-open}\} \\ &= \cap\{X \setminus U : U \subseteq A, U \text{ is b-+-open}\} \\ &= \cap\{X \setminus U : X \setminus A \subseteq X \setminus U, U \text{ is b-+-open}\} = Cl_{b+(X\setminus A)}. \end{aligned}$$

(3) From (2), $Cl_{b+(X\setminus A)} = X \setminus Int_{b+(A)}$ i.e. $Int_{b+(A)} + Cl_{b+(X\setminus A)} = X$ which implies $X \setminus Cl_{b+(X\setminus A)} = Int_{b+(A)}$.

(4) From (1), $Int_{b+(X\setminus A)} = X \setminus Cl_{b+(A)}$ i.e. $Int_{b+(X\setminus A)} + Cl_{b+(A)} = X$ which implies $X \setminus Int_{b+(X\setminus A)} = Cl_{b+(A)}$. □

Theorem 2.9. *Let A be a subset of a topological space (X, τ). Then, $x \in Cl_{b+(A)}$ if and only if for every b-+-open subset M of X containing x, $A \cap M \neq \emptyset$.*

Proof. Let x $\in$ $\mathrm{Cl}_{b+(A)}$ and suppose that M $\cap$ A $= \emptyset$ for some b-+-open set M which contains x. Then, (X $\setminus$ M) is b-+-closed and A $\subset$ (X $\setminus$ M), thus $\mathrm{Cl}_{b+(A)}$ $\subset$ (X $\setminus$ M). But this implies that x $\in$ (X $\setminus$ M), a contradiction. Thus, A $\cap$ M $\neq$ $\emptyset$.

Conversely, let A $\subseteq$ X and x $\in$ X such that for each b-+-open set M_1 which contains x, $M_1 \cap \neq \emptyset$. If x $\notin$ $\mathrm{Cl}_{b+(A)}$, there is a b-+-closed set F such that A $\subseteq$ F and x $\notin$ F. Then, (X $\setminus$ F) is a b-+-open set with x $\in$ (X $\setminus$ F), and thus (X $\setminus$ F) $\cap$ A $\neq \emptyset$, which is a contradiction. □

Theorem 2.10. *Let (X, τ) be a topological space $M \subseteq X$. Then M is b-+-open if and only if for each $s \in M$, there exists a b-+-open set D such that $s \in D \subseteq M$.*

Proof. It follows from Definition 2.2 and Theorem 2.5. □

Definition 2.4. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be b-+-irresolute if f^{-1}(A) is b-+-open set in (X, τ) for every b-+-open set A of (Y, σ).

Theorem 2.11. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function, then the following statements are equivalent:*

(1) *f is b-+-irresolute.*
(2) *$f(Cl_{b+(A)}) \subseteq Cl_{b+(f(A))}$ holds for every subset A of X.*
(3) *$f^{-1}(A)$ is b-+-closed set in X, for every b-+-closed subset A of Y.*

Proof. (2) $\Rightarrow$ (3): Let A be a b-+-closed set in Y. Then $Cl_{b+(A)} = A$. By using (2), we have $f(Cl_{b+f^{-1}(A)}) \subseteq Cl_{b+(A)} = A$. Thus, $(Cl_{b+f^{-1}(A)}) \subseteq f^{-1}(A)$ and hence $f^{-1}(A)$ is b-+-closed in X.

(3) $\Rightarrow$ (2): If $A \subseteq K$, then $Cl_{b+(f(A))}$ is b-+-closed in Y and by (3) holds $f^{-1}(Cl_{b+(f(A))})$ is b-+-closed in X. Furthermore,

$$A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(Cl_{b+(f(A))}).$$

Thus, $Cl_{b+(A)} \subseteq f^{-1}(Cl_{b+(f(A))})$, consequently,

$$f(Cl_{b+(A)}) \subseteq f(f^{-1}(Cl_{b+(f(A))})) \subseteq Cl_{b+(f(A))}.$$

(3) $\Leftrightarrow$ (1): Obvious. □

Definition 2.5. A function $f : X \rightarrow Y$ is said to be b-+-continuous at a point $x \in X$ if for each open subset K of Y containing $f(x)$, there exists a b-+-open subset L of X containing x such that $f(L) \subseteq K$. The function f is said to be b-+-continuous if it has this property at each $x \in X$.

