BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., Vol. **12**(1)(2022), 51-60
DOI: 10.7251/BIMVI2201051A

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

PRIME AND MAXIMAL IDEAL BASED ON SOFT INTERSECTIONAL RINGS

Recep Aydın and Filiz Çıtak

Abstract. In this paper, we firstly introduced the soft intersectional prime ideal, and studied some of its properties. We then defined soft intersectional maximal ideal. We finally discussed some basic properties of this concept.

1. Introduction

Scientists have been interested in the complex structures causing uncertainty in the economy, engineering, environmental science, social sciences, health science and many other areas. Since it is difficult to solve them by classical methods, different methods have been developed to model the structures had these uncertainties and to create a systematic solution apart from classical methods for solving such problems. Some of the theories dealing with such methods like probability theory, fuzzy set theory, rough set theory and soft set theory. Fuzzy set theory was first introduced in 1965 by Zadeh [**31**]. According to Zadehs fuzzy set theory, everything is indicated by a certain degree in the interval [0,1]. In 1971, Rosenfeld was first studied algebraic structures in fuzzy sets. In his study, he investigated some algebraic properties by making the definition of fuzzy group [**22**]. In 1979, Giles pointed out a strong relationship between fuzzy sets and many-valued logic [**13**]. It has been used in many areas like fuzzy logic, electronic control systems, automotive braking systems, home electronics. In 1982, Pawlak wrote an article on the theory of rough sets [**21**]. The soft set theory was proposed by Molodtsov in 1999 [**20**]. In 2002, Maji et al. made some definitions on soft sets and included some operations on soft sets [**19**]. Successful results have been achieved with these operations in

2010 *Mathematics Subject Classification.* Primary 3E99; Secondary 3E75.
Key words and phrases. Soft sets, soft intersectional ring, soft intersectional ideal, soft intersectional prime ideal, soft intersectional maximal ideal.
Communicated by Daniel A. Romano.

many areas. Çağman and Enginoğlu redefined Molodstovs concept of soft sets for the development of several new results in 2010 [**7**]. Çağman and Enginoğlu also defined soft matrices and its some operations [**8**]. Ali et al. introduced some different operations related to soft sets in 2009 [**4**]. Also, Ali et al. examined the relationship among fuzzy sets, rough sets and soft sets [**5**]. Kharal and Ahmad studied mappings on soft classes [**18**]. Aktaş and Çağman compared soft sets with fuzzy sets and rough sets by defining soft group in 2007 [**3**]. Also, they investigated some algebraic properties of soft group. With this study, soft set theory paved the way for many studies in the field of algebra [**10, 11, 14, 23, 27, 28, 29, 30, 32, 33, 34, 35, 37**]. Sezgin defined LA semigroups by soft sets and AG groupoids by soft sets [**24, 25**]. Soft rings and some studies on algebraic operations of soft rings have been made by Acar et al. in 2010 [**2**]. Jun et al. analyzed BCK/BCI ideals by soft set theory [**15, 16**]. Feng et al. introduced soft semiring and soft ideals on soft semiring [**12**]. In 2012, Çağman et al. described soft intersection groups and researched its algebraic operations [**6**]. Also, Çıtak and Çağman investigated its algebraic properties by defining soft intersection rings in 2015 [**9**]. Soft algebraic studies continue to increase in recent years [**1, 26, 17, 36**].

In this paper, we firstly introduced the soft intersectional prime ideal, and studied some of its properties. Then, we defined soft intersectional maximal ideal. We finally some basic properties of this concept are discussed and the related result are investigated.

2. Preliminary

Throughout this chapter, U refers to an initial universe, X is a set of parameters, $P(U)$ is the power set of U.

Definition 2.1. ([**20**]) A soft set $(\widetilde{\varphi}, X)$ over U is a function defined by

$$\widetilde{\varphi} : X \to P(U)$$

A soft set $(\widetilde{\varphi}, X)$ over U can be represented by the set of ordered pairs

$$(\widetilde{\varphi}, X) = \{(e, \widetilde{\varphi}(e)) : e \in X\}$$

Definition 2.2. ([**19**]) Let $(\widetilde{\varphi}, X)$ be a soft set over U. If $\widetilde{\varphi}(e) = \emptyset$ for all $e \in X$, then $(\widetilde{\varphi}, X)$ is called an empty soft set, denoted by $\widetilde{\Phi}$.

If $\widetilde{\varphi}(e) = U$ for all $e \in X$, then $(\widetilde{\varphi}, X)$ is called universal soft set, denoted by $\widetilde{E}$.

Definition 2.3. ([**7**]) Let $(\widetilde{\varphi}, X)$ and $(\widetilde{\chi}, X)$ be soft sets over U. Then, $(\widetilde{\varphi}, X)$ is a soft subset of $(\widetilde{\chi}, X)$, denoted by $(\widetilde{\varphi}, X)\widetilde{\subseteq}(\widetilde{\chi}, X)$, if $\widetilde{\varphi}(e) \subseteq \widetilde{\chi}(e)$ for all $e \in X$.

$(\widetilde{\varphi}, X)$ is called a soft proper subset of $(\widetilde{\chi}, X)$, denoted by $(\widetilde{\varphi}, X)\widetilde{\subset}(\widetilde{\chi}, X)$, if $\widetilde{\varphi}(e) \subseteq \widetilde{\chi}(e)$ for all $e \in E$ and $\widetilde{\varphi}(e) \neq \widetilde{\chi}(e)$ for at least one $e \in E$.

$(\widetilde{\varphi}, X)$ and $(\widetilde{\chi}, X)$ are equal, denoted by $(\widetilde{\varphi}, X) = (\widetilde{\chi}, X)$ if $\widetilde{\varphi}(e) = \widetilde{\chi}(e)$ for all $e \in X$.

