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NEUTROSOPHIC BIMINIMAL α-OPEN SETS

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Abstract. In this article, we have introduced the notions of N^j_{mX}-α-open sets, α-interior and α-closure operators in neutrosophic biminimal structures. We investigate some basic properties and theorems of such notions. Also we have introduced the notion of N^j_{mX}-α-continuous maps and study characterizations of N^j_{mX}-α-continuous maps by using the α-interior and α-closure operators in neutrosophic biminimal structures.

1. Introduction

Zadehs [**14**] Fuzzy set laid the foundation of many theories such as intuitionistic fuzzy set and neutrosophic set, rough sets etc. Later, researchers developed K. T. Atanassovs [**1**] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computer science and so on. F. Smarandache [**12, 13**] found that some objects have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. Q. H. Imran et al [**6**] introduced and studied neutrosophic semi-α-open sets. R. Dhavaseelan et al [**2**] introduced and studied neutrosophic α^m-continuity. C. Maheswari and S. Chandrasekar [**8**] introduced and studied neutrosophic gb-closed sets and neutrosophic gb-continuity. Q. H. Imran et al [**7**] introduced and studied neutrosophic generalized alpha generalized continuity. M. H. Page and Q. H. Imran [**9**] introduced and studied neutrosophic generalized homeomorphism. The concept of minimal structure (in short, m-structure) was introduced by V. Popa and T. Noiri [**10**] in 2000. Also they introduced the notion of m_x-open set and m_x-closed set and characterize those sets using m_x-closure and m_x-interior operators respectively. Further they

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introduced $\mathcal{M}$-continuous functions and studied some of it is basic properties. S. Ganesan et al [**4**] introduced and studied the notion of neutrosophic biminimal structure spaces and also applications of neutrosophic biminimal structure spaces. S. Ganesan and F. Smarandache [**5**] introduced and studied neutrosophic biminimal semi-open sets. The main objective of this study is to introduce a new hybrid intelligent structure called neutrosophic biminimal α-open set. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results. The rest of this article is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concept of N^{j}_{mX}-α-open set is investigated some properties with suitable example.

2. Preliminaries

Definition 2.1. [**10**] A subfamily m_x of the power set $\wp$(X) of a nonempty set X is called a minimal structure (in short, m-structure) on X if $\emptyset \in m_x$ and X $\in m_x$. By (X, m_x), we denote a nonempty set X with a minimal structure m_x on X and call it an m-space.
Each member of m_x is said to be m_x-open (or in short, m-open) and the complement of an m_x-open set is said to be m_x-closed (or in short, m-closed).

Definition 2.2. ([**12, 13**]) A neutrosophic set (in short ns) K on a set X $\neq \emptyset$ is defined by K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ where P_K : X $\to$ [0,1], Q_K : X $\to$ [0,1] and R_K : X $\to$ [0,1] denotes the membership of an object, indeterminacy and non-membership of an object, for each a $\in$ X to K, respectively and 0 $\leqslant P_K$(a) + Q_K(a) + R_K(a) $\leqslant$ 3 for each a $\in$ X.

Definition 2.3. ([**11**]) Let K = $\{\prec$ a, P_K(a), Q_K(a), R_K(a) $\succ$: a $\in$ X$\}$ be a ns.

(1) A ns K is an empty set i.e., $K = 0_\sim$ if 0 is membership of an object and 0 is an indeterminacy and 1 is an non-membership of an object respectively. i.e., $0_\sim = \{x, (0, 0, 1) : x \in X\}$;

(2) A ns K is a universal set i.e., $K = 1_\sim$ if 1 is membership of an object and 1 is an indeterminacy and 0 is an non-membership of an object respectively. $1_\sim = \{x, (1, 1, 0) : x \in X\}$;

(3) $K_1 \cup K_2 =$
$\{a, max\{P_{K_1}(a), P_{K_2}(a)\}, max\{Q_{K_1}(a), Q_{K_2}(a)\}, min\{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$;

(4) $K_1 \cap K_2 =$
$\{a, min\{P_{K_1}(a), P_{K_2}(a)\}, min\{Q_{K_1}(a), Q_{K_2}(a)\}, max\{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$;

(5) K^C = $\{\prec$ a, R_K(a), 1 − Q_K(a), P_K(a) $\succ$: a $\in$ X$\}$.

