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SOLUTION OF DUFFING EQUATION WITH FOURIER DECOMPOSITION METHOD

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Abstract. In this article, we aim to find an alternative solution to the Duffing Equation, which is an important ordinary differential equation in mathematical physics. We used for solution Fourier Transform and Adomian decomposition method. Adomian polynomials have been used for nonlinear term. Finally we obtained the complete solution by using the Pade Approach from the approximate solutions founded with the Fourier Adomian Decomposition Method(FADM).

1. Introduction

Various natural systems are modelled by differential equations and most of them are nonlinear. However, solving such nonlinear equations is not easy in general. Therefore, investigation of various efficient methods to solve these equations have been an important topic of the research. In recent years, many methods have been developed to obtain exact and approximate solutions of such equations. But the point reached for the solution of such equations is not enough. Because for all nonlinear equations there is no method that gives the exact solution or the closest solution. Therefore, the method that gives the closest solution to the exact solution may be differ to equation from the equation. Some methods used for such equations are Adomian decomposition method, Homotopy perturbation method, Variational iteration method, tanh method, differential transform method, etc. The solution of such equations cannot be obtained by using integral transforms. However, nonlinear equations can be solved by combining integral transforms with the abovementioned methods. For example, nonlinear equations are solved by using Laplace

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and Adomain,Elzaki and Homotopy perturbation, Fourier and Adomian method together. [**14, 7, 8, 5**]

One of the most common physical nonlinear ordinary differential equations, governs many oscillative systems, is the Duffing equations. These equations can be found in a wide variety of engineering and scientific applications. There are many articles about the Duffing equation [**14, 11, 12, 10, 4, 6, 9, 13, 1**]. In this study, we tried to find approximate solution of Duffing equation by using FADM. We have applied the Pade approach to the Fourier transform of this approximate solution. We obtained the complete solution by taking the inverse Fourier transform of the obtained function. The Duffing equation is described by second order ordinary differential equation with the common form

$$y'' + py' + p_1 y + p_2 y^3 = f(x)$$
$$y(0) = \alpha, y'(0) = \beta$$

where $p, p_1, p_2, \alpha, \beta$ are real constants.

We found that our results are consistent with the literature.

2. Basic Definitions and Theorems

2.1. Fourier Transform.

Definition 2.1. Let f be an absolutely integrable on the real line and piecewise continuous on every finite interval. Then Fourier Transform is defined by

$$\mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-iwt}dt = F(w)$$

and similarly inverse Fourier transform is

$$\mathcal{F}^{-1}[F(w)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{iwt}dt = F(w)$$

Theorem 2.1 ([**2**]). *Let $f(t)$ be continuous or partly continuous in the interval $(-\infty, \infty)$ and*

$$f(t), f'(t), f''(t), ..., f^{(n-1)}(t) \to 0 \text{ for } |t| \to \infty.$$

If $f(t), f'(t), f''(t), , f^{(n-1)}(t)$ are absolutely integrable in the interval $(-\infty, \infty)$, then

$$\mathcal{F}[f^{(n)}(t)] = (iw)^n \mathcal{F}[f(t)].$$

Definition 2.2. The Dirac delta distribution can be rigorously thought of as a distribution on real line which is zero every where except at the origin, where it is infinite,

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

Some properties ([**2**]) of the Dirac delta are

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$$

$$\int_{-\infty}^{\infty} f(t)\delta^{(n)}(t-t_0)dt = (-1)^n.f^{(n)}(t_0)$$

$$(t-t_o)^n\delta^{(n)}(t-t_0) = (-1)^n.n!.\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} \frac{\delta(t-t_o)f(w)}{(t-t_o)^n}dw = \frac{1}{n!}\frac{d^n f(w)}{dw^n}(t=t_0)$$

$$f(w).\delta(w-w_0) = f(w+w_0)\delta(w)$$

THEOREM 2.2. [**3**] *If Fourier transforms of some functions are following*
(i) $\mathcal{F}[1] = 2\pi\delta(w)$
(ii) $\mathcal{F}[t^n] = 2\pi.i^n\delta^{(n)}(w)$
(iii) $\mathcal{F}[e^{iw_0t}] = 2\pi\delta(w-w_0)$

3. FADM for Duffing Equation

In this section, FADM is applied to the Duffing equation. Let us consider following general Duffing equation with initial conditions

$$y'' + \alpha y' + \beta y + \gamma y^3 = f(x), y(0) = A, y'(0) = B$$

Let we apply Fourier transform to this equation.

