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ON THE ZAGREB INDICES OF THE LINE CUT-VERTEX GRAPHS OF THE SUBDIVISION GRAPHS

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Abstract. The first Zagreb index $M_1(G)$ is equal to the sum of squares of the degrees of the vertices and the second Zagreb index $M_2(G)$ is equal to the sum of the products of the degrees of pairs of adjacent vertices of the underlying molecular graph G. In this paper, we obtain the Zagreb indices and coindices of the line cut-vertex graph of the tadpole graph, ladder graph, wheel graph, star graph, and path graph using the subdivision concept.

1. Introduction

Let $G = (V, E)$ be a simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$, where $|V(G)| = n$ and $|E(G)| = m$. These two basic parameters n and m are called the *order* and *size* of G, respectively. The edge connecting the vertices u and v will be denoted by uv. The *degree* of a vertex v, written $d_G(v)$, is the number of edges of G incident with v, each loop counting as two edges.

Among the oldest and most studied topological indices, there are two classical vertex-degree based topological indices - the first Zagreb index and second Zagreb index. These two indices first appeared in [**4**], and were elaborated in [**5**]. The main properties of $M_1(G)$ and $M_2(G)$ were summarized in [**7, 13**]. The first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ of a graph G are defined, respectively, as

$$M_1 = M_1(G) = \sum_{v \in V(G)} d_G(v)^2 \tag{1.1}$$

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$$M_2 = M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v) \tag{1.2}$$

In fact, one can rewrite the first Zagreb index as

$$M_1 = M_1(G) = \sum_{uv\in E(G)} [d_G(u) + d_G(v)] \tag{1.3}$$

During the past decades, numerous results concerning Zagreb indices have been put forward, see [**2, 3, 6, 8, 9**]; for historical details see [**10**].

In 2008, bearing in mind expression (1.3), Došlić put forward the first Zagreb coindex, defined as [**1**]

$$\overline{M_1} = \overline{M_1}(G) = \sum_{uv\notin E(G)} [d_G(u) + d_G(v)] \tag{1.4}$$

In view of expression (1.4), the second Zagreb coindex is defined analogously as [**1**]

$$\overline{M_2} = \overline{M_2}(G) = \sum_{uv\notin E(G)} d_G(u)d_G(v) \tag{1.5}$$

In expressions (1.4) and (1.5), it is assumed that $u \neq v$.

The *subdivision graph* of a graph G, written $S(G)$, is the graph obtained from G by replacing each of its edges by a path of length 2, or equivalently by inserting an additional vertex into each edge of G. The Tadpole graph $T_{n,k}$ is the graph obtained by joining a cycle graph C_n to a path graph of length k. The n-ladder graph can be defined as $L_n = P_2 \times P_n$, where P_n is a path graph of order n. For example, $L_1 = P_2$ and $L_2 = C_4$. A wheel graph of order n, written W_n, is a graph that contains a cycle of order $(n-1)$, and for which every graph vertex in the cycle is connected to one other graph vertex.

There are many graph operators (or graph valued functions) with which one can construct a new graph from a given graph, such as the line graphs, the total graphs, and their generalizations. One such a graph operator is called the *line cut-vertex graph* of a graph G.

The *line graph* of a graph G, written $L(G)$, is the graph whose vertices are the edges of G, with two vertices of $L(G)$ adjacent whenever the corresponding edges of G have a vertex in common. In [**14**], the Zagreb indices and coindices of the line graphs of the subdivision graphs were studied.

The authors in [**12**] gave the following definition. The *line cut-vertex graph* of G, written $L_c(G)$, is the graph whose vertices are the edges and cut-vertices of G, with two vertices of $L_c(G)$ adjacent whenever the corresponding edges of G have a vertex in common; or one corresponds to an edge e_i of G and the other corresponds to a cut-vertex c_j of G such that e_i is incident with c_j.

See Figure.1 for an example of a graph G and its line cut-vertex graph $L_c(G)$.

