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# NIRMALA INDEX OVER A GRAPH OF A MONOGENIC SEMIGROUPS

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Abstract. Inspired by the Sombor index, the nirmala index is defined as follows:

$$N(G) = \sum_{uv \in E(G)} \sqrt{(d_u) + (d_v)}.$$

In this paper, the nirmala index of monogenic semigroup graphs, which is an important class of algebraic structures, is calculated.

## 1. Introduction and Preliminaries

The monogenic semigroup graph is influenced by zero divisor graphs. Our focal point is based on zero divisor graphs. The first study on zero divisor graphs was described by Beck on commutative rings in [**10**]. Afterwards, some studies were carried out by Anderson et al. on commutative and non-commutative rings ([**7, 8, 9**]). De Meyer et al. has been the leader in research linking zero divisor graph to commutative and non-commutative semigroups ([**14, 15**]). In the study [**12**], the authors defined monogenic semigroup graphs, inspired by zero-divisor graphs. While keeping the initial idea the authors in [**12**] employed the adjacement rule of vertices. The authors determined that a finite multiplicative monogenic semigroup with 0 as follows:

$$S_M = \left\{0, x, x^2, x^3, ..., x^n\right\}. \tag{1.1}$$

As the definition utilized on [**14, 15**], the authors procured results associated with monogenic semigroups in [**12**]. The vertices of this graph are all nonzero elements

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in $S_M$ and for any two different vertices $x^i$ and $x^j$ where $(1 \leqslant i, j \leqslant n)$ are linked to each other, if and only if $i + j > n$. There are many studies concerning monogenic semigroup graphs were published by Akgüneş et al. (see for example [**2, 3, 4**].)

For centuries, topological indices have been instrumental in the advancement of science, chemistry, and mathematics. The structural and molecular properties are associated with these indices. Several graph indices which are defined as molecular graphs derived its properties from molecules which are further replaced by atoms with vertices and bonds between them with edge. All chemical molecules employ the use of graphs to acquire research in connections to the composition of molecules which would not recquire expensive research. Many indices are used to calculate the physical, chemical and biological properties of graphs. Different properties of the chemical structure in chemical graph theory utilize topological indices (see [**23**]). The Sombor index was introduced by Gutman in [**16**]. The topological index is a relatively new vertex degree about molecular structure descriptor. Lower and upper bounds on the Sombor index of a graph may be found in [**13, 17, 20, 21**] and elsewhere. For chemical applications of the Sombor index see [**5, 6, 11, 19, 22**].

For a graph $G$, its edge set and vertex set are denoted by $E(G)$ and $V(G)$, respectively. The theory of vertex-degree-based topological indices, in [**16**] Gutman discovered a new topological index defined by

$$SO(G) = \sum_{uv \in E(G)} \sqrt{(d_u)^2 + (d_v)^2}.$$

Inspired by work on Sombor indices, Kulli in [**18**] put forward the nirmala index of a graph G as follows:

$$N(G) = \sum_{uv \in E(G)} \sqrt{(d_u) + (d_v)}$$

Also as a reminder, for a real number $r$, we identify by $\lfloor r \rfloor$ the greatest integer $\leqslant r$ and by $\lceil r \rceil$, the least integer $\geqslant r$. It is clear that $r - 1 < \lfloor r \rfloor \leqslant r$ and $r \leqslant \lceil r \rceil < r + 1$. However, for a natural number $n$, we have

$$\left\lfloor \frac{n}{2} \right\rfloor = \begin{cases} \frac{n}{2} & if \quad \text{n} \quad is \quad even \\ \frac{n-1}{2} & if \quad \text{n} \quad is \quad odd \end{cases} \tag{1.2}$$

In this paper we focus on determining the explicit formula of nirmala index of the monogenic semigroup graph.

## 2. An Algorithm

The authors in [**2**] to simplify their research they gave the algorithm concerning the neighboorhood of vertices by utilising the initial statement of monogenic semigroup graph. We will use this algorithm in our main theorem in the next section.

$I_n$: The vertex $x^n$ is adjoining to every vertex $x^{i_1}$ $(1 \leqslant i_1 \leqslant n - 1)$ except itself.

$I_{n-1}$: The vertex $x^{n-1}$ is adjoining to every vertex $x^{i_2}$ $(2 \leqslant i_2 \leqslant n - 2)$ except itself and the vertex $x^n$.

$I_{n-2}$: The vertex $x^{n-2}$ is adjoining to every vertex $x^{i_3}$ $(3 \leqslant i_3 \leqslant n-3)$ except itself and the vertices $x^n$ and $x^{n-1}$.

Carrying on the algorithm this way we get the following result, depending on whether the number n is odd or even.

If $n$ is even:

$I_{\frac{n}{2}+2}$: The vertex $x^{\frac{n}{2}+2}$ is adjoining not only to the vertices $x^{\frac{n}{2}-1}$, $x^{\frac{n}{2}}$ and $x^{\frac{n}{2}+1}$ also adjoining to the vertices $x^n$, $x^{n-1}$, $x^{n-2}$, $\ldots$, $x^{\frac{n}{2}+3}$.

