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NORMAL UP-FILTERS OF UP-ALGEBRAS

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Abstract. The concept of normal UP-filters of UP-algebras is introduced and analyzed. Some characterizations of normal UP-filters are derived with the using of some other filter types in such algebras.

1. Introduction

Prabpayak and Leerawat [**6, 7**] introduced the notion of a KU-algebra. In [**1**], Iampan introduced a new algebraic structure, called a UP-algebra, which is a generalization of a KU-algebra. Somjanta et al. [**12**] introduced the concept of UP-filters. Then Jun and Iampan ([**2, 3**]) developed several types of filters in these algebras such as comparative and implicative UP-filters.

In this paper, we introduce the notion of normal UP-filters and relate it to other types of UP-filters in UP-algebras. The condition by which the normal UP-filter is determined is taken from the literature on BL-algebras. Additionally, the concept of normal UP-filters has been shown as a well link between implicative and comparative UP-filters in UP-algebras. In addition to the above, it has been shown that in meet-commutative UP-algebras any UP-filter is a normal UP-filter.

2. Preliminaries

In this section, taking them from the literature, we will list some terms and statements used in this article.

An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra (see [**1**]) if it satisfies the following axioms:

(UP-1) $(\forall x, y, \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$;

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(UP-2) $(\forall x \in A)(0 \cdot x = x)$;
(UP-3) $(\forall x \in A)(x \cdot 0 = 0)$; and
(UP-4) $(\forall x, y \in A)((x \cdot y = 0 \wedge y \cdot x = 0) \implies x = y)$.

The UP-algebra A is said to be a meet-commutative UP-algebra ([**10**]) if the following

$$(\forall x, y \in A)((x \cdot y) \cdot y = (y \cdot x) \cdot x)$$

is valid. Some important properties of mee-commutative UP-algebras and the UP-filters in them can be found in [**5, 8, 9**].

In this algebra, the order relation '$\leqslant$' is defined as follows

$$(\forall x, y \in A)(x \leqslant y \iff x \cdot y = 0).$$

A subset F of A is called a UP-filter of A (see [**12**]) if it satisfies the following conditions:
(F-1) $0 \in F$; and
(F-2) $(\forall x, y \in A)((x \in F \wedge x \cdot y \in F) \implies y \in F)$.

Obviously, the following applies
(1) $(\forall x, y \in A)((x \in F \wedge x \leqslant y) \implies y \in F)$.

The family $\mathfrak{F}(A)$ of all UP-filters of a UP-algebra A is not empty and forms a complete lattice.

Notions, notations and statements used in this paper that have not been previously determined can be found in [**1, 2, 3**].

3. Concept of normal UP-filters

In this section we introduce the concept of 'normal UP-filter of a UP-algebra' looking at the notion of a normal filter in BL-algebras.

3.1. Normal filter in BL-algebras. The term 'normal filter in BE/BL-algebras' was introduced independently of each other by A. B. Saeid and S. Motamed 2009 in article [**11**] and A. Walendziak 2012 in article [**13**].

Definition 3.1. ([**11**], Definition 3.1) A filter F of a BE-algebra A is said to be normal if it satisfies the following condition:
(NF_W) $(\forall x, y, z \in A)(x \cdot y \in F \implies ((z \cdot x) \cdot (z \cdot y) \in F \wedge (y \cdot z) \cdot (x \cdot z) \in F))$.

This definition due to (UP-1) leads us to the fact that each UP-filter is a normal UP-filter in the UP-algebra that satisfies the additional condition
(KU) $(\forall x, y, z \in A)(x \cdot (y \cdot z) = y \cdot (x \cdot z))$.

Saeid and Motamed gave a different definition of the term 'normal filter in BL-algebra' than the previous one.

Definition 3.2. ([**11**], Definition 9) Let F be a filter of of a BL-algebra A. F is called a *normal* filter of A if it satisfies:
(NF_S) $(\forall x, y, z \in A)((z \in F \wedge z \cdot ((y \cdot x) \cdot x) \in F) \implies (x \cdot y) \cdot y \in F)$.

Taking into account the way of determining previously introduced UP-filters of UP-algebras such as implicative and comparative UP-filters ([**2, 3**]), we decided to use this second determination for introducing the concept of normal UP-filters of a UP-algebra.

3.2. Concept of normal UP-filter of UP-algebras.

DEFINITION 3.3. Let F be a UP-filter of of a UP-algebra A. F is called a *normal UP-filter* of A if it satisfies:
(NF) $(\forall x, y, z \in A)((z \in F \wedge z \cdot ((y \cdot x) \cdot x) \in F) \Longrightarrow (x \cdot y) \cdot y \in F)$.

In the following example, we show one example of a normal UP-filter and show that, generally speaking, a UP-filter does not have to be a normal UP-filter.

EXAMPLE 3.1. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

·	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	0	0	0
c	0	a	c	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. The subsets $F := \{0\}$ and $G := \{0, a\}$ are UP-filters of A. The filter F is not a normal UP-filter of A because, for example, for $z = 0$, $y = a$ and $x = b$, we have $z \cdot ((y \cdot x) \cdot x) = 0 \cdot (a \cdot b) \cdot b) = (a \cdot b) \cdot b = 0 \in F$ but $(x \cdot y) \cdot y = (b \cdot a) \cdot a = 0 \cdot a = a \notin F$. By direct verification it can prove that G is a normal UP-filter of A.

It is obvious that the following statement is true.