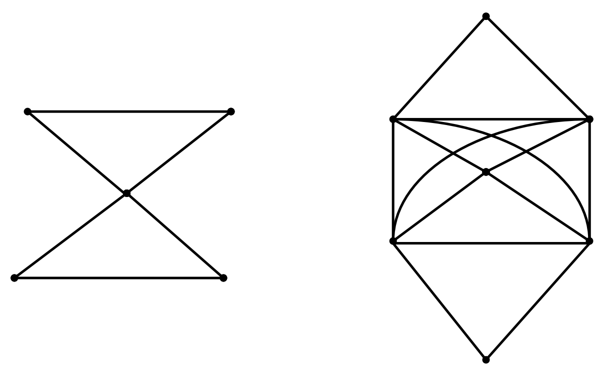

Figure.1

In this paper we study the line cut-vertex graph of the subdivision graph of the tadpole graph $T_{n,k}$, wheel graph W_n, and ladder graph L_n, star graph $K_{1,n}$, and path graph P_n; and calculate the Zagreb indices and coindices of the graphs $L_c(S(T_{n,k}))$, $L_c(S(W_n))$, $L_c(S(L_n))$, $L_c(S(K_{1,n}))$, and $L_c(S(P_n))$. Notations and definitions not introduced here can be found in [**11**].

2. Zagreb indices of the line cut-vertex graph of the subdivision graph of the tadpole graph $T_{n,k}$

In this section we calculate the Zagreb indices and coindices of the line cut-vertex graph of the subdivision graph of the tadpole graph.

THEOREM 2.1. *Let G be the line cut-vertex graph of the subdivision graph of the tadpole graph. Then $M_1(G) = 8n + 40k + 26$ and*

$$M_2(G) = \begin{cases} 8n + 119 & \text{if } k = 1 \\ 8n + 64k + 61 & \text{if } k \geqslant 2 \end{cases}$$

PROOF. The subdivision graph $S(T_{n,k})$ contains $2n + 2k$ edges, so that the line cut-vertex graph of $S(T_{n,k})$ contains $2n + 2k + 2k = 2(n + 2k)$ vertices, out of which $2k$ vertices are of degree four; $2(n + k - 1)$ vertices are of degree two; and the remaining 2 vertices are of degree three and five, respectively. Therefore, $M_1(G) = 2k(16) + 8(n + k - 1) + 34 = 8n + 40k + 26$.

The size of $L_c(S(T_{n,k})) = 2n + 6k + 2$. In other words, there exists exactly one copy of K_4 (i.e., a complete graph of order four); $2k - 1$ copies of K_3 (i.e., a complete graph of order three); 2 edges whose neighbors have degree two and three, respectively; and $2(n - 2) + 1$ number of edges whose neighbors have degree two. Therefore, $M_2(G) = 8n + 64k + 111 - 50 = 8n + 64k + 61, k \geqslant 2$. For $k = 1$, $M_2(G) = 8n + 16 + 95 + 20 - 12 = 8n + 119$. $\square$

Gutman et al. in [**8**] established a complete set of relations between first and second Zagreb index and coindex of a graph as follows:

Theorem 2.2. *Let G be a graph with n vertices and m edges. Then*

$$\overline{M_1}(G) = 2m(n-1) - M_1(G).$$

Theorem 2.3. *Let G be a graph with n vertices and m edges. Then*

$$\overline{M_2}(G) = 2m^2 - \tfrac{1}{2}M_1(G) - M_2(G).$$

We now give the expressions for the first and second Zagreb coindices of the line cut-vertex graph of the tadpole graph using Theorem 2.2 and Theorem 2.3.

Theorem 2.4. *Let G be the line cut-vertex graph of the subdivision graph of the tadpole graph. Then $\overline{M_1}(G) = 8n^2 + 48k^2 + 40nk - 4n - 36k - 30$.*

Proof. The order and size of G are $2n+4k$ and $2n+6k+2$, respectively. By Theorem 2.1, $M_1(G) = 8n + 40k + 26$. Then Theorem 2.2 implies that

$$\begin{aligned}
\overline{M_1}(G) &= 2[2n+6k+2][2n+4k-1] - [8n+40k+26] \\
&= 2[4n^2 + 8nk - 2n + 12nk + 24k^2 - 6k + 4n + 8k - 2] - [8n+40k+26] \\
&= 2[4n^2 + 24k^2 + 20nk + 2n + 2k - 2] - [8n+40k+26] \\
&= 8n^2 + 48k^2 + 40nk + 4n + 4k - 4 - 8n - 40k - 26 \\
&= 8n^2 + 48k^2 + 40nk - 4n - 36k - 30.
\end{aligned}$$