Theorem 2.12. *A function $f : X \rightarrow Y$ is b-+-continuous if and only if the inverse image of every open set in Y is b-+-open in X.*

Proof. Let f be b-+-continuous and K be any open set in Y. If $f^{-1}(K) = \emptyset$, then $f^{-1}(K)$ is a b-+-open set in X but if $f^{-1}(K) \neq \emptyset$, then there exists $x \in f^{-1}(K)$ which implies $f(x) \in K$. Since f is b-+-continuous, then there exists a b-+-open set L in X containing x such that $f(L) \subseteq K$. This implies that $x \in L \subseteq f^{-1}(K)$ and hence $f^{-1}(K)$ is b-+-open.

Conversely, let K be any open set in Y containing f(x), then $x \in f^{-1}(K)$ and by hypothesis $f^{-1}(K)$ is a b-+-open set in X containing x, so $f(f^{-1}(K)) \subseteq K$. Thus, f is b-+-continuous. □

Theorem 2.13. *For a function $f : X \rightarrow Y$, the following statements are equivalent:*

(1) *f is b-+-continuous.*
(2) *$f^{-1}(K)$ is a b-+-open set in X, for each open subset K of Y.*
(3) *$f^{-1}(F)$ is a b-+-closed set in X, for each closed subset F of Y.*
(4) *$f(Cl_{b+(A)}) \subseteq Cl(f(A))$, for each subset A of X.*
(5) *$Cl_{b+(f^{-1}(B))} \subseteq f^{-1}(Cl(B))$, for each subset B of Y.*
(6) *$f^{-1}(Int(B)) \subseteq Int_{b+(f^{-1}(B))}$, for each subset B of Y.*

Proof. (1) $\Rightarrow$ (2): Directly from Theorem 2.12.

(2) $\Rightarrow$ (3): Let F be any closed subset of Y. Then, $Y \setminus F$ is a open subset of Y. By (2), $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$ is a b-+-open set in X and hence $f^{-1}(F)$ is a b-+-closed set in X.

(3) $\Rightarrow$ (4): Let A be any subset of X. Then, $f(A) \subseteq Cl(f(A))$ and $Cl(f(A))$ is a closed set in Y. Hence, $A \subseteq f^{-1}(Cl(f(A)))$. By (3), we have $f^{-1}(Cl(f(A)))$ is a b-+-closed set in X. Therefore, $Cl_{b+(A)} \subseteq f^{-1}(Cl(f(A)))$. Hence, $f(Cl_{b+(A)}) \subseteq Cl(f(A))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. Then, $f^{-1}(B)$ is a subset of X. By (4), we have $f(Cl_{b+(f^{-1}(B))}) \subseteq Cl(f(f^{-1}(B))) \subseteq Cl(B)$. Hence, $Cl_{b+(f^{-1}(B))} \subseteq f^{-1}(Cl(B))$.

(5) $\Leftrightarrow$ (6): Let B be any subset of Y. Then, apply (5) to $Y \setminus B$ we obtain $Cl_{b+(f^{-1}(Y \setminus B))} \subseteq f^{-1}(Cl(Y \setminus B)) \Leftrightarrow Cl_{b+(X \setminus f^{-1}(B))} \subseteq f^{-1}(X \setminus Int(B)) \Leftrightarrow X \setminus Int_{b+(f^{-1}(B))} \subseteq X \setminus f^{-1}(int(B)) \Leftrightarrow f^{-1}(Int(B)) \subseteq Int_{b+(f^{-1}(B))}$. Thus, $f^{-1}(Int(B))) \subseteq Int_{b+(f^{-1}(B))}$.