$\mathcal{F}(y'') + \alpha\mathcal{F}(y') + \beta\mathcal{F}(y) + \gamma\mathcal{F}(y^3) = \mathcal{F}[f(x)]$

$(iw)^2\mathcal{F}(y) + \alpha(iw)\mathcal{F}(y) + \beta\mathcal{F}(y) + \gamma\mathcal{F}(y^3) = \mathcal{F}[f(x)]$

$\mathcal{F}(y) = \frac{-(\alpha iw+\beta)\mathcal{F}(y)-\gamma\mathcal{F}(y^3)+\mathcal{F}[f(x)]}{-w^2}$ and $\mathcal{F}(y) = \frac{(\alpha iw+\beta)\mathcal{F}(y)+\gamma\mathcal{F}(y^3)-\mathcal{F}[f(x)]}{w^2}$

If we apply inverse Fourier transform to above equality, than we get that:

$$\mathcal{F}^{-1}[\mathcal{F}(y)] = \mathcal{F}^{-1}(\frac{(\alpha iw+\beta)\mathcal{F}(y)}{w^2}) + \mathcal{F}^{-1}(\frac{\mathcal{F}(\gamma y^3))}{w^2} - \mathcal{F}^{-1}(\frac{\mathcal{F}(f(x))}{w^2})$$

So we get the following iteration relation:

$$y_{n+1} = \mathcal{F}^{-1}[\frac{(\alpha iw+\beta)\mathcal{F}(y_n)}{w^2}] + \mathcal{F}^{-1}(\frac{\gamma\mathcal{F}(A_n)}{w^2})$$

where A_n's Adomian polynomials.

$A_0 = y_0^3$, $A_1 = 3y_1 \cdot (y_0)^2$, $A_2 = 3y_2 \cdot (y_0)^2 + 3y_0(y_1)^2$,

$A_3 = 3y_3(y_0)^2 + 6y_0y_1 \cdot y_2 + (y_1)^3$,...

$y_0 = Bx + A - \mathcal{F}^{-1}(\frac{F[f(x)}{w^2})$, $y_1 = \mathcal{F}^{-1}(\frac{(\alpha iw+\beta)\mathcal{F}(y_0)}{w^2} + \mathcal{F}^{-1}(\frac{\gamma F(A_0)}{w^2}$

$y_2 = \mathcal{F}^{-1}(\frac{(\alpha iw+\beta)\mathcal{F}(y_1)}{w^2} + \mathcal{F}^{-1}(\frac{\gamma F(A_1)}{w^2}$.

Since the complicated excitation term $f(x)$ can cause difficult integrations and proliferation of terms, we can express $f(x)$ in Taylor series at $x_0 = 0$, which is truncated for simplification. If we replace in place of $f(x)$

$$f \approx \sum_{i=0}^{K} a_i x^i$$

than as the approximate solution with the first n terms, the sum of the terms up to the mth degree in the expression $y = y_0 + y_+ y_2 + ... y_{n-1}$ can be taken. Here m is the order of the first term (y_0). It is clear that $m = K + 2$.

EXAMPLE 3.1. ([**14, 9**]) Consider the Duffings equation in the following type:

$$y'' + 3y - 2y^3 = cosx \cdot sin2x$$

with initial conditions $y(0) = 0, y'(0) = 1$.