□

Theorem 2.5. *Let G be the line cut-vertex graph of the subdivision graph of the tadpole graph. Then*

$$\overline{M_2}(G) = \begin{cases} 8n^2 + 72k^2 + 48nk + 28k + 4n - 124 & \text{if } k = 1 \\ 8n^2 + 72k^2 + 48nk - 36k + 4n - 66 & \text{if } k \geqslant 2 \end{cases}$$

Proof. For $k = 1$, by Theorem 2.1 and Theorem 2.3,

$$\begin{aligned}
\overline{M_2}(G) &= 2[2n+6k+2]^2 - [4n+20k+13] - [8n+119] \\
&= 2[4n^2 + 36k^2 + 4 + 24nk + 24k + 8n] - [4n+20k+13] - [8n+119] \\
&= 8n^2 + 72k^2 + 48nk + 28k + 4n - 124.
\end{aligned}$$

For $k \geqslant 2$, again by Theorem 2.1 and Theorem 2.3,

$$\begin{aligned}
\overline{M_2}(G) &= 2[2n+6k+2]^2 - [4n+20k+13] - [8n+64k+61] \\
&= 2[4n^2 + 36k^2 + 4 + 24nk + 24k + 8n] - [4n+20k+13] \\
&\quad - [8n+64k+61] \\
&= 8n^2 + 72k^2 + 48nk - 36k + 4n - 66.
\end{aligned}$$

□

Note that for wheel graphs and ladder graphs (except L_1), $L_c(S(W_n)) \simeq L(S(W_n))$ and $L_c(S(L_n)) \simeq L(S(L_n))$, where $L(S(W_n))$ and $L(S(L_n))$ is the line graph of $S(W_n)$ and $S(L_n)$, respectively.

3. Zagreb indices of the line cut-vertex graph of the subdivision graph of the wheel graph W_n

In this section we calculate the Zagreb indices and coindices of the line cut-vertex graph of the subdivision graph of the wheel graph.

The *corona* $G_1 \odot G_2$ of two graphs G_1 (with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

THEOREM 3.1. *Let G be the line cut-vertex graph of the subdivision graph of the wheel graph. Then $M_1(G) = 36n - 36$ and $M_2(G) = n^3 + 33n - 34$.*

PROOF. The wheel graph W_n has n vertices and $2n - 2$ edges, so that $S(W_n)$ has $n + 2(n - 1) = 3n - 2$ vertices and $4n - 4$ edges. Hence $L_c(S(W_n))$ contains $4n - 4$ vertices and each vertex is of degree three. Hence $M_1(G) = 36n - 36$. The size of $L_c(S(W_n))$ is

$$|E(L_c(S(W_n))| = 2(n-1) + 3(n-1) + K_{n-1} = 2n - 2 + 3n - 3 + \frac{(n-1)(n-2)}{2}$$
$$= (n-1)\left[5 + \frac{n-2}{2}\right]$$

In other words, there exists exactly one copy of $K_{n-1} \odot K_1$; $(n - 1)$ copies of K_3; and $(n - 1)$ number of edges whose end vertices have degree two. Therefore,

$$\begin{aligned} M_2(G) &= 3[n-1]^2 + [n-1]^3 + 27n - 27 + 9n - 9 \\ &= 3[n^2 + 1 - 2n] + [n-1]\left[n^2 + 1 - 2n\right] + 27n - 27 + 9n - 9 \\ &= n^3 + 33n - 34. \end{aligned}$$

□

THEOREM 3.2. *Let G be the line cut-vertex graph of the subdivision graph of the wheel graph. Then $\overline{M_1}(G) = 4n^3 + 23n^2 - 103n + 76$.*

PROOF. The order and size of G are $4n - 4$ and $(n-1)\left(5 + \frac{n-2}{2}\right)$, respectively. By Theorem 3.1, $M_1(G) = 36n - 36$. Then Theorem 2.2 implies that

$$\begin{aligned} \overline{M_1}(G) &= 2[n-1][4n-4]\left[5 + \frac{n-2}{2}\right] - 2[n-1]\left[5 + \frac{n-2}{2}\right] - 36[n-1] \\ &= 4[n-1][n+8][n-1] - [n-1][n+8] - 36[n-1] \\ &= 4[n-1]^2[n+8] - [n-1][n+8] - 36n + 36 \\ &= [n-1][n+8][4n-5] - 36n + 36 \\ &= [n^2 + 7n - 8][4n-5] - 36n + 36 \\ &= 4n^3 + 23n^2 - 103n + 76. \end{aligned}$$

□

THEOREM 3.3. *Let G be the line cut-vertex graph of the subdivision graph of the wheel graph. Then* $\overline{M_2}(G) = \frac{1}{2}[n^4 + 12n^3 + 33n^2 - 214n + 168]$.