$I_{\frac{n}{2}+1}$: The vertex $x^{\frac{n}{2}+1}$ is adjoining not only to the single vertex $x^{\frac{n}{2}}$ also adjoining to the vertices $x^n$, $x^{n-1}$, $x^{n-2}$, $\ldots$, $x^{\frac{n}{2}+2}$.

If $n$ is odd:

$I_{\frac{n+1}{2}}$: The vertex $x^{\frac{n+1}{2}+2}$ is adjoining not only to the vertices $x^{\frac{n+1}{2}-2}$, $x^{\frac{n+1}{2}-1}$, $x^{\frac{n+1}{2}}$ and $x^{\frac{n+1}{2}+1}$ also adjoining to the vertices $x^n$, $x^{n-1}$, $x^{n-2}$, $\ldots$, $x^{\frac{n+1}{2}+3}$.

$I_{\frac{n+1}{2}+1}$: The vertex $x^{\frac{n+1}{2}+1}$ is adjoining not only to the vertices $x^{\frac{n+1}{2}-1}$ and $x^{\frac{n+1}{2}}$ also adjoining to the vertices $x^{n-1}$, $x^{n-2}$, $\ldots$, $x^{\frac{n+1}{2}+2}$.

In the lemma given below, the degrees of vertices $x^1, x^2, \ldots, x^n \in \Gamma(S_M)$ are denoted by $d_1, d_2, \ldots, d_n$. There are many studies on the degree series. Regarding this, you can refer to [**1, 12**] references and references cited in these studies. In fact, in the lemma below, it is mentioned that there is an ordering between the degrees $d_1, d_2, \ldots, d_n$. You can reach the proof of this lemma from [**12**], as well as from the algorithm given above. (See [**2**]).

Lemma 2.1.

$$d_1 = 1,\ d_2 = 2,\ \ldots\ d_{\lfloor \frac{n}{2} \rfloor} = \left\lfloor \frac{n}{2} \right\rfloor,\ d_{\lfloor \frac{n}{2} \rfloor + 1} = \left\lfloor \frac{n}{2} \right\rfloor,\ d_{\lfloor \frac{n}{2} \rfloor + 2} = \left\lfloor \frac{n}{2} \right\rfloor + 1,\ \ldots\ d_n = n-1 \tag{2.1}$$

Remark 2.1. Pay attention in Lemma 2.1 the repeated terms which are given in the following

$$d_{\lfloor \frac{n}{2} \rfloor} = \left\lfloor \frac{n}{2} \right\rfloor = d_{\lfloor \frac{n}{2} \rfloor + 1}.$$

Therefore, the degree of $d_n$ is denoted by $n-1$, although the number of vertices is $n$.

## 3. Calculating Nirmala Index of $\Gamma(S_M)$

In this section we will calculate an exact formula of nirmala index over monogenic semigroup graph.

Theorem 3.1. *For any monogenic semigroup $S_M$ as given in (1.1), the nirmala index of the graph $\Gamma(S_M)$ is*

$$N(\Gamma(S_M)) = \begin{cases} \sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} \sqrt{(n-k)+i} + \sum_{k=1}^{\frac{n}{2}} \sqrt{(n-k)+\left(\frac{n}{2}\right)} & if \quad n \quad is \quad even \\ \sum_{k=1}^{\frac{n-1}{2}} \sum_{i=k}^{n-k-1} \sqrt{(n-k)+i} + \sum_{k=1}^{\frac{n-1}{2}} \sqrt{(n-k)+\left(\frac{n}{2}\right)} & if \quad n \quad is \quad odd \end{cases}$$

PROOF. We will show the calculation of formula of $N(\Gamma(S_M))$ in terms of the total number of degrees. First we calculate sum as the sum of different blocks thereafter each one is calculated seperately. Our calculations will utilise the algorithm in Section 2 because it has a methodical plan of calculating the degree of vertices while making use of equations (1.2), (2.1) and Remark 2.2.

If $n$ is odd:

$$\begin{aligned}
[N]\left(\Gamma(S_M)\right) = {} & \sqrt{d_n + d_1} + \sqrt{d_n + d_2} + \sqrt{d_n + d_3} + \ldots + \sqrt{d_n + d_{n-2}} + \sqrt{d_n + d_{n-1}} + \\
& + \sqrt{d_{n-1} + d_2} + \sqrt{d_{n-1} + d_3} + \ldots + \sqrt{d_{n-1} + d_{n-2}} + \\
& + \ldots + \\
& + \sqrt{d_{\frac{n+1}{2}+2} + d_{\frac{n+1}{2}-2}} + \sqrt{d_{\frac{n+1}{2}+2} + d_{\frac{n+1}{2}-1}} + \sqrt{d_{\frac{n+1}{2}+2} + d_{\frac{n+1}{2}}} \\
& + \sqrt{d_{\frac{n+1}{2}+2} + d_{\frac{n+1}{2}+1}} + \sqrt{d_{\frac{n+1}{2}+1} + d_{\frac{n+1}{2}-1}} + \sqrt{d_{\frac{n+1}{2}+1} + d_{\frac{n+1}{2}}}
\end{aligned}$$