The analytic solution of this equation is

$$y(x) = sinx.$$

Approximation value at $x = 0$ of $f(x) = cosx \cdot sin2x$ is

$$f(x) \approx 2x - \frac{7x^3}{3} + \frac{61x^5}{60} - \frac{547x^7}{2520}.$$

Coefficients of equation which we investigate are $\alpha = 0, \beta = 3, \gamma = -2$.

$$y_0 = x - \mathcal{F}^{-1}\left(\frac{\mathcal{F}(2x - \frac{7x^3}{3} + \frac{61x^5}{60} - \frac{547x^7}{2520})}{w^2}\right)$$

$$y_0 = x - \mathcal{F}^{-1}\left(\frac{2\pi(i2\delta' - \frac{7i^3\delta^{(3)}}{3} + \frac{61i^5\delta^{(v)}}{60} - \frac{547i^7\delta^{(7)}}{2520})}{w^2}\right)$$

From properties of Dirac delta function y_0 can be written as following

$$y_0 = x + 2i\int_{-\infty}^{\infty} \frac{\delta e^{iwx} dw}{w^3} + 14i\int_{-\infty}^{\infty} \frac{\delta e^{iwx} dw}{w^5} + 122i\int_{-\infty}^{\infty} \frac{\delta e^{iwx} dw}{w^7} + \frac{547i}{2520}\int_{-\infty}^{\infty} \frac{7!\delta e^{iwx} dw}{w^9}.$$

$$= x + 2i\frac{(ix)^3}{3!} + 14i\frac{(ix)^5}{5!} + 122i\frac{(ix)^7}{7!} + \frac{547i}{2520}\frac{(ix)^9}{72}$$

$$= x + \frac{x^3}{3} - \frac{7x^5}{60} + \frac{61x^7}{2520} - \frac{547x^9}{181440}$$

$$y_1 = \mathcal{F}^{-1}\left(\frac{3\mathcal{F}(y_0) - 2\mathcal{F}(y_0^3)}{w^2}\right).$$

$$y_1 \approx \mathcal{F}^{-1}\left(\frac{\mathcal{F}(3x - x^3 - \frac{47x^5}{20} + \frac{2119x^7}{20000})}{w^2}\right)$$

$$y_1 \approx 2\pi\mathcal{F}^{-1}\left(\frac{3i\delta' - i^3\delta^{(3)} - \frac{47i^5\delta^{(5)}}{20} + \frac{2119i^7\delta^{(7)}}{20000}}{w^2}\right)$$

$$y_1 \approx 2\pi\mathcal{F}^{-1}\left(\frac{-3i\delta}{w^3} - \frac{6i\delta}{w^5} + \frac{282i\delta}{w^7} + \frac{133497i\delta}{250w^9}\right)$$

$$\approx -3i\frac{(ix)^3}{3!} - 6i\frac{(ix)^5}{5!} + 282i\frac{(ix)^7}{7!} + \frac{133497i}{250}\frac{(ix)^9}{9!}$$

$$\approx -\frac{x^3}{2} + \frac{x^5}{20} + \frac{47x^7}{840} - \frac{89x^9}{60480}$$

$$y_2 = \mathcal{F}^{-1}\left(\frac{3\mathcal{F}(y_1) - 2\mathcal{F}(3y_0^2 y_1)}{w^2}\right)$$

$$y_2 \approx \mathcal{F}^{-1}\left(\frac{\mathcal{F}(-\frac{3x^3}{2} + \frac{63x^5}{20} + \frac{523x^7}{280})}{w^2}\right)$$

$\approx 2\pi\mathcal{F}^{-1}(\frac{\frac{-3i^3\delta'''}{2}+\frac{63i^5\delta^{(5)}}{20}+\frac{523i^7\delta^{(7)}}{280}}{w^2})$

$\approx 2\pi\mathcal{F}^{-1}(\frac{-18i\delta(w)}{2w^5} - \frac{63i5!\delta(w)}{20w^7} + \frac{523i7!\delta(w)}{280w^9})$

$\approx \frac{3x^5}{40} - \frac{3x^7}{40} - \frac{523x^9}{20160}$

$y_3 = \mathcal{F}^{-1}(\frac{3\mathcal{F}(y_2)-2\mathcal{F}(3y_0^2y_2+3y_0y_1^2)}{w^2})$

$y_3 \approx \mathcal{F}^{-1}(\frac{\mathcal{F}(\frac{9x^5}{40}-\frac{87x^7}{40})}{w^2})$

$y_3 \approx \mathcal{F}^{-1}(\frac{9.2\pi i^5\delta^{(5)}}{40w^2}) - \mathcal{F}^{-1}(\frac{87.2\pi i^7\delta^{(7)}}{40w^2})$