PROOF. By Theorem 3.1, $M_1(G) = 36n - 36$ and $M_2(G) = n^3 + 33n - 34$. Then Theorem 2.3 implies that

$$\begin{aligned}
\overline{M_2}(G) &= 2\left[\frac{(n-1)(n+8)}{2}\right]^2 - 18n + 18 - n^3 - 33n + 34 \\
&= \frac{1}{2}\left[n^2 + 7n - 8)\right]^2 - 18n + 18 - n^3 - 33n + 34 \\
&= \frac{1}{2}[n^4 + 14n^3 + 33n^2 - 112n + 64] - n^3 - 51n + 52 \\
&= \frac{1}{2}[n^4 + 14n^3 + 33n^2 - 112n + 64 - 2n^3 - 102n + 104] \\
&= \frac{1}{2}[n^4 + 12n^3 + 33n^2 - 214n + 168].
\end{aligned}$$

□

4. Zagreb indices of the line cut-vertex graph of the subdivision graph of the ladder graph L_n

In this section we calculate the Zagreb indices and coindices of the line cut-vertex graph of the subdivision graph of the ladder graph.

The graph obtained by joining two disjoint cycles $u_1u_2\dots u_nu_1$ and $v_1v_2\dots v_nv_1$ with an edge u_1v_1 is called the *dumbbell graph* Db_n.

THEOREM 4.1. *Let G be the line cut-vertex graph of the subdivision graph of the ladder graph. Then* $M_1(G) = 54n - 76$ *and* $M_2(G) = 81n - 132$, $n \geqslant 3$.

PROOF. Since $L_1 = P_2$, $M_1(G) = M_2(G) = 12n = 12$. Similarly, $L_2 = C_4$. Then $M_1(G) = M_2(G) = 4(4n) = 32$. For $n \geqslant 3$, L_n has $2n$ vertices and $3n-2$ edges, so that $S(L_n)$ contains $2(3n-2) = 6n-4$ edges. Therefore, $L_c(S(L_n))$ contains $6n-4$ vertices, out of which 8-vertices are of degree two and the remaining $6n-12$ vertices are of degree three. Thus $M_1(G) = 54n - 76$, $n \geqslant 3$.

Now, $|E(L_c(S(L_n))| = |E(S(L_n))| + |E(L_n)| - 4 = 6n - 4 + 3n - 2 - 6 = 9n - 10$, out of which 6 edges whose end vertices have degree two; $n-2$ copies of dumbbell graph Db_3, where each dumbbell graph has 7 edges and the degree of the end vertices of each of these edges is three; and 4 edges whose end vertices have degree two and three, respectively. Also, for $n \geqslant 3$, there are $2(\alpha - 1)$ number of edges whose end vertices have degree three, where $\alpha = (n-2), n \geqslant 3$. Therefore, $M_2(G) = 63(n-2) + 18(n-3) + 48 = 81n - 132$, $n \geqslant 3$. □

THEOREM 4.2. *Let G be the line cut-vertex graph of the subdivision graph of the ladder graph. Then*

$$\overline{M_1}(G) = \begin{cases} 0 & \text{if } n = 1 \\ 80 & \text{if } n = 2 \\ 108n^2 - 264n + 176 & \text{if } n \geqslant 3 \end{cases}$$