Definition 2.4. ([**7**]) Let $(\widetilde{\varphi}, X)$ and $(\widetilde{\chi}, X)$ be two soft sets over U. Then, union $(\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\chi}, X)$ and intersection $(\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\chi}, X)$ of $(\widetilde{\varphi}, X)$ and $(\widetilde{\chi}, X)$ are defined by,

$$(\widetilde{\varphi} \cup \widetilde{\chi})(e) = \widetilde{\varphi}(e) \cup \widetilde{\chi}(e), \quad (\widetilde{\varphi} \cap \widetilde{\chi})(e) = \widetilde{\varphi}(e) \cap \widetilde{\chi}(e)$$

for all $e \in X$, respectively.

Definition 2.5. ([**7**]) Let $(\widetilde{\varphi}, X)$ be a soft set over U. Then, complement $(\widetilde{\varphi}^{\tilde{c}}, X)$ of $(\widetilde{\varphi}, X)$ is defined by,

$$\widetilde{\varphi}^{c}(e) = U \setminus \widetilde{\varphi}(e)$$

for all $e \in X$.

It is easy to see that $((\widetilde{\varphi}, X)^{\tilde{c}})^{\tilde{c}} = (\widetilde{\varphi}, X)$ and $\widetilde{\Phi}^{\tilde{c}} = \widetilde{X}$.

Proposition 2.1 ([**7**]). *Let $(\widetilde{\varphi}, X)$ be a soft set over U. Then,*

(1) $(\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\varphi}, X) = (\widetilde{\varphi}, X)$, $(\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\varphi}, X) = (\widetilde{\varphi}, X)$
(2) $(\widetilde{\varphi}, X)\widetilde{\cup}\widetilde{\Phi} = (\widetilde{\varphi}, X)$, $(\widetilde{\varphi}, X)\widetilde{\cap}\widetilde{\Phi} = \widetilde{\Phi}$
(3) $(\widetilde{\varphi}, X)\widetilde{\cup}\widetilde{X} = \widetilde{X}$, $(\widetilde{\varphi}, X)\widetilde{\cap}\widetilde{X} = (\widetilde{\varphi}, X)$
(4) $(\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\varphi}, X)^{\tilde{c}} = \widetilde{X}$, $(\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\varphi}, X)^{\tilde{c}} = \widetilde{\Phi}$

Proposition 2.2 ([**7**]). *Let $(\widetilde{\varphi}, X), (\widetilde{\chi}, X)$ and $(\widetilde{\psi}, X)$ be soft sets over U. Then,*

(1) $(\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\chi}, X) = (\widetilde{\chi}, X)\widetilde{\cup}(\widetilde{\varphi}, X)$,
$(\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\chi}, X) = (\widetilde{\chi}, X)\widetilde{\cap}(\widetilde{\varphi}, X)$
(2) $((\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\chi}, X))^{\tilde{c}} = (\widetilde{\chi}, X)^{\tilde{c}}\widetilde{\cap}(\widetilde{\varphi}, X)^{\tilde{c}}$,
$((\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\chi}, X))^{\tilde{c}} = (\widetilde{\chi}, X)^{\tilde{c}}\widetilde{\cup}(\widetilde{\varphi}, X)^{\tilde{c}}$
(3) $((\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\chi}, X))\widetilde{\cup}(\widetilde{\psi}, X) = (\widetilde{\varphi}, X)\widetilde{\cup}((\widetilde{\chi}, X)\widetilde{\cup}(\widetilde{\psi}, X))$,
$((\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\chi}, X))\widetilde{\cap}(\widetilde{\psi}, X) = (\widetilde{\varphi}, X)\widetilde{\cap}((\widetilde{\chi}, X)\widetilde{\cap}(\widetilde{\psi}, X))$
(4) $(\widetilde{\varphi}, X)\widetilde{\cup}((\widetilde{\chi}, X)\widetilde{\cap}(\widetilde{\psi}, X)) = ((\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\chi}, X))\widetilde{\cap}((\widetilde{\varphi}, X)\widetilde{\cup}(\widetilde{\psi}, X))$,
$(\widetilde{\varphi}, X)\widetilde{\cap}((\widetilde{\chi}, X)\widetilde{\cup}(\widetilde{\psi}, X)) = ((\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\chi}, X))\widetilde{\cup}((\widetilde{\varphi}, X)\widetilde{\cap}(\widetilde{\psi}, X))$

Definition 2.6. ([**9**]) Let R be a ring and $(\widetilde{\varphi}, R)$ be a soft set over U. Then, $(\widetilde{\varphi}, R)$ is called a soft intersectional ring over U iff

(1) $\widetilde{\varphi}(r_1 - r_2) \supseteq \widetilde{\varphi}(r_1) \cap \widetilde{\varphi}(r_2)$
(2) $\widetilde{\varphi}(r_1 r_2) \supseteq \widetilde{\varphi}(r_1) \cap \widetilde{\varphi}(r_2)$

for all $r_1, r_2 \in R$.

Definition 2.7. ([**9**]) Let R be a ring and $(\widetilde{\varphi}, R)$ be a soft set over U. Then, $(\widetilde{\varphi}, R)$ is called a soft intersectional ideal over U iff

(1) $\widetilde{\varphi}(r_1 - r_2) \supseteq \widetilde{\varphi}(r_1) \cap \widetilde{\varphi}(r_2)$
(2) $\widetilde{\varphi}(r_1 r_2) \supseteq \widetilde{\varphi}(r_1) \cup \widetilde{\varphi}(r_2)$

for all $r_1, r_2 \in R$.