Definition 2.4. ([**11**]) A neutrosophic topology (nt) in Salamas sense on a nonempty set X is a family τ of ns in X satisfying three axioms:
(1) Empty set ($0_\sim$) and universal set ($1_\sim$) are members of τ;
(2) $K_1 \cap K_2 \in \tau$ where K_1, $K_2 \in \tau$;

(3) $\cup K_\delta \in \tau$ for every $\{K_\delta : \delta \in \Delta\} \leqslant \tau$.

Each ns in nt are called neutrosophic open sets. Its complements are called neutrosophic closed sets.

Definition 2.5. ([**4**]) Let X be a nonempty set and N^1_{mX}, N^2_{mX} be nms on X. A triple (X, N^1_{mX}, N^2_{mX}) is called a neutrosophic biminimal structure space (in short, nbims)

Definition 2.6. [**4**] Let (X, N^1_{mX}, N^2_{mX}) be a nbims and S be any neutrosophic set. Then

(1) Every S $\in N^j_{mX}$ is open and its complement is closed, respectively, for j = 1, 2.
(2) $N_m cl_j$(S) = min {L : L is N^j_{mX}-closed set and L $\geqslant$ S}, respectively, for j = 1, 2.
(3) $N_m int_j$(S) = max {T : T is N^j_{mX}-open set and T $\leqslant$ S}, respectively, for j = 1, 2.

Proposition 2.1 ([**4**]). *Let (X, N^1_{mX}, N^2_{mX}) be a nbims and $A \leqslant X$. Then*

(1) $N_m int_j(0_\sim) = 0_\sim$
(2) $N_m int_j(1_\sim) = 1_\sim$
(3) $N_m int_j(A) \leqslant A$.
(4) *If* $A \leqslant B$, *then* $N_m int_j(A) \leqslant N_m int_j(B)$.
(5) *A is* N^j_{mX}*-open if and only if* $N_m int_j(A) = A$.
(6) $N_m int_j(N_m int_j(A)) = N_m int_j(A)$.
(7) $N_m cl_j(X - A) = X - N_m int_j(A)$ *and* $N_m int_j(X - A) = X - N_m cl_j(A)$.
(8) $N_m cl_j(0_\sim) = 0_\sim$
(9) $N_m cl_j(1_\sim) = 1_\sim$
(10) $A \leqslant N_m cl_j(A)$.
(11) *If* $A \leqslant B$, *then* $N_m cl_j(A) \leqslant N_m cl_j(B)$.
(12) *F is* N^j_{mX}*-closed if and only if* $N_m cl_j(F) = F$.
(13) $N_m cl_j(N_m cl_j(A)) = N_m cl_j(A)$.

Definition 2.7. ([**4**]) Let (X, N^1_{mX}, N^2_{mX}) be a nbims and A be a subset of X. Then A is $N^1_{mX}N^2_{mX}$-closed if and only if $N_m cl_1$(A) = A and $N_m cl_2$(A) = A.

Proposition 2.2 ([**4**]). *Let N^1_{mX} and N^2_{mX} be nms on X satisfying (Union Property). Then A is a $N^1_{mX}N^2_{mX}$-closed subset of a nbims (X, N^1_{mX}, N^2_{mX}) if and only if A is both N^1_{mX}-closed and N^2_{mX}-closed.*

Proposition 2.3 ([**4**]). *Let (X, N^1_{mX}, N^2_{mX}) be a nbims. If A and B are $N^1_{mX}N^2_{mX}$-closed subsets of (X, N^1_{mX}, N^2_{mX}), then $A \wedge B$ is $N^1_{mX}N^2_{mX}$-closed.*

Proposition 2.4 ([**4**]). *Let (X, N^1_{mX}, N^2_{mX}) be a nbims. If A and B are $N^1_{mX}N^2_{mX}$-open subsets of (X, N^1_{mX}, N^2_{mX}), then $A \vee B$ is $N^1_{mX}N^2_{mX}$-open.*

Definition 2.8. ([**5**]) A map $f : (X, N^1_{mX}, N^2_{mX}) \to (Y, N^1_{mY}, N^2_{mY})$ is called N^j_{mX}-continuous map if and only if $f^{-1}(V) \in N^j_{mX}$-open whenever $V \in N^j_{mY}$.