As a result, the nirmala index of $\Gamma(S_M)$ is written as the sum below

$$[N]\left(\Gamma(S_M)\right) = \sum_{ij \in E(G)} \sqrt{d_i + d_j} = [N]_n + [N]_{n-1} + \ldots + [N]_{\frac{n+1}{2}+2} + [N]_{\frac{n+1}{2}+1}$$

When calculating the nirmala index sum, we will write the smallest degree at the end of the line, so we will get a second total and this will provide us with ease of operation. By the way, while making these calculations, we use the equation $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n-1}{2}$ given in (1.2) for the case where $n$ is odd.

$$\begin{aligned}
[N]_n = {} & \sqrt{(n-1)+1} + \sqrt{(n-1)+2} + \sqrt{(n-1)+3} + \ldots + \sqrt{(n-1) + \left\lfloor \frac{n}{2} \right\rfloor} + \ldots + \\
& + \sqrt{(n-1)+(n-2)} + \sqrt{(n-1) + \left\lfloor \frac{n}{2} \right\rfloor} \\
= {} & \sum_{i=1}^{n-2} \sqrt{(n-i)+i} + \sqrt{(n-1) + (\frac{n-1}{2})}
\end{aligned}$$

If similar operations applied in $[N]_n$ are applied in $[N]_{n-1}$, we obtain

$[N]_{n-1} = \sum_{i=2}^{n-3} \sqrt{(n-i)+i} + \sqrt{(n-2) + (\frac{n-1}{2})}$,

$[N]_{\frac{n+1}{2}+2} =$

$\sqrt{(\frac{n+3}{2}) + (\frac{n-3}{2})} + \sqrt{(\frac{n+3}{2}) + (\frac{n-1}{2})} + \sqrt{(\frac{n+3}{2}) + (\frac{n-1}{2})} + \sqrt{(\frac{n+3}{2}) + (\frac{n+1}{2})}$

and finally

$$[N]_{\frac{n+1}{2}+1} = \sqrt{(\frac{n+1}{2}) + (\frac{n-1}{2})} + \sqrt{(\frac{n+1}{2}) + (\frac{n-1}{2})}.$$

Hence

$$\begin{gathered}
[N]_n + [N]_{n-1} + \ldots + [N]_{\frac{n+1}{2}+2} + [N]_{\frac{n+1}{2}+1} = \\
\sum_{k=1}^{\frac{n-1}{2}} \sum_{i=k}^{n-k-1} \sqrt{(n-k)+i} + \sum_{k=1}^{\frac{n-1}{2}} \sqrt{(n-k) + \frac{n}{2}}
\end{gathered}$$

□

If we follow similar steps as if $n$ is odd, we will get the following sum if $n$ is even

$$[N]_n + [N]_{n-1} + \ldots + [N]_{\frac{n}{2}+2} + [N]_{\frac{n}{2}+1} =$$

$$\sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} \sqrt{(n-k)+i} + \sum_{k=1}^{\frac{n}{2}} \sqrt{(n-k)+\frac{n}{2}}$$

We will give the following example to reinforce Theorem 3.1.

Example 3.1. Consider the monogenic semigroup $S_{M_6}$ given below and calculate the nirmala index of $\Gamma(S_{M_6})$ graph by applying the rule given in Theorem 3.1.

$$S_{M_6} = \left\{x, x^2, x^3, x^4, x^5, x^6\right\} \cup \{0\}$$

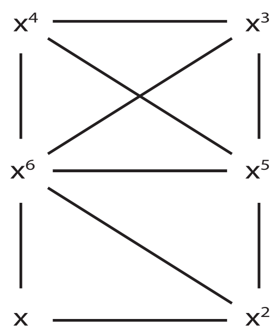


Figure 1. $S_{M_6}$ monogenic semigroup graph

$$\begin{aligned}
N(\Gamma(S_M)) &= \sum_{k=1}^{2} \sum_{i=k}^{5-k} \sqrt{(6-k)+i} + \sum_{k=1}^{3} \sqrt{(6-k)+(3)} \\
&= \sqrt{5+1} + \sqrt{5+2} + \sqrt{5+3} + \sqrt{5+4} + \sqrt{4+2} + \sqrt{4+3} \\
&+ \sqrt{5+3} + \sqrt{4+3} + \sqrt{3+3} \\
&= 3 + 4\sqrt{2} + 3\sqrt{6} + 3\sqrt{7}
\end{aligned}$$

As can be seen, the nirmala index of a monogenic semigroup graph can be calculated very easily with the given formula in Theorem 3.1.

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