Proposition 2.3 ([**9**]). *If $(\widetilde{\varphi}, R)$ is a soft intersectional ring/ideal over U, then $\widetilde{\varphi}(0_R) \supseteq \widetilde{\varphi}(x)$ for all $x \in R$.*

PROPOSITION 2.4 ([**9**]). *Let R be a ring with identity. If $(\widetilde{\varphi}, R)$ is a soft intersectional ideal over U, then $\widetilde{\varphi}(x) \supseteq \widetilde{\varphi}(1_R)$ for all $x \in R$.*

DEFINITION 2.8. Let P be an ideal of a ring R. λ_P is called soft characteristic function, defined by

$$\lambda_P(\mathrm{r}) = \begin{cases} U, & r \in P, \\ \emptyset, & r \notin P. \end{cases}$$

where $\lambda_P : R \longrightarrow P(U)$ for all $r \in R$.

THEOREM 2.1. *Let P be an ideal of a ring R. Then, characteristic function λ_p is a soft intersectional ring over U.*

PROOF. Firstly, we will show that $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$ for all $r_1, r_2 \in P$. Since P is an ideal of R, then $r_1 - r_2 \in P$ for all $r_1, r_2 \in P$. Then, $\lambda_P(r_1) = U$ and $\lambda_P(r_2) = U$, and $\lambda_P(r_1 - r_2) = U$. Thus, $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$. And, $\lambda_P(r_1) = \emptyset$, $\lambda_P(r_2) = \emptyset$ for all $r_1, r_2 \notin P$. Therefore, $\lambda_P(r_1) \cap \lambda_P(r_2) = \emptyset$ and so, $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$. And, $\lambda_P(r_1) = U$, $\lambda_P(r_2) = \emptyset$ for all $r_1 \in P, r_2 \notin P$. Hence, $\lambda_P(r_1) \cap \lambda_P(r_2) = \emptyset$ and so, $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$.

Now, we will show that iin $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$ for all $r_1, r_2 \in R$. Since P is an ideal of R, then $r_1 \cdot r_2 \in P$ for all $\forall r_1, r_2 \in P$. Therefore, $\lambda_P(r_1) = U$ and $\lambda_P(r_2) = U$, and $\lambda_P(r_1 \cdot r_2) = U$ Thus, $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$. Since P is an ideal of R, then $r_1 \cdot r_2 \in P$ ve $r_1 \cdot r_2 \in P$ for all $r_1, r_2 \notin P$. Then, $\lambda_P(r_1 \cdot r_2) = U$ and $\lambda_P(r_1) = U$, and $\lambda_P(r_2) = \emptyset$. Thus, $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$. □

THEOREM 2.2. *Let P be an ideal of a ring R. Then, characteristic function λ_p is a soft intersectional ideal over U.*

PROOF. Firstly, we will show that $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$ for all $r_1, r_2 \in P$. Since P is an ideal of R, then $r_1 - r_2 \in P$ for all $r_1, r_2 \in P$. It follows that $\lambda_P(r_1) = U$ and $\lambda_P(r_2) = U$, and $\lambda_P(r_1 - r_2) = U$. Thus, $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$. And, $\lambda_P(r_1) = \emptyset$, $\lambda_P(r_2) = \emptyset$ for all $r_1, r_2 \notin P$. Then, $\lambda_P(r_1) \cap \lambda_P(r_2) = \emptyset$ and so, $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$. And, $\lambda_P(r_1) = U$, $\lambda_P(r_2) = \emptyset$ for all $r_1 \in P, r_2 \notin P$. Thus, $\lambda_P(r_1) \cap \lambda_P(r_2) = \emptyset$ and so, $\lambda_P(r_1 - r_2) \supseteq \lambda_P(r_1) \cap \lambda_P(r_2)$.

Now, we will show that $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_1)$ ve $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_2)$ for all $r_1, r_2 \in R$. Since P is an ideal of R, then $r_1 \cdot r_2 \in P$ for all $r_1, r_2 \in P$. Therefore, $\lambda_P(r_1) = U$ and $\lambda_P(r_2) = U$, and $\lambda_P(r_1 \cdot r_2) = U$. Thus, $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_1)$ and $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_2)$. Since P is an ideal of R, then $r_1 \cdot r_2 \in P$ and $r_1 \cdot r_2 \in P$ for all $r_1, r_2 \notin P$. Therefore, $\lambda_P(r_1 \cdot r_2) = U$ and $\lambda_P(r_1) = U$, and $\lambda_P(r_2) = \emptyset$. Thus, $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_1)$ and $\lambda_P(r_1 \cdot r_2) \supseteq \lambda_P(r_2)$. □

3. Soft Intersectional Prime Ideals

In this section, soft intersectional prime ideals will be defined by a ring. Also, some properties of soft intersectional prime ideals will be investigated. Throughout this section, we assume that R is a ring.

DEFINITION 3.1. Let $(\widetilde{\varphi}, R)$ and $(\widetilde{\chi}, R)$ be two soft set over U. $(\widetilde{\varphi}, R)$ and $(\widetilde{\chi}, R)$ are called a common soft set if $\widetilde{\varphi}(r_1) \cap \widetilde{\chi}(r_2) \neq \emptyset$ for all $r_1, r_2 \in R$.

DEFINITION 3.2. $(\widetilde{\varphi}, R)$ be a soft intersectional ideal over U. A soft intersectional ideal $(\widetilde{\varphi}, R)$ is called a soft intersectional prime ideal over U, if

$$(\widetilde{\chi}, R)(\widetilde{\psi}, R)\widetilde{\subseteq}(\widetilde{\varphi}, R) \text{ implies either } (\widetilde{\chi}, R)\widetilde{\subseteq}(\widetilde{\varphi}, R) \text{ or } (\widetilde{\psi}, R)\widetilde{\subseteq}(\widetilde{\varphi}, R)$$

where $(\widetilde{\chi}, R)$ and $(\widetilde{\psi}, R)$ are common soft intersectional ideals over U.