(6) $\Rightarrow$ (1): Let $x \in X$ and K be any open subset of Y containing $f(x)$. By (6), we have $f^{-1}(Int(K)) \subseteq Int_{b+(f^{-1}(K))}$ implies that $f^{-1}(K) \subseteq Int_{b+(f^{-1}(K))}$. Hence, $f^{-1}(K)$ is a b-+-open set in X which contains x and clearly $f(f^{-1}(K)) \subseteq K$. Thus, f is b-+-continuous. □

Definition 2.6. Let (X, τ) be a topological space and $A \subseteq X$. Then, A is said to be a generalized b-+-closed set or simply gb-+-closed set if $Cl_{b+(A)} \subseteq U$, whenever $A \subseteq U$ and U is a open set. The complement of a gb-+-closed set is called gb-+-open set. The collection of all gb-+-closed sets and gb-+-open sets are denoted by gb+c(X, τ) and gb+o(X, τ), respectively.

Proposition 2.1. *Every b-+-closed set is gb-+-closed set.*

Proof. The proof is followed by the Definition 2.6. □

The following example shows that the converse of the above Proposition, it is not always true.

Example 2.6. Let X and $\tau(X)$ as in the Example 2.1. Then b-+-closed sets are $\emptyset$, X, {a}, {b}, {a, b}; gb+-closed sets are power set of X. Here {a, c} is a gb-+-closed set but it is not a b-+-closed set.

Theorem 2.14. *Let A be a gb+-closed subset of X. Then, $Cl_{b+(A)} - A$ does not contain any non-empty closed sets.*

Proof. Let F be a closed set of X such that F $\subseteq$ Cl$_{b+(A)}$ $-$ A. Since X $-$ F is a open set, then A $\subseteq$ X $-$ F and A is gb+-closed, it follows Cl$_{b+(A)}$ $\subseteq$ X $-$ F, in consequence F $\subseteq$ X $-$ Cl$_{b+(A)}$. This implies that F $\subseteq$ (X $-$ Cl$_{b+(A)}$) $\cap$ (Cl$_{b+(A)}$ $-$ A) = $\emptyset$, therefore F = $\emptyset$. □

Corollary 2.1. *Let A be a gb-+-closed set. Then, A is b-+-closed if and only if $Cl_{b+(A)} - A$ is a closed set.*

Proof. Let A be gb-+-closed set. If A is b-+-closed, it has Cl$_{b+(A)}$ $-$ A = $\emptyset$ which is a closed set.

Conversely, let Cl$_{b+(A)}$ $-$ A be a closed set. Then, by the Theorem 2.26, Cl$_{b+(A)}$ $-$ A does not contain any non-empty closed set and Cl$_{b+(A)}$ is a closed set of itself. Thus, Cl$_{b+(A)}$ $-$ A = $\emptyset$. Therefore, A = Cl$_{b+(A)}$, in consequence A is a b-+-closed set. □

Corollary 2.2. *Let A be an open set and gb-+-closed set. Then, $A \cap J$ is gb-+-closed set whenever b-+-closed set J of X.*

Proof. Since A is gb-+-closed and open set, then Cl$_{b+(A)}$ $\subseteq$ A and so A is a b-+-closed. Therefore, A $\cap$ J is b-+-closed set of X and this implies that A $\cap$ J is gb-+-closed set of X. □

Theorem 2.15. *Let (X, τ) be a topological space and A, B $\subseteq$ X. If A is a gb-+-closed set and B is any set such that A $\subseteq$ B $\subseteq$ $Cl_{b+(A)}$, then B is a gb-+-closed set of X.*

Proof. Let B $\subseteq$ V where V is an open set of X. Since A is a gb-+-closed set and A $\subseteq$ V , then $\mathrm{Cl}_{b+(A)} \subseteq$ V and so $\mathrm{Cl}_{b+(A)} = \mathrm{Cl}_{b+(B)}$. Therefore, $\mathrm{Cl}_{b+(B)} \subseteq$ V and hence B is a gb-+-closed set of X. □

Theorem 2.16. *Let (X, τ) be a topological space and A $\subset$ X. A is a gb-+-open set if and only if $J \subseteq Int_{b+(A)}$ whenever J closed set and $J \subseteq A$.*

Proof. Let A be a gb-+-open set and let $J \subseteq A$ where J is a closed set. Then, $X - A$ is a gb-+-closed set contained in the open set $X - J$. Therefore, $Cl_{b+(X-A)} \subseteq X - J$ and $X - Int_{b+(A)} \subseteq X - J$. In consequence, $J \subseteq Int_{b+(A)}$.