PROOF. Since the order and size of $L_c(S(L_1))$ is three; and $M_1(L_c(S(L_1)) = 12$, Theorem 2.1 implies that, $\overline{M_1}(G) = 2(3)(2) - 12 = 0$. Now, for L_2, the size of $L_c(S(L_2))$ is eight; and $M_1(L_c(S(L_1)) = M_2(L_c(S(L_1)) = 32$. By Theorem 2.2, $\overline{M_1}(G) = 2(64) - 16 - 32 = 80$. For $n \geqslant 3$, the order and size of G are $6n - 4$ and $9n - 10$, respectively. By Theorem 4.1, $M_1(G) = 54n - 76$. Then Theorem 2.2 implies that

$$\begin{aligned}\overline{M_1}(G) &= 2[9n - 10][6n - 5] - [54n - 76]\\ &= [18n - 20][6n - 5] - [54n - 76]\\ &= 108n^2 - 90n - 120n + 100 - 54n + 76\\ &= 108n^2 - 264n + 176, n \geqslant 3.\end{aligned}$$

□

THEOREM 4.3. *Let G be the line cut-vertex graph of the subdivision graph of the ladder graph. Then*

$$\overline{M_2}(G) = \begin{cases} 0 & \text{if } n = 1\\ 80 & \text{if } n = 2\\ 162n^2 - 468n + 370 & \text{if } n \geqslant 3 \end{cases}$$

PROOF. Since the order and size of $L_c(S(L_1))$ is three; and $M_1(L_c(S(L_1)) = M_2(L_c(S(L_1)) = 12$, Theorem 2.2 implies that, $\overline{M_2}(G) = 2(9) - 6 - 12 = 0$. Similarly, for L_2, the size of $L_c(S(L_2))$ is eight; and $M_1(L_c(S(L_1)) = M_2(L_c(S(L_1)) = 32$, Theorem 2.2 implies that, $\overline{M_1}(G) = 2(64) - 16 - 32 = 80$. For $n \geqslant 3$, by Theorem 4.1, $M_1(G) = 54n - 76$ and $M_2(G) = 81n - 132$. Then Theorem 2.3 implies that

$$\begin{aligned}\overline{M_2}(G) &= 2[9n - 10]^2 - [27n - 38] - [81n - 132]\\ &= 2[81n^2 - 180n + 100] - 27n + 38 - 81n + 132\\ &= 162n^2 - 468n + 370, n \geqslant 3.\end{aligned}$$

□

5. Zagreb indices of the line cut-vertex graph of the subdivision graph of the star graph $K_{1,n}$ $(n \geqslant 3)$

In this section we calculate the Zagreb indices and coindices of the line cut-vertex graph of the subdivision graph of the star graph.

THEOREM 5.1. *Let G be the line cut-vertex graph of the subdivision graph of the star graph. Then $M_1(G) = n^3 + 5n^2 + 12n$ and $M_2(G) = \frac{1}{2}(n^4 + 5n^3 + 12n^2 + 20n)$.*

PROOF. The subdivision graph $S(K_{1,n})$ contains $2n + 1$ vertices and $2n$ edges, so that the line cut-vertex graph of $S(K_{1,n})$ contains $2n+n+1 = 3n+1$ vertices, out of which $2n$ vertices are of degree two; one vertex is of degree n; and the remaining

n vertices are of degree $(n+2)$. Therefore, $M_1(G) = 4n + 4n + n^2 + n(n+2)^2 = n^3 + 5n^2 + 12n$. The size of $L_c(S(K_{1,n}))$ is

$$\begin{aligned}|E(L_c(S(K_{1,n})| &= 3n + \frac{n(n+1)}{2} \\ &= \frac{n^2+7n}{2}.\end{aligned}$$

In other words, there are n number of edges whose end vertices have degree two; $2n$ number of edges whose end vertices have degree two and $n+2$; $\frac{n(n-1)}{2}$ edges whose end vertices have degree $n+2$; and the remaining n edges whose end vertices have degree n and $n+2$. Therefore,

$$\begin{aligned}M_2(G) &= 4n + 4n^2 + 8n + \frac{n(n-1)(n-2)^2}{2} + n^3 + 2n^2 \\ &= n^3 + 6n^2 + 12n + \frac{n^4+3n^3-4n}{2} \\ &= \frac{1}{2}(n^4 + 5n^3 + 12n^2 + 20n)\end{aligned}$$

□

THEOREM 5.2. *Let G be the line cut-vertex graph of the subdivision graph of the star graph. Then $\overline{M_1}(G) = 2n^3 + 16n^2 - 12n$.*