Theorem 2.1 ([**5**]). *Let $f: X \to Y$ be a map on two nbims (X, N^1_{mX}, N^2_{mX}) and (Y, N^1_{mY}, N^2_{mY}). Then the following statements are equivalent:*

(1) *Identity map from (X, N^1_{mX}, N^2_{mX}) to (Y, N^1_{mY}, N^2_{mY}) is a nbims map.*
(2) *Any constant map which map from (X, N^1_{mX}, N^2_{mX}) to (Y, N^1_{mY}, N^2_{mY}) is a nbims map.*

Definition 2.9. ([**5**]) Let (X, N^1_{mX}, N^2_{mX}) be a nbims and A $\leqslant$ X. A subset A of X is called an $N^1_{mX}N^2_{mX}$-semi-open (in short, N^j_{mX}-semi-open) set if A $\leqslant$ $\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\mathrm{A}))$, respectively, for j = 1, 2.
The complement of an N^j_{mX}-semi-open set is called an N^j_{mX}-semi-closed set.

Definition 2.10. ([**5**]) A map $f: (X, N^1_{mX}, N^2_{mX}) \to (Y, N^1_{mY}, N^2_{mY})$ is called N^j_{mX}-semi-continuous map if and only if $f^{-1}(V) \in N^j_{mX}$-semi-open whenever $V \in N^j_{mY}$.

3. $N^1_{mX}N^2_{mX}$-α-open sets

Definition 3.1. Let (X, N^1_{mX}, N^2_{mX}) be a nbims and A $\leqslant$ X. A subset A of X is called an $N^1_{mX}N^2_{mX}$-α-open (in short, N^j_{mX}-α-open) set if

$$\mathrm{A} \leqslant \mathrm{N}_m int_j(\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\mathrm{A}))), \text{ respectively, for j = 1, 2.}$$

The complement of an N^j_{mX}-α-open set is called an N^j_{mX}-α-closed set.

Remark 3.1. Let (X, N_{mX}) be a nms and A $\leqslant$ X. A is called an N_m-α-open set [**3**] if A $\leqslant$ $\mathrm{N}_m\mathrm{int}(\mathrm{N}_m\mathrm{cl}(\mathrm{N}_m\mathrm{int}(\mathrm{A})))$. If the nms N_{mX} is a topology, clearly an N^j_{mX}-α-open set is N_m-α-open.

From Definition of 3.1, obviously the following statement are obtained.

Lemma 3.1. *Let (X, N^1_{mX}, N^2_{mX}) be a nbims. Then*

(1) *Every N^j_{mX}-open set is N^j_{mX}-α-open.*
(2) *A is an N^j_{mX}-α-open set if and only if $A \leqslant N_m int_j(N_m cl_j(N_m int_j(A)))$.*
(3) *Every N^j_{mX}-closed set is N^j_{mX}-α-closed.*
(4) *A is an N^j_{mX}-α-closed set if and only if $N_m cl_j(N_m int_j(N_m cl_j(A))) \leqslant A$.*

Theorem 3.1. *Let (X, N^1_{mX}, N^2_{mX}) be a nbims. Any union of N^j_{mX}-α-open sets is N^j_{mX}-α-open.*

Proof. Let A_δ be an N^j_{mX}-α-open set for $\delta \in \Delta$. From Definition 3.1 and Proposition 2.1(4), it follows

$$\mathrm{A}_\delta \leqslant \mathrm{N}_m int_j(\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\mathrm{A}_\delta))) \leqslant \mathrm{N}_m int_j(\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\bigcup \mathrm{A}_\delta))).$$

This implies

$$\bigcup \mathrm{A}_\delta \leqslant \mathrm{N}_m int_j(\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\bigcup \mathrm{A}_\delta))).$$

Hence $\bigcup \mathrm{A}_\delta$ is an N^j_{mX}-α-open set. □

REMARK 3.2. Let (X, N^1_{mX}, N^2_{mX}) be a nbims. The intersection of any two N^j_{mX}-α-open sets may not be N^j_{mX}-α-open set as shown in the next example.