THEOREM 3.1. *If P is a prime ideal of R, then the characteristic function λ_P is a soft intersectional prime ideal over U.*

PROOF. Let $(\widetilde{\chi}, R)$ and $(\widetilde{\psi}, R)$ be common soft intersectional ideals over U such that $(\widetilde{\chi}, R)(\widetilde{\psi}, R)\widetilde{\subseteq}\lambda_P$, $(\widetilde{\chi}, R)\widetilde{\nsubseteq}\lambda_P$ and $(\widetilde{\psi}, R)\widetilde{\nsubseteq}\lambda_P$. Thus, there exist $r_1, r_2 \in R$ such that $\widetilde{\chi}(r_1)\widetilde{\nsubseteq}\lambda_P(r_1)$ and $\widetilde{\psi}(r_2)\widetilde{\nsubseteq}\lambda_p(r_2)$. Therefore, $\widetilde{\chi}(r_1) \neq \emptyset$ and $\widetilde{\psi}(r_2) \neq \emptyset$. Also, $\lambda_P(r_1) \neq U$ and $\lambda_P(r_2) \neq U$. Hence $\lambda_P(r_1) = \emptyset$ and $\lambda_P(r_2) = \emptyset$. Thus, $r_1 \notin P$ and $r_2 \notin P$. Since P is a prime ideal of R, then $r_1r_2 \notin P$ and so, $\lambda_p(r_1r_2) = \emptyset$. Since $(\widetilde{\chi}, R)(\widetilde{\psi}, R)\widetilde{\subseteq}\lambda_p$, then $\widetilde{\chi}\widetilde{\psi}(r_1r_2) = \emptyset$.

Let $r_3 = r_1r_2$. Then $\widetilde{\chi}\widetilde{\psi}(r_3) = \bigcup_{r_3=r_1r_2}(\widetilde{\chi}(r_1) \cap \widetilde{\psi}(r_2)) \supseteq \widetilde{\chi}(r_1) \cap \widetilde{\psi}(r_2)$. Since $(\widetilde{\chi}, R)$ and $(\widetilde{\psi}, R)$ are common soft intersectional ideals over U, then $\widetilde{\chi}(r_1) \cap \widetilde{\psi}(r_2) \neq \emptyset$. Thus, $\widetilde{\chi}\widetilde{\psi}(r_1r_2) \neq \emptyset$. But, this is a contradicts. Therefore, λ_P is a soft intersectional prime ideals over U. □

THEOREM 3.2. *Let P be an ideal of R. If λ_P is a soft intersectional prime ideal over U, then P is a prime ideal of R.*

PROOF. Let λ_P be a soft intersectional prime ideal over U. Let A ve B be two ideals of R such that $AB \subseteq P$. If $\lambda_A\lambda_B = \emptyset$ for $r_1 \in R$, then $(\lambda_A\lambda_B)(r_1) \subseteq \lambda_P(r_1)$. Suppose that $(\lambda_A\lambda_B)(r_1) \neq \emptyset$. Then,

$$(\lambda_A\lambda_B)(r_1) = \bigcup_{r_1=r_2r_3} \{\lambda_A(r_2) \cap \lambda_B(r_3)\}$$

Since $(\lambda_A\lambda_B)(r_1) \neq \emptyset$, then

$$\bigcup_{r_1=r_2r_3} \{\lambda_A(r_2) \cap \lambda_B(r_3)\} \neq \emptyset$$

Thus, there exist $r_2, r_3 \in R$ such that $r_1 = r_2r_3$ and $\lambda_A(r_2) \neq \emptyset$, $\lambda_B(r_3) \neq \emptyset$. Hence, $\lambda_A(r_2) = U$ and $\lambda_B(r_3) = U$. This implies $r_2 \in A$ and $r_3 \in B$. Then, $r_1 = r_2r_3 \in AB \subseteq P$. Therefore, $\lambda_P(r_1) = U$. It follows that $(\lambda_A\lambda_B)(r_1) \subseteq \lambda_P(r_1)$ for all $r_1 \in R$. Then, $\lambda_A\lambda_B\widetilde{\subseteq}\lambda_P$. Since λ_P is a soft intersectional prime ideal over U, then $\lambda_A\widetilde{\subseteq}\lambda_P$ or $\lambda_B\widetilde{\subseteq}\lambda_P$. Thus, $\lambda_A(r_1) \subseteq \lambda_P(r_1)$ or $\lambda_B(r_1) \subseteq \lambda_P(r_1)$ for all $r_1 \in R$. If $r_1 \in A$ then $\lambda_A(r_1) = U$. And, $\lambda_P(r_1) = U$. So, $r_1 \in P$.

Similarly, if $r_1 \in B$ then $r_1 \in P$. That is, $A \subseteq P$ or $B \subseteq P$. Hence, P is a prime ideal of R. □

4. Soft Intersectional Maximal Ideals

In this section, soft intersectional maximal ideals will be defined by a ring. Also, some properties of soft intersectional maximal ideals will be investigated. Throughout this section, we assume that R is a ring.

DEFINITION 4.1. Let $(\widetilde{\varphi}, R)$ be a soft set over $U$and $\emptyset \subseteq P \subseteq U$. Then, the set $(\widetilde{\varphi}, R)^P = \{r \in R : \widetilde{\varphi}(r) \supseteq P\}$ is called a level set of $(\widetilde{\varphi}, R)$.