PROOF. The order and size of G are $3n+1$ and $\frac{n^2+7n}{2}$, respectively. By Theorem 5.1, $M_1(G) = n^3 + 5n^2 + 12n$. Then Theorem 2.2 implies that

$$\begin{aligned}\overline{M_1}(G) &= 2\left[\frac{n^2+7n}{2}\right][3n] - [n^3 + 5n^2 + 12n] \\ &= 2n^3 + 16n^2 - 12n.\end{aligned}$$

□

THEOREM 5.3. *Let G be the line cut-vertex graph of the subdivision graph of the star graph. Then $\overline{M_2}(G) = 4n^3 + 16n^2 - 16n$.*

PROOF. The order and size of G are $3n+1$ and $\frac{n^2+7n}{2}$, respectively. By Theorem 5.1, $M_2(G) = \frac{1}{2}(n^4 + 5n^3 + 12n^2 + 20n)$. Then Theorem 2.3 implies that

$$\begin{aligned}\overline{M_2}(G) &= 2\left[\frac{n^2+7n}{2}\right]^2 - \frac{1}{2}[n^3 + 5n^2 + 12n] - \frac{1}{2}[n^4 + 5n^3 + 12n^2 + 20n] \\ &= \frac{1}{2}[n^4 + 49n^2 + 14n^3 - n^3 - 5n^2 - 12n - n^4 - 5n^3 - 12n^2 - 20n] \\ &= \frac{1}{2}[8n^3 + 32n^2 - 32n] \\ &= 4n^3 + 16n^2 - 16n.\end{aligned}$$

□

6. Zagreb indices of the line cut-vertex graph of the subdivision graph of the path graph P_n $(n \geqslant 2)$

In this section we calculate the Zagreb indices and coindices of the line cut-vertex graph of the subdivision graph of the path graph.

THEOREM 6.1. *Let G be the line cut-vertex graph of the subdivision graph of the path graph. Then $M_1(G) = 40n - 68$ and $M_2(G) = 64n - 120$.*

PROOF. The subdivision graph $S(P_n)$ contains $2n-1$ vertices and $2n-2$ edges, so that the line cut-vertex graph of $S(P_n)$ contains $2n-2+2n-3 = 4n-5$ vertices, out of which $2n-1$ vertices are of degree two and the remaining $2n-4$ vertices are of degree four. Therefore, $M_1(G) = 4(2n-1) + 16(2n-4) = 40n - 68$.

For P_2, $M_2(G) = 12$. Now, the size of $L_c(S(P_n)) = 6n - 9$ $(n \geqslant 3)$. That is, there are 2 edges whose end vertices have degree 2; $2n-5$ number of edges whose end vertices have degree 4; and $4n-6$ number of edges whose end vertices have degree 2 and 4, respectively. Therefore, $M_2(G) = 8+(2n-5)16+(4n-6)8 = 64n-120$. □

THEOREM 6.2. *Let G be the line cut-vertex graph of the subdivision graph of the path graph. Then $\overline{M_1}(G) = 48n^2 - 184n + 176$.*

PROOF. The order and size of G are $4n-5$ and $6n-9$, respectively. By Theorem 6.1, $M_1(G) = 40n - 68$. Then Theorem 2.2 implies that

$$\begin{aligned}\overline{M_1}(G) &= 2[6n-9][4n-6] - [40n-68]\\ &= 48n^2 - 184n + 176.\end{aligned}$$

□

THEOREM 6.3. *Let G be the line cut-vertex graph of the subdivision graph of the path graph. Then $\overline{M_2}(G) = 72n^2 - 84n + 100$.*

PROOF. The order and size of G are $4n-5$ and $6n-9$, respectively. By Theorem 6.1, $M_1(G) = 40n - 68$ and $M_2(G) = 64n - 120$. Then Theorem 2.3 implies that

$$\begin{aligned}\overline{M_2}(G) &= 2[6n-9]^2 - \frac{1}{2}(40n-68) - (64n-120)\\ &= 72n^2 - 84n + 100.\end{aligned}$$

□

7. Conclusion

In this paper we have investigated the Zagreb indices and coindices of the line cut-vertex graph of the subdivision graph of the tadpole graphs, wheel graphs, ladder graphs, star graphs, and path graphs. However, to determine the Zagreb indices and coindices of graph operators (or graph valued functions) still remains open and challenging problem for researchers.

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