EXAMPLE 3.1. Let $X = \{a, b, c\}$ with

$$N^1_{mX} = \{0_\sim, U, 1_\sim\},\ (N^1_{mX})^C = \{1_\sim, V, 0_\sim\} \text{ and}$$
$$N^2_{mX} = \{0_\sim, O, 1_\sim\},\ (N^2_{mX})^C = \{1_\sim, P, 0_\sim\}$$

where

$$\text{U} = \prec (0.7, 0.4, 0.9), (0, 0.8, 0.2), (0.4, 0.6, 0.7)\succ$$
$$\text{O} = \prec (0.5, 0.6, 0.8), (0.2, 0.4, 0.6), (0.7, 0.5, 0)\succ$$
$$\text{V} = \prec (0.9, 0.6, 0.7), (0.2, 0.2, 0), (0.7, 0.4, 0.4)\succ$$
$$\text{P} = \prec (0.8, 0.4, 0.5), (0.6, 0.2, 0.2), (0, 0.5, 0.3)\succ$$

We know that

$$0_\sim = \{\prec \text{x}, 0, 0, 1 \succ : \text{x} \in \text{X}\},\ 1_\sim = \{\prec \text{x}, 1, 1, 0 \succ : \text{x} \in \text{X}\}$$

and

$$0_\sim^C = \{\prec \text{x}, 1, 1, 0 \succ : \text{x} \in \text{X}\},\ 1_\sim^C = \{\prec \text{x}, 0, 0, 1 \succ : \text{x} \in \text{X}\}.$$

Now we define the two N^j_{mX}-α-open sets as follows:

$$R_1 = \prec (0.7, 0.5, 0.6), (0.4, 0.8, 0.2), (0.8, 0.7, 0.5)\succ$$
$$R_2 = \prec (0.6, 0.3, 0.4), (0, 0.2, 0.1), (0.6, 0.6, 0.4)\succ$$

Here $\text{N}_m int_j(\text{N}_m cl_j(\text{N}_m int_j(R_1))) = 0_\sim$ and $\text{N}_m int_j(\text{N}_m cl_j(\text{N}_m int_j(R_2))) = 0_\sim$. But $R_1 \wedge R_2 = \prec (0.6, 0.3, 0.6), (0, 0.2, 0.2), (0.6, 0.6, 0.5)\succ$ is not a N^j_{mX}-α-open set in X. □

PROPOSITION 3.1. *Let (X, N^1_{mX}, N^2_{mX}) be a nbims. If A is a N^j_{mX}-α-open set then it is a N^j_{mX}-semi-open set.*

PROOF. The proof is straightforward from the definitions. □

DEFINITION 3.2. Let (X, N^1_{mX}, N^2_{mX}) be a nbims and S be any neutrosophic set. Then

(1) Every $S \in N^j_{mX}$ is α-open and its complement is α-closed, respectively, for $j = 1, 2$.
(2) $N_m\alpha cl_j(S) = min\{L : LisN^j_{mX}$-$\alpha$-closed set and $L \geqslant S\}$, respectively, for $j = 1, 2$.
(3) $N_m\alpha int_j(S) = max\{T : TisN^j_{mX}$-$\alpha$-open set and $T \leqslant S\}$, respectively, for $j = 1, 2$.

THEOREM 3.2. *Let (X, N^1_{mX}, N^2_{mX}) be a nbims and $A \leqslant X$. Then:*

(1) $N_m\alpha int_j(0_\sim) = 0_\sim$;
(2) $N_m\alpha int_j(1_\sim) = 1_\sim$;
(3) $N_m\alpha int_j(A) \leqslant A$;
(4) *If $A \leqslant B$, then $N_m\alpha int_j(A) \leqslant N_m\alpha int_j(B)$;*
(5) *A is $N^j_{mX} - \alpha$-open if and only if $N_m\alpha int_j(A) = A$;*
(6) $N_m\alpha int_j(N_m\alpha int_j(A)) = N_m\alpha int_j(A)$;
(7) $N_m\alpha cl_j(X - A) = X - N_m\alpha int_j(A)$.