Let $(\widetilde{\varphi}, R)$ be a soft intersectional left (right) ideal over U. $(\widetilde{\varphi}, R)^P$ is a left (right) ideal over U for any $\emptyset \subseteq P \subseteq \widetilde{\varphi}(O_R)$. $(\widetilde{\varphi}, R)^P$ is called a level left (right) ideal over U with respect to (φ, R). If $P_1, P_2 \in Im(\widetilde{\varphi}, R)$, then $(\widetilde{\varphi}, R)^{P_1} = (\widetilde{\varphi}, R)^{P_2}$ if and only if $P_1 = P_2$.

DEFINITION 4.2. Let $(\widetilde{\varphi}, R)$ be a soft set over U. A set $(\widetilde{\varphi}, R_*)$ is denoted by $(\widetilde{\varphi}, R_*) = \{r \in R : \widetilde{\varphi}(r) = \widetilde{\varphi}(O_R)\}$. If $(\widetilde{\varphi}, R)$ is a soft intersectional left (right) ideal over U, then $(\widetilde{\varphi}, R_*)$ is a left (right) ideal of R.

PROPOSITION 4.1. *Let $(\widetilde{\varphi}, R)$ and $(\widetilde{\chi}, R)$ be two soft intersectional ideals over U. Then, $(\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*) \subseteq ((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_*$.*

PROOF. Let $r \in (\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*)$ for all $r \in R$. Then, $\widetilde{\varphi}(r) = \widetilde{\varphi}(O_R)$ and $\widetilde{\chi}(r) = \widetilde{\chi}(O_R)$. It follows

$$\begin{aligned} (\widetilde{\varphi} \cap \widetilde{\chi})(r) &= \widetilde{\varphi}(r) \cap \widetilde{\chi}(r) \\ &= \widetilde{\varphi}(O_R) \cap \widetilde{\chi}(O_R) \\ &= (\widetilde{\varphi} \cap \widetilde{\chi})(O_R) \end{aligned}$$

Thus, we have $r \in ((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_*$. It is clear that

$$(\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*) \subseteq ((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_*$$

□

In general, the equality in the above proposition need not hold, as shown by the following example.

EXAMPLE 4.1. Let R be a ring. Let $(\widetilde{\varphi}, R)$ and $(\widetilde{\chi}, R)$ be two soft sets over U such that $\widetilde{\varphi}(r) = \emptyset$ for all $r \in R$ and if $r \neq O_R$ then $\widetilde{\chi}(r) = \emptyset$, if $r = O_R$ then $\widetilde{\chi}(r) = U$. Then, $(\widetilde{\varphi}, R)$ and $(\widetilde{\chi}, R)$ be soft intersectional ideals over U. Thus, $(\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*) = R \cap \{O_R\} = \{O_R\}$ and $((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_* = R$.

PROPOSITION 4.2. *Let $(\widetilde{\varphi}, R)$ and $(\widetilde{\chi}, R)$ be two soft intersetional ideals over U such that $\widetilde{\varphi}(O_R) = U = \widetilde{\chi}(O_R)$. Then, $(\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*) = ((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_*$.*

PROOF. Suppose that $r \in ((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_*$. Then, $(\widetilde{\varphi} \cap \widetilde{\chi})(r) = (\widetilde{\varphi} \cap \widetilde{\chi})(O_R)$. Thus, $\widetilde{\varphi}(r) \cap \widetilde{\chi}(r) = \widetilde{\varphi}(O_R) \cap \widetilde{\chi}(O_R) = U$. Therefore, $\widetilde{\varphi}(r) = U = \widetilde{\chi}(r)$. Then, $r \in (\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*)$. Thus, $((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_* \subseteq (\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*)$. Also, $(\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*) \subseteq ((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_*$ by Proposition 4.1. It follows that $(\widetilde{\varphi}, R_*) \cap (\widetilde{\chi}, R_*) = ((\widetilde{\varphi}, R) \cap (\widetilde{\chi}, R))_*$. □

THEOREM 4.1. *Let $(\widetilde{\varphi}, R)$ be a soft intersectional left (right) ideal over U such that $\widetilde{\varphi}(r_1) \neq U$ for some $r_1 \in R$. Then, there exist a soft intersectional left (right) ideal $(\widetilde{\chi}, R)$ over U such that $\widetilde{\chi}(r_2) \neq U$ for some $r_2 \in R$ and $(\widetilde{\varphi}, R) \widetilde{\subseteq} (\widetilde{\chi}, R)$.*

PROOF. **Case 1**: Let $\widetilde{\varphi}(O_R) \neq U$ and $\widetilde{\varphi}(O_R) \subset P \subset U$. Let $(\widetilde{\chi}, R)$ be a soft set over U such that $\widetilde{\chi}(r) = P$ for all $r \in R$ .Then, $(\widetilde{\chi}, R)$ is a soft intersectional left ideal over U such that $\widetilde{\chi}(r) \neq U$ and $(\widetilde{\varphi}, R) \widetilde{\subseteq} (\widetilde{\chi}, R)$ for all $r \in R$.