PROPOSITION 3.1. *If A is a meet-commutative UP-algebra, then any UP-filter of A is a normal UP-filter of A.*

PROOF. Let F be a UP-filter of a meet-commutative UP-algebra A. Suppose that $x, y, z \in A$ are elements of A such that $z \in F$ and $z \cdot ((y \cdot x) \cdot x) \in F$. Then $(y \cdot x) \cdot x \in F$ by (F-2). Since $(y \cdot x) \cdot x = (x \cdot y) \cdot y$ because A is a meet-commutative UP-algebra, we conclude that $(x \cdot y) \cdot y \in F$ is valid. So F is a normal UP-filter. □

EXAMPLE 3.2. Let A be as in the Example 3.1. The UP-algebra A is not a meet-commutative UP-algebra, because for example, $(a \cdot b) \cdot b = b \cdot b = 0$ but $(b \cdot a) \cdot a = 0 \cdot a = a$ holds.

EXAMPLE 3.3. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

·	0	a	b	c
0	0	a	b	c
a	0	0	0	0
b	0	c	0	c
c	0	b	b	0

Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subsets $\{0\}$, $\{0, b\}$ and $\{0, c\}$ are normal UP-filters of A.

The following theorem gives one criterion that a UP-filter of a UP-algebra is a normal UP-filter.

THEOREM 3.1. *Let F be a UP-filter of a UP-algebra A. Then F is a normal UP-filter if only if the following holds*
(2) $(\forall x, y \in A)((y \cdot x) \cdot x \in F \implies (x \cdot y) \cdot y \in F)$.

PROOF. Let F be a normal filter of a UP-algebra A and let $x, y, z \in A$ be elements such that $(y \cdot x) \cdot x \in F$. Since $(y \cdot x) \cdot x = 0 \cdot (y \cdot x) \cdot x \in F$ and $0 \in F$, thus $(x \cdot y) \cdot y \in F$ because F is a normal UP-filter of A.

Suppose that the UP-filter F of a UP-algebra A satisfies the condition (2). Let $z \cdot ((y \cdot x) \cdot x) \in F$ and $z \in F$ be holds for all $x, y, z \in A$. Since F is a filter, then $(y \cdot x) \cdot x \in F$ by (F-2). Thus hypothesis we get that $(x \cdot y) \cdot y \in F$ by the hypothesis (2). Hence F is a normal UP-filter of A. □

In [**3**], the concept of comparative UP-filters was introduced by the following definition:

DEFINITION 3.4. ([**3**], Definition 2) A subset F of a UP-algebra A is called a comparative UP-filter of A if it satisfies the conditions (F-1) and
(CF) $(\forall x, y, z \in A)((x \in F \wedge x \cdot ((y \cdot z) \cdot y) \in F) \implies y \in F)$.

THEOREM 3.2. *Let F be a comparative UP-filter of a UP-algebra A which additionally satisfies the condition (KU). Then F is a normal UP-filter of A.*

PROOF. In [**2**] it is shown (Theorem 4) that F is a comparative UP-filter of A if and only if it satisfies the condition (2). Therefore, F is a normal UP-filter of A. □

EXAMPLE 3.4. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	a	b
b	0	0	0	b
c	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. Subset $F := \{0, a, b\}$ is a comparative UP-filter of A. Therefore, F is a normal UP-filter of A according to the previous theorem.

A normal UP-filter of a UP-algebra does not have to be a comparative UP-filter.

EXAMPLE 3.5. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	0	b
b	0	b	0	a
c	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. Subset $F := \{0\}$ is a normal UP-filter of A while it is not a comparative UP-filter, since for $x = 0$, $y = b$ and $z = c$ we have $0 \in F$ and $0 \cdot ((b \cdot c) \cdot b) = a \cdot b = 0 \in F$ but $b \notin F$.

In [**2**] the concept of the implicative UP-filter of a UP-algebra by the following definition was introduced:

Definition 3.5. A subset F of a UP-algebra A is called an implicative UP-filter of A if it satisfies the conditions (F-1) and
(IF) $(\forall x, y, z \in A)((x \cdot (y \cdot z) \in F \wedge x \cdot y \in F) \implies x \cdot z \in F)$.

Any normal UP-filter need not be an implicative UP-filter.

Example 3.6. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	0	a
b	0	a	0	c
c	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. Subset $F := \{0, b\}$ is a normal UP-filter of A while it is not a implicative UP-filter, because for $x = a$, $y = a$ and $z = c$ we have $a \cdot (a \cdot c) = a \cdot a = 0 \in F$ and $a \cdot a = 0 \in F$ but $a \cdot c = a \notin F$.

Theorem 3.3. *Let F be an implicative UP-filter of a UP-algebra A which additionally satisfies the condition (KU). Then F is a comparative UP-filter of A if and only if it is a normal UP-filter.*

Proof. In [**3**] it is shown (Theorem 4) that an implicative UP-filter is a comparative UP-filter of A if and only if it satisfies the condition (2). □

Theorem 3.4. *Let F be a UP-filer of a UP-algebra A which additionally satisfies the condition (KU). If the following holds*
(3) $(\forall x, y \in A)((x \cdot y) \cdot x) \in F \implies x \in F)$,
then F is a normal UP-filter of A.

Proof. In [**3**] it is shown (Theorem 2) that a UP-filter of UP-algebra A is a comparative UP-filter of A if and only if it satisfies the condition (3). Thus, the UP-filter that satisfies condition (3) is a normal UP filter according to Theorem 3.2. □

4. Final Comments

UP-algebras are a generalization of KU-algebras. This class of logical algebras has been researched by several authors. Many authors have focused on various types of filters in such algebras. Within this class of logical algebras, a special subclass consists of meet-commutative UP-algebras, introduced in the article [**10**]. An interested reader can find more about this subclass of UP-algebras in the articles [**5, 8, 9**]. In connection with the previous one, one can look at the published review article [**4**], although chronologically it comes after this paper since this text is incorporated into it. It seems to the author that the publication of this article completes the presentation of various types of UP filters, as has already been done in the article [**4**].

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