PROOF. (1), (2), (3), (4) Obvious.

(5) It follows from Theorem 3.1.

(6) It follows from (5).

(7) For $A \leqslant X$, we have

$X - N_m\alpha int_j(A) = X - max\{U : U \leqslant A, U \text{ is } N^j_{mX} - \alpha - \text{open}\}$
$= min\{X - U : U \leqslant A, U \text{ is } N^j_{mX} - \alpha - \text{open}\}$
$= min\{XU : X - A \leqslant X - U, U \text{ is } N^j_{mX} - \alpha - \text{open}\} = N_m\alpha cl_j(X - A)$. □

THEOREM 3.3. *Let (X, N^1_{mX}, N^2_{mX}) be a nbims and $A \leqslant X$. Then:*

(1) $N_m\alpha cl_j(0_\sim) = 0_\sim$;
(2) $N_m\alpha cl_j(1_\sim) = 1_\sim$;
(3) $A \leqslant N_m\alpha cl_j(A)$;
(4) *If* $A \leqslant B$, *then* $N_m\alpha cl_j(A) \leqslant N_m\alpha cl_j(B)$;
(5) F *is* N^j_{mX}-α-*closed if and only if* $N_m\alpha cl_j(F) = F$;
(6) $N_m\alpha cl_j(N_m\alpha cl_j(A)) = N_m\alpha cl_j(A)$;
(7) $N_m\alpha int_j(X - A) = X - N_m\alpha cl_j(A)$.

PROOF. It is similar to the proof of Theorem 3.2. □

THEOREM 3.4. *Let (X, N^1_{mX}, N^2_{mX}) be a nbims and $A \leqslant X$. Then:*

(1) $x \in N_m\alpha cl_j(A)$ *if and only if* $A \cap V \neq \emptyset$ *for every* $N^j_{mX} - \alpha$-*open set* V *containing* x.
(2) $x \in N_m\alpha int_j(A)$ *if and only if there exists an* $N^j_{mX} - \alpha$-*open set* U *such that* $U \leqslant A$.

PROOF. (1) Suppose there is an $N^j_{mX} - \alpha$-open set V containing x such that $A \cap V = \emptyset$. Then $X - V$ is an $N^j_{mX} - \alpha$-closed set such that $A \leqslant X - V$, $x \notin X - V$. This implies $x \notin N_m\alpha cl_j(A)$. The reverse relation is obvious.

(2) Obvious. □

LEMMA 3.2. *Let (X, N^1_{mX}, N^2_{mX}) be a nbims and $A \leqslant X$. Then*

(1) $N_mcl_j(N_mint_j(N_mcl_j(A))) \leqslant N_mcl_j(N_mint_j(N_mcl_j(N_m\alpha cl_j(A))) \leqslant N_m\alpha cl_j(A)$.
(2) $N_m\alpha int_j(A) \leqslant N_mint_j(N_mcl_j(N_mint_j(N_m\alpha int_j(A)))) \leqslant N_mint_j(N_mcl_j(N_mint_j(A)))$.

PROOF. (1) For $A \leqslant X$, by Theorem 3.3, $N_m\alpha cl_j(A)$ is an $N^j_{mX} - \alpha$-closed set. Hence from Lemma 3.1, we have

$$N_mcl_j(N_mint_j(N_mcl_j(A))) \leqslant N_mcl_j(N_mint_j(N_mcl_j(N_m\alpha cl_j(A)))) \leqslant N_m\alpha cl_j(A).$$

(2) It is similar to the proof of (1). □

DEFINITION 3.3. A map $f : (X, N^1_{mX}, N^2_{mX}) \to (Y, N^1_{mY}, N^2_{mY})$ is called $N^j_{mX} - \alpha$-continuous map if and only if $f^{-1}(V) \in N^j_{mX} - \alpha$-open whenever $V \in N^j_{mY}$.