Case 2: Let $\widetilde{\varphi}(O_R) = U$. By the hypothesis, there exist $r \in R$ such that $\widetilde{\varphi}(r) \neq U$. Let $\widetilde{\varphi}(r) \subset P \subset \widetilde{\varphi}(O_R)$. Thus, $(\widetilde{\varphi}, R)^P$ is a left ideal of R. Let $(\widetilde{\chi}, R)$ be a soft set uver U such that $\widetilde{\chi}(u) = U$ if $u \in (\widetilde{\varphi}, R)^P$ and $\widetilde{\chi}(u) = P$ if $u \notin (\widetilde{\varphi}, R)^P$. Then, $(\widetilde{\chi}, R)$ is a soft intersectional left deal over U. Since $r \notin (\widetilde{\varphi}, R)^P$, then $\widetilde{\chi}(r) = P \neq U$. Also, it can be easily checked that $(\widetilde{\varphi}, R)\widetilde{\subseteq}(\widetilde{\chi}, R)$. □

Definition 4.3. Let $(\widetilde{\varphi}, R)$ be a soft intersectional ideal over U. $(\widetilde{\chi}, R)$ is called a soft intersectional maximal left (right) ideal over U if $\widetilde{\varphi}$ is not constant and for any soft intersectional left (right) ideal $(\widetilde{\varphi}, R)$ over U, if $(\widetilde{\varphi}, R)\widetilde{\subseteq}(\widetilde{\chi}, R)$ then either $(\widetilde{\varphi}, R_*) = (\widetilde{\chi}, R_*)$ or $(\widetilde{\chi}, R) = \lambda_R$.

Theorem 4.2. *Let $(\widetilde{\varphi}, R)$ be a soft intersectional maximal left (right) ideal over U. Then, $\widetilde{\varphi}(O_R) = U$.*

Proof. Suppose that $\widetilde{\varphi}(O_R) \neq U$. Let $\widetilde{\varphi}(O_R) \subset P \subset U$ and $(\widetilde{\chi}, R)$ be a soft set over U such that $\widetilde{\chi}(r) = P$ for all $r \in R$. Then, $(\widetilde{\chi}, R)$ is a soft intersectional ideal over U. It can be easily checked that $(\widetilde{\varphi}, R)\widetilde{\subset}(\widetilde{\chi}, R)$, $(\widetilde{\chi}, R_*) = R$ and $(\widetilde{\chi}, R) \neq \lambda_R$. Thus, $(\widetilde{\varphi}, R)\widetilde{\subset}(\widetilde{\chi}, R)$ but $(\widetilde{\varphi}, R_*) \neq (\widetilde{\chi}, R_*)$ and $(\widetilde{\chi}, R) \neq \lambda_R$. This contradicts the hypothesis that $(\widetilde{\varphi}, R)$ is a soft intersectional maximal left ideal over U. Hence, $\widetilde{\varphi}(O_R) = U$. □

Theorem 4.3. *Let $(\widetilde{\varphi}, R)$ be a soft intersectional maximal left (right) ideal over U. Then, $|Im(\widetilde{\varphi}, R)| = 2$.*

Proof. By Teorem 4.2, it follows that $\widetilde{\varphi}(O_R) = U$. We claim that if

$$P \in Im(\widetilde{\varphi}, R) \text{ then } (\widetilde{\varphi}, R)^P = R \text{ for } \emptyset \subseteq P \subset U.$$

Let $\emptyset \subseteq P \subset U$ and $P \in Im(\widetilde{\varphi}, R)$. Then, $(\widetilde{\varphi}, R)^P$ is a left ideal of R and since $P \subset U$ then $(\widetilde{\varphi}, R_*) \subset (\widetilde{\varphi}, R)^P$. Let $(\widetilde{\chi}, R)$ be a soft set over U such that $\widetilde{\chi}(r) = U$ if $r \in (\widetilde{\varphi}, R)^P$ and $\widetilde{\chi}(r) = P$ if $r \notin (\widetilde{\varphi}, R)^P$. Then, $(\widetilde{\chi}, R)$ is a soft intersectional left ideal over U and so, $(\widetilde{\chi}, R_*) = (\widetilde{\varphi}, R)^P$. Clearly, $(\widetilde{\varphi}, R)\widetilde{\subseteq}(\widetilde{\chi}, R)$. Since $(\widetilde{\varphi}, R)$ is a soft intersectional maximal ideal over U and $(\widetilde{\varphi}, R_*) \subset (\widetilde{\varphi}, R)^P = (\widetilde{\chi}, R_*)$, then we have $(\widetilde{\chi}, R) = \lambda_R$. Thus, $\widetilde{\chi}(r) = U$ for all $r \in R$. Hence, $(\widetilde{\varphi}, R)^P = (\widetilde{\chi}, R_*) = R$. This proves our claim. Now, we have $(\widetilde{\varphi}, R)^{P_1} = R = (\widetilde{\varphi}, R)^{P_2}$ iff $P_1 = P_2$ for any $P_1, P_2 \in Im(\widetilde{\varphi}, R)$, $\emptyset \subseteq P_1, P_2 \subset U$. Thus, $(\widetilde{\varphi}, R)$ is two-valued. □

Theorem 4.4. *Let $(\widetilde{\varphi}, R)$ be a soft intersectional maximal left (right) ideal over U. Then, $(\widetilde{\varphi}, R_*)$ is a maximal left (right) ideal of R.*

Proof. Since $(\widetilde{\varphi}, R)$ is not constant, then $(\widetilde{\varphi}, R_*) \neq R$. By Teorem 4.3, $(\widetilde{\varphi}, R)$ two-valued. Let $Im(\widetilde{\varphi}, R) = \{P, U\}$ where $\emptyset \subseteq P \subset U$. Let M be a left ideal of R such that $(\widetilde{\varphi}, R_*) \subseteq M$. Let $(\widetilde{\chi}, R)$ be a soft set over U such that $\widetilde{\chi}(m) = U$ if $m \in M$ and $\widetilde{\chi}(m) = K$ if $m \notin M$ for $P \subset K \subset U$. Then, $(\widetilde{\chi}, R)$ is a soft intersectional left ideal over U. Clearly, $(\widetilde{\varphi}, R)\widetilde{\subseteq}(\widetilde{\chi}, R)$. Since $(\widetilde{\varphi}, R)$ is a soft intersectional maximal left ideal over U, then $(\widetilde{\varphi}, R_*) = (\widetilde{\chi}, R_*)$ or $(\widetilde{\chi}, R) = \lambda_R$. If $(\widetilde{\varphi}, R_*) = (\widetilde{\chi}, R_*)$ then $(\widetilde{\chi}, R_*) = M$ since $(\widetilde{\varphi}, R_*) = M$. If $(\widetilde{\chi}, R) = \lambda_R$ then $M = R$. Thus, $(\widetilde{\varphi}, R_*)$ is a maximal left ideal of R. □