Conversely, if A is a closed set with $J \subseteq Int_{b+(A)}$ and $J \subseteq A$, then $X - Int_{b+(A)} \subseteq X - J$. Therefore, $Cl_{b+(X-A)} \subseteq X - J$. Hence, $X - A$ is a gb-+-closed set and A is a gb-+-open set of X. □

Definition 2.7. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be gb-+-continuous if $f^{-1}(A)$ is gb-+-closed set in (X, τ) for every closed set A of (Y, σ).

Theorem 2.17. *Every b-+-continuous is gb-+-continuous but not conversely.*

Proof. The proof follows from Proposition 2.1. □

Definition 2.8. Let (X, τ) be a topological space and A $\subseteq$ X. Then, A is said to be a regular generalized b-+-closed set or simply rgb-+-closed set if $\mathrm{Cl}_{b+(A)} \subseteq$ U, whenever A $\subseteq$ U and U is a regular open set. The complement of a rgb-+-closed set is called rgb-+-open set. The collection of all rgb-+-closed sets and rgb-+-open sets are denoted by $rgb + c(X, \tau)$ and $rgb + o(X, \tau)$, respectively.

Proposition 2.2. *Every closed set is rgb-+-closed set.*

Proof. Let B be any closed set of X such that B $\subseteq$ V . where V is a regular open set. Since $\mathrm{Cl}_{b+(B)} \subseteq$ Cl(B) = B. Therefore, $\mathrm{Cl}_{b+(B)} \subseteq$ V . In consequence, B is a rgb+-closed set. □

The following example shows that the converse of the above Theorem need not be true.

Example 2.7. Let X and $\tau(X)$ as in the Example 2.6. Then rgb-+-closed sets are power set of X. Here, {a, b} is rgb-+-closed set but it is not closed set.

Proposition 2.3. *Every b-+-closed set is rgb-+-closed set.*

Proof. Let B be any b-+-closed set of X such that V is any regular open set containing B. Since B is a b-+- closed set, then $\mathrm{Cl}_{b+(B)}$ = B. Therefore, $\mathrm{Cl}_{b+(B)} \subseteq$ V . Hence, B is a rgb-+-closed set. □

The following example shows that the converse of the above Theorem need not be true.

Example 2.8. Let X and $\tau(X)$ as in the Example 2.7. Here, {a, c} is rgb-+-closed set but it is not b-+-closed set.

Proposition 2.4. *Every gb-+-closed set is rgb-+-closed set but not conversely.*

Proof. If A is a gb-+-closed subset of (X, τ) and G is any regular open set containing A, since every regular open set is open, we have G $\supseteq$ $\mathrm{Cl}_{b+(A)}$. Hence A is rgb-+-closed in (X, τ). □

Example 2.9. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}\}$. Then gb-+-closed sets are $\emptyset, X, \{a\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$; rgb-+-closed sets are power of X. Here $\{a, b, c\}$ is rgb-+-closed set but it is not a gb-+-closed set.

Definition 2.9. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be regular generalized b-+-continuous (berifly, rgb-+-continuous) if f^{-1}(A) is rgb-+-closed set in (X, τ) for every closed set A of (Y, σ).

Theorem 2.18. *Every semi-+-open set is b-+-open set but not conversely.*

Proof. The proof follows from Theorem 2.2. □

Theorem 2.19. *Every α-+-continuous set is b-+-continuous set but not conversely.*

Proof. The proof follows from Theorem 2.3. □

Theorem 2.20. *Every pre-+-continuous set is b-+-continuous set but not conversely.*

Proof. The proof follows from Theorem 2.4. □

Theorem 2.21. *Every continuous is rgb-+-continuous but not conversely.*

Proof. The proof follows from Proposition 2.2. □

Theorem 2.22. *Every b-+-continuous set is rgb-+-continuous but not conversely.*

Proof. The proof follows from Proposition 2.3. □

Proposition 2.5. *Every gb-+-continuous is rgb-+-continuous but not conveserly.*

Proof. The proof follows from Proposition 2.4. □

3. Conclusion

In this paper we introduced a new operator of b-+-open sets, generalized b-+-closed sets and regular generalized b-+-closed sets in topological spaces. Also we characterize the relations between them and the related properties. In future, we have extended this work in various topological fields.

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