THEOREM 3.5. (1) *Every* N^j_{mX}*-continuous is* N^j_{mX}*-*α*-continuous but the conversely.*

(2) *Every* N^j_{mX}*-*α*-continuous is* N^j_{mX}*-semi-continuous but not conversely.*

PROOF. (1) The proof follows from Lemma 3.1 (1).

(2) The proof follows from Proposition 3.1. □

THEOREM 3.6. *Let* $f : X \to Y$ *be a map on two nbims* (X, N^1_{mX}, N^2_{mX}) *and* (Y, N^1_{mY}, N^2_{mY})*. Then the following statements are equivalent:*

(1) f *is* N^j_{mX}*-*α*-continuous.*
(2) $f^{-1}(V)$ *is an* N^j_{mX}*-*α*-open set for each* N^j_{mX}*-open set* V *in* Y*.*
(3) $f^{-1}(B)$ *is an* N^j_{mX}*-*α*-closed set for each* N^j_{mX}*-closed set* B *in* Y*.*
(4) $f(N_m\alpha cl_j(A)) \leqslant N_m cl_j(f(A))$ *for* $A \leqslant X$*.*
(5) $N_m\alpha cl_j(f^{-1}(B)) \leqslant f^{-1}(N_m cl_j(B))$ *for* $B \leqslant Y$*.*
(6) $f^{-1}(N_m int_j(B)) \leqslant N_m\alpha int_j(f^{-1}(B))$ *for* $B \leqslant Y$*.*

PROOF. (1) $\Rightarrow$ (2) Let V be an N^j_{mX}-open set in Y and x $\in$ f^{-1}(V). By hypothesis, there exists an N^j_{mX}-α-open set U$_x$ containing x such that f(U) $\leqslant$ V. This implies x $\in$ U$_x$ $\leqslant$ f^{-1}(V) for all x $\in$ f^{-1}(V). Hence by Theorem 3.1, f^{-1}(V) is N^j_{mX}-α-open.

(2) $\Rightarrow$ (3) Obvious.

(3) $\Rightarrow$ (4) For $A \leqslant X$, $f^{-1}(N_m cl_j(f(A))) = f^{-1}(min\{F \leqslant Y : f(A) \leqslant F$ and F is N^j_{mX}-closed$\}) = min\{f^{-1}(F) \leqslant X : A \leqslant f^{-1}(F)$ and F is $N^j_{mX} - \alpha$-closed$\} \geqslant min\{K \leqslant X : A \leqslant K$ and K is $N^j_{mX} - \alpha$-closed$\} = N_m\alpha cl_j(A)$. Hence $f(N_m\alpha cl_j(A)) \leqslant N_m cl_j(f(A))$.

(4) $\Rightarrow$ (5) For $A \leqslant X$, from (4), it follows

$$f(N_m\alpha cl_j(f^{-1}(A))) \leqslant N_m cl_j(f(f^{-1}(A))) \leqslant N_m cl_j(A).$$

Hence, we get (5).

(5) $\Rightarrow$ (6) For $B \leqslant Y$, from $N_m int_j(B) = Y - N_m cl_j(Y - B)$ and (5), it follows $f^{-1}(N_m int_j(B)) = f^{-1}(Y - N_m cl_j(Y - B)) = X - f^{-1}(N_m cl_j(Y - B)) \leqslant X - N_m\alpha cl_j(f^{-1}(Y - B)) = N_m\alpha int_j(f^{-1}(B))$. Hence (6) is obtained.