Theorem 4.5. *Let* $(\widetilde{\varphi}, R)$ *be a soft intersectional left (right) ideal over* U*. If* $(\widetilde{\varphi}, R_*)$ *is a maximal left (right) ideal of* R *then* $(\widetilde{\varphi}, R)$ *is two-valued.*

Proof. Since $(\widetilde{\varphi}, R_*)$ is a maximal left ideal of R, then $(\widetilde{\varphi}, R_*) \neq R$. Thus, there exist $r \in R$ such that $\widetilde{\varphi}(r) \neq \widetilde{\varphi}(O_R)$. Hence, $(\widetilde{\varphi}, R)$ is at least two-valued. Let $\emptyset \subseteq P \subset \widetilde{\varphi}(O_R)$ and $P \in Im(\widetilde{\varphi}, R)$. Then, $(\widetilde{\varphi}, R)^P$ is a left ideal of R such that $(\widetilde{\varphi}, R_*) \subset (\widetilde{\varphi}, R)^P$. Since $(\widetilde{\varphi}, R_*)$ is a maximal left ideal, then $(\widetilde{\varphi}, R)^P = R$. Thus, if $P_1, P_2 \in Im(\widetilde{\varphi}, R)$ and $P_1 \neq \widetilde{\varphi}(O_R)$, $P_2 \neq \widetilde{\varphi}(O_R)$ then $(\widetilde{\varphi}, R)^{P_1} = R = (\widetilde{\varphi}, R)^{P_2}$ iff $P_1 = P_2$. Thus, $(\widetilde{\varphi}, R)$ is two-valued. □

Theorem 4.6. *Let* $(\widetilde{\varphi}, R)$ *be a soft intersectional left (right) ideal over* U*. If* $(\widetilde{\varphi}, R_*)$ *is a maximal left (right) ideal of* R *and* $\widetilde{\varphi}(O_R) = U$*, then* $(\widetilde{\varphi}, R)$ *is a soft intersectional maximal left (right) ideal over* U*.*

Proof. By Theorem 4.5, $(\widetilde{\varphi}, R)$ is two-valued. Let $Im(\widetilde{\varphi}, R) = \{P, U\}$ where $\emptyset \subseteq P \subset U$. Let $(\widetilde{\chi}, R)$ be a soft intersectional left ideal over U such that $(\widetilde{\varphi}, R)\widetilde{\subseteq}(\widetilde{\chi}, R)$. Then, $\widetilde{\chi}(O_R) = U$. Let $r \in (\widetilde{\varphi}, R_*)$. Thus, $U = \widetilde{\varphi}(O_R) = \widetilde{\varphi}(r) \subseteq \widetilde{\chi}(r)$. So, $\widetilde{\chi}(r) = U = \widetilde{\chi}(O_R)$ and hence $r \in (\widetilde{\chi}, R_*)$. Thus, $(\widetilde{\varphi}, R_*) \subseteq (\widetilde{\chi}, R_*)$. Since $(\widetilde{\varphi}, R_*)$ is a maximal left ideal of R, then $(\widetilde{\varphi}, R_*) = (\widetilde{\chi}, R_*)$ or $(\widetilde{\chi}, R_*) = R$. If $(\widetilde{\chi}, R_*) = R$ then $(\widetilde{\chi}, R) = \lambda_R$. Hence, $(\widetilde{\varphi}, R)$ is a soft intersectional maximal left ideal over U. □

5. Conclusion

In this study, we defined prime and maximal ideal based on soft intersectional rings. Then, we defined prime ideal based on soft intersectional rings and investigated their some properties. Also, we defined maximal ideal based on soft intersectional rings and investigated their some properties. To extend our work, further research could be done in other algebraic structures such as semiring.

References

[1] M. Abbas, M. I. Ali and S. Romaguerac. Generalized operations in soft set theory via relaxed conditions on parameters. *Filomat*, **31**(19)(2017), 5955-5964.

[2] U. Acar, F. Koyuncu and B. Tanay. Soft sets and soft rings. *Comput. Math. Appl.*, **59**(11)(2010), 3458–3463.

[3] H. Akta and N. Çağman. Soft sets and soft groups. *Inf. Sci.*, **177**(13)(2007), 2726–2735.

[4] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir. On some new operations in soft set theory. *Comput. Math. Appl.*, **57**(9)(2009), 1547–1553.

[5] M. I. Ali, B. Davvaz, C. Li and F. Feng. Soft sets combined with fuzzy sets and rough sets: a tentative approach. *Soft Comput.*, **14**(9)(2010), 899–911.

[6] N. Çağman, F. Çıtak and H. Aktaş. Soft int-group and its applications to group theory. *Neural Comput. Appl.*, **21**(1)(2012), 151–158.

[7] N. Çağman and S. Enginoğlu. Soft set theory and uni-int decision making. *Eur. J. Oper. Res.*, **207**(2)(2010), 847–855.

[8] N. Çağman and S. Enginoğlu. Soft matrix theory and decision making. *Comput. Math. Appl.*, **59**(10)(2010), 3308–3314.