$y_3 \approx -\frac{3x^7}{560} + \frac{29x^9}{960}$

$y_4 = \mathcal{F}^{-1}(\frac{3\mathcal{F}(y_3)-2\mathcal{F}(3y_0^2y_3+6y_0y_1y_2+y_1^3)}{w^2})$

$y_4 \approx \mathcal{F}^{-1}(\frac{\mathcal{F}(\frac{-9x^7}{560})}{w^2}) = \frac{-9}{560}\mathcal{F}^{-1}(\frac{2\pi i^7\delta^{(7)}}{w^2})$

$\approx \frac{-9}{560}\mathcal{F}^{-1}(\frac{2\pi i^7 7!\delta}{w^9}) = \frac{x^9}{4480}$

$y \approx y_0 + y_1 + y_2 + y_3 + y_4$

$y \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} = \phi_5(x)$

$\phi_5(x)$ is the sum of the terms up to the 9th order of the first 5 terms If we take Fourier transform of $\phi_5(x)$ than we get

$$\mathcal{F}[\phi_5(x)] = 2\pi\delta' - \frac{2\pi i^3\delta'''}{3!} + \frac{2\pi i^5\delta^{(5)}}{5!} - \frac{2\pi i^7\delta^{(7)}}{7!} + \frac{2\pi i^9\delta^{(9)}}{9!}$$
$$= -2\pi i\left(\frac{\delta}{w} + \frac{\delta}{w^3} + \frac{\delta}{w^5} + \frac{\delta}{w^7} + \frac{\delta}{w^9}\right)$$

All of the [L/M] pade approximation of $\mathcal{F}[\phi_5(x)]$, yields $[L/M] = 2\pi i\frac{w\delta(w)}{1-w^2}$.

$$\frac{2\pi iw\delta(w)}{1-w^2} = \frac{2\pi i\delta(w)}{2}(\frac{1}{1-w} - \frac{1}{1+w})$$
$$= \pi i(\frac{\delta(w)}{1-w} - \frac{\delta(w)}{1+w}) = \pi i(\frac{\delta(w+1)}{-w} - \frac{\delta(w-1)}{w})$$

By using the inverse Fourier transformation to $[L/M]$,

$$\mathcal{F}^{-1}[\pi i(\frac{\delta(w+1)}{-w} - \frac{\delta(w-1)}{w})] = \frac{i}{2}\int_{-\infty}^{\infty}(\frac{\delta(w+1)}{-w} - \frac{\delta(w-1)}{w})e^{iwx}dw$$
$$= \frac{i}{2}(e^{-ix} - e^{ix}) = sinx.$$

Thus, a complete solution was obtained.

EXAMPLE 3.2. [**11, 13**] Let us consider the Duffings equation

$$y'' + y' + y + y^3 = cos^3x - sinx \text{ with } y(0) = 1,\ y'(0) = 0.$$

The exact solution of the initial value problem is y(x)=cosx. Approximation value of $f(x)$ can be found by using Maclaurin expansion as

$$f(x) = cos^3x - sinx \approx 1 - x - \frac{3x^2}{2} + \frac{x^3}{6} + \frac{7x^4}{8}.$$

Coefficients of the equation which we investigate are $\alpha = \beta = \gamma = 1$.