(6) $\Rightarrow$ (1) Let $x \in X$ and V an N^j_{mX}-open set containing $f(x)$. Then from (6) and Proposition 2.1, it follows

$$x \in f^{-1}(V) = f^{-1}(N_m int_j(V)) \leqslant N_m\alpha int_j(f^{-1}(V)).$$

So from Theorem 3.4, we can say that there exists an $N^j_{mX} - \alpha$-open set U containing x such that $x \in U \leqslant f^{-1}(V)$. Hence, f is $N^j_{mX} - \alpha$-continuous. □

THEOREM 3.7. *Let* $f : X \to Y$ *be a map on two nbims* (X, N^1_{mX}, N^2_{mX}) *and* (Y, N^1_{mY}, N^2_{mY})*. Then the following statements are equivalent:*

(1) f *is* N^j_{mX}*-*α*-continuous.*
(2) $f^{-1}(V) \leqslant N_m cl_j(N_m int_j(f^{-1}(V)))$ *for each* N^j_{mX}*-open set* V *in* Y*.*
(3) $N_m cl_j(N_m int_j(N_m cl_j(f^{-1}(F)))) \leqslant f^{-1}(F)$ *for each* N^j_{mX}*-closed set* F *in* Y*.*

(4) $f(N_m cl_j(N_m int_j(N_m cl_j(A)))\leqslant N_m cl_j(f(A))$ for $A\leqslant X$.
(5) $N_m cl_j(N_m int_j(N_m cl_j(f^{-1}(B))))\leqslant f^{-1}(N_m cl_j(B))$ for $B\leqslant Y$.
(6) $f^{-1}(N_m int_j(B))\leqslant N_m int_j(N_m cl_j(N_m int_j(f^{-1}(B))))$ for $B\leqslant Y$.

Proof. (1) $\Leftrightarrow$ (2) It follows from Theorem 3.6 and Definition of N^j_{mX}-α-open sets.

(1) $\Leftrightarrow$ (3) It follows from Theorem 3.6 and Lemma 3.1.

(3) $\Rightarrow$ (4) Let $A\leqslant X$. Then from Theorem 3.6(4) and Lemma 3.2, it follows $N_m cl_j(N_m int_j(N_m cl_j(A)))\leqslant N_m\alpha cl_j(A))\leqslant f^{-1}(N_m cl_j(f(A)))$. Hence

$$f(N_m cl_j(N_m int_j(N_m cl_j(A))))\leqslant N_m cl_j(f(A)).$$

(4) $\Rightarrow$ (5) Obvious.

(5) $\Rightarrow$ (6) From (5) and Proposition 2.1, it follows: $\mathrm{f}^{-1}(\mathrm{N}_m int_j(\mathrm{B}))=\mathrm{f}^{-1}(\mathrm{Y}-\mathrm{N}_m cl_j(\mathrm{Y}-\mathrm{B}))=\mathrm{X}-\mathrm{f}^{-1}(\mathrm{N}_m cl_j(\mathrm{Y}-\mathrm{B}))\leqslant \mathrm{X}-\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\mathrm{N}_m cl_j(\mathrm{f}^{-1}(\mathrm{Y}-\mathrm{B}))))=\mathrm{N}_m int_j(\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\mathrm{f}^{-1}(\mathrm{B}))))$. Hence, (6) is obtained.

(6) $\Rightarrow$ (1) Let V be an N^j_{mX}-open set in Y. Then by (6) and Proposition 2.1, we have $\mathrm{f}^{-1}(\mathrm{V})=\mathrm{f}^{-1}(\mathrm{N}_m int_j(\mathrm{V}))\leqslant \mathrm{N}_m int_j(\mathrm{N}_m cl_j(\mathrm{N}_m int_j(\mathrm{f}^{-1}(\mathrm{V}))))$. This implies $\mathrm{f}^{-1}(\mathrm{V})$ is an N^j_{mX}-α-open set. Hence by (2), f is N^j_{mX}-α-continuous. □

4. Conclusion

Neutrosophic set is a general formal framework, which generalizes the concept of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval intuitionistic fuzzy set. Since the world is full of indeterminacy, the neutrosophic biminimal structures found its place into contemporary research world. This article can be further developed into several possible such as Geographical Information Systems (GIS) field including remote sensing, object reconstruction from airborne laser scanner, real time tracking, routing applications and modeling cognitive agents. In GIS there is a need to model spatial regions with indeterminate boundary and under indeterminacy. Hence this N^j_{mX}-α-open set can also be extended to a neutrosophic spatial region.

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