[9] F. Çıtak and N. Çağman. Soft int-rings and its algebraic applications. *J. Intell. Fuzzy Syst.*, **28**(3)(2015), 1225–1233.

[10] F. Çıtak and N. Çağman. Soft k-int-ideals of semiring and its algebraic structures. *Ann. Fuzzy Math. Inform.*, **13**(4)(2017), 531–538.

[11] F. Çıtak. Soft k-uni ideals of semirings and its algebraic applications. *Journal of the Institute of Science and Technology*, **8**(4)(2018), 281–294.
[12] F. Feng, Y.-B. Jun and X. Zhao. Soft semiring. *Comput. Math. Appl.*, **56**(10)(2008), 2621–2628.
[13] R. Giles. A formal system for fuzzy reasoning. *Fuzzy Sets Syst.*, **2**(3)(1979), 233–257.
[14] C. Jana and M. Pal. Application of new soft intersection set on groups. *Ann. Fuzzy Math. Inform.*, **11**(6)(2016), 923–944.
[15] Y. B. Jun. Soft BCK/BCI-algebras. *Comput. Math. Appl.*, 56(5)(2008), 1408-1413.
[16] Y. B. Jun, K. J. Lee and E. H. Roh. Intersectional soft BCK/BCI-ideals. *Ann. Fuzzy Math. Inform.*, **4**(1)(2012), 1-7.
[17] A. Z. Khameneh and A. Kilicman. Multi-attribute decision-making based on soft set theory: a systematic review. *Soft Comput.*, **23**(16)(2019), 6899–6920.
[18] A. Kharal and B. Ahmad. Mappings on soft classes. *New Math. Nat. Comput.*, **7**(3)(2011), 471-481.
[19] P. K .Maji, R. Biswas and A. R. Roy. Soft set theory. *Comput. Math. Appl.*, **45**(4-5)(2003), 555–562.
[20] D. Molodtsov. Soft set theory-first result. *Comput. Math. Appl.*, **37**(4-5)(1999), 19–31.
[21] Z. Pawlak. Rough sets. *International Journal of Information and Computer Sciences*, **11**(5)(1982), 341–356.
[22] A. Rosenfeld. Fuzzy groups. *J. Math. Anal. Appl.*, **35**(3)(1971), 512–517.
[23] A. Sezgin Sezer. A new to ring theory via soft union rings, ideal and bi-ideals. *Knowledge-Based Systems*, **36**(2012), 300-314.
[24] A. Sezgin Sezer. Certain characterizations of LA semigroups by soft sets. *J. Intell. Fuzzy Syst.*, **27**(2)(2014), 1035-1046.
[25] A. Sezgin. A new view on AG groupoid theory via soft sets for uncertainty modeling. *Filomat*, **32**(8)(2018), 2995-3030.
[26] A. Sezgin. Soft union interior ideals, quasi-ideals and generalized bi-ideals of rings. *Filomat*, **32**(6)(2018), 1991-2018.
[27] A. Sezgin, N. Çağman and F. Çıtak. α-inclusions applied to group theory via soft and logic. *Commun. Fac. Sci. Univ. Ankara, Ser. A1, Math. Stat.*, **68**(1)(2018), 334–352.
[28] Q.-M. Sun, Z.-L. Zhang and J. Liu. Soft sets and soft modules. In: Wang G., Li T., Grzymala-Busse J.W., Miao D., Skowron A., Yao Y. (eds) *Rough Sets and Knowledge Technology.* RSKT 2008. Lecture Notes in Computer Science, vol 5009(2008) (pp. 403–409). Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-79721-0-56.
[29] M. Tuncay and A. Sezgin. Soft union ring and its applications to ring theory. *International Journal of Computer Application*, **151**(9)(2016), 7–13.
[30] Y. Yin, Y. B. Jun and J. Zhang. Vague soft hemirings. *Comput. Math. Appl.*, **62**(1)(2011), 199–213.
[31] L. A. Zadeh. Fuzzy Sets. *Inf. Control*, **8**(3)(1965), 338–353.
[32] J. Zhan, N. Çağman and A. S. Sezer. Applications of soft union sets to hemirings via SU-h-ideals. *J. Intell. Fuzzy Syst.*, **26**(3)(2014), 1363–1370.
[33] J. Zhan, Q. Liu and T. Herawan. A novel soft rough set: soft rough hemirings and its multicriteria group decision making. *Applied Soft Computing*, **54**(2017), 393–402.
[34] J. Zhan and K. Zhu. Z-soft rough fuzzy ideals of hemirings and corresponding decision making. *Soft Comput.*, **21**(8)(2017), 1923–1936.
[35] J. Zhan, M. I. Ali and N. Mehmood. On a novel uncertain of set model: Z-soft fuzzy rough set model and corresponding decision making methods. *Applied Soft Computing*, **56**(2017) 446–457.
[36] L. Zhang, J. Zhan, J. C. R. Alcantud. Novel classes of fuzzy soft β-coverings-based fuzzy rough sets with applications to multi-criteria fuzzy group decision making. *Soft Comput.*, **23**(14)(2019), 5327–5351.
[37] Y. Zou, S. Xia, Z. Xiao and K. Gong. Exclusive disjunctive soft sets. *Comput. Math. Appl.*, **59**(6)(2010), 2128–2137.

Received by editors 16.06.2021; Revised version 27.07.2021; Available online 02.08.2021.

Department of Mathematics, Tokat Gaziosmanpaa University, Tokat, Turkey
E-mail address: pfs.25a@gmail.com

Department of Mathematics, Tokat Gaziosmanpaa University, Tokat, Turkey
E-mail address: filiz.citak@gop.edu.tr