$y_0 = 1 - \mathcal{F}^{-1}(\frac{1-x-\frac{3x^2}{2}+\frac{x^3}{6}+\frac{7x^4}{8}}{w^2}) = 1 + \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{8} + \frac{x^5}{120} + \frac{21x^6}{720}$
$y_1 = \mathcal{F}^{-1}(\frac{1}{w^2}(iw+1)\mathcal{F}(y_0)) + \mathcal{F}^{-1}(\frac{1}{w^2}\mathcal{F}(y_0^3)) \approx -x^2 - \frac{x^3}{6} - \frac{x^4}{8} + \frac{7x^5}{120} - \frac{7x^6}{720}.$
$y_2 \approx \mathcal{F}^{-1}(\frac{1}{w^2}(iw+1)\mathcal{F}(y_1)) + \mathcal{F}^{-1}(\frac{1}{w^2}\mathcal{F}(3y_0^2y_1))$
$y_2 \approx \mathcal{F}^{-1}[\frac{1}{w^2}(iw+1)\mathcal{F}(-x^2 - \frac{x^3}{6} - \frac{x^4}{8} + \frac{7x^5}{120} - \frac{7x^6}{720})]$
$+\mathcal{F}^{-1}[\frac{1}{w^2}\mathcal{F}(-3x^2 - \frac{x^3}{2} - \frac{27x^4}{8} + \frac{27x^5}{40} + \frac{19x^6}{80})]$
$= \frac{2x^3}{6} + \frac{9x^4}{24} + \frac{7x^5}{120} + \frac{77x^6}{720}.$
$y_3 \approx \mathcal{F}^{-1}(\frac{1}{w^2}(iw+1)\mathcal{F}(y_2)) + \mathcal{F}^{-1}(\frac{1}{w^2}\mathcal{F}(3y_0^2y_2 + 3y_0y_1^2))$
$y_3 = \mathcal{F}^{-1}[\frac{1}{w^2}(iw+1)\mathcal{F}(\frac{2x^3}{6} + \frac{9x^4}{24} + \frac{7x^5}{120} + \frac{77x^6}{720})]$
$+\mathcal{F}^{-1}[\frac{1}{w^2}(\mathcal{F}(x^3 + \frac{33x^4}{8} + \frac{87x^5}{40} + \frac{827x^6}{240})]$
$\approx -\frac{2x^4}{24} - \frac{17x^5}{120} - \frac{115x^6}{720}.$
$y_4 \approx \mathcal{F}^{-1}(\frac{1}{w^2}(iw+1)\mathcal{F}(y_3)) + \mathcal{F}^{-1}(\frac{1}{w^2}\mathcal{F}(3y_0^2y_3 + 6y_0y_1y_2 + y_1^3))$
$y_4 = \mathcal{F}^{-1}[\frac{1}{w^2}(iw+1)\mathcal{F}(\frac{-2x^4}{24} - \frac{17x^5}{120} - \frac{115x^6}{720})]$
$+\mathcal{F}^{-1}[\frac{1}{w^2}\mathcal{F}(-\frac{x^4}{4} - \frac{97x^5}{40} - \frac{69x^6}{16})]$
$\approx \frac{2x^5}{120} + \frac{25x^6}{720}.$
$y_5 \approx \mathcal{F}^{-1}(\frac{1}{w^2}(iw+1)\mathcal{F}(y_4)) + \mathcal{F}^{-1}(\frac{1}{w^2}\mathcal{F}(3y_0^2y_4 + 6y_0y_1y_3 + 3y_0y_2^2 + 3y_1^2y_2))$
$\approx -\frac{2x^6}{720}.$
$y \approx y_0 + y_1 + y_2 + y_3 + y_4 + y_5$
$\approx 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} = \phi_5(x).$

All of the [L/M] pade approximation of $\mathcal{F}[\phi_5(x)]$, yields $[L/M] = 2\pi\delta(w)\frac{w^2}{w^2-1}$. Let us use specials of dirac delta function for $\frac{2\pi\delta(w)w^2}{w^2-1}$.

$$\frac{2\pi\delta(w)w^2}{w^2-1} = 2\pi w\frac{w\delta(w)}{w^2-1} = \pi w(\frac{\delta(w)}{w-1} + \frac{\delta(w)}{w+1})$$

.

$$= \pi w(\frac{\delta(w+1)}{w} + \frac{\delta(w-1)}{w})$$
$$= \pi(\delta(w+1) + \delta(w-1)).$$

If we take inverse fourier transform of $\pi(\delta(w+1) + \delta(w-1))$, we get that

$$\mathcal{F}^{-1}(\pi(\delta(w+1) + \delta(w-1))) = \int_{-\infty}^{\infty} \frac{\delta(w+1)+\delta(w-1)}{2}e^{iwx}dw$$
$$= \frac{e^{ix}+e^{-ix}}{2} = cosx.$$

Thus, a complete solution was obtained.

4. conclusion

In this article, approximate solution of Duffing equation is obtained by using Fourier Transform and Adomian decomposition method. Some samples, which were solved by other methods, were examined by this method and it is shown that results are consistent. Since the technique is direct and powerful it can be used to handle a variety of equations which appears in applications in several branch of the nonlinear equations.

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