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ON NANO g_s-CLOSED SETS AND NANO g_s-CONTINUOUS FUNCTIONS

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Abstract. In this article, a new arrival of sets is called nano g_s-closed sets are introduced in nanotopological spaces. The properties of the nano g_s-closed sets are investigated and they are compared with the obtainable related nano generalized closed sets and continuous function between nano topological spaces are also defined and their properties are investigated.

1. Introduction and Preliminary

The concept of nano topology was introduced by M. Lellis Thivagar *et al* (2013). which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano-interior and nano-closure. K. Bhuvaneswari *et al* (2014) introduced and studied the concept of nano generalised closed sets in nano topological spaces.

Since then many mathematicians introduced and investigated different types of nano generalized closed sets in nanotopological spaces.

In this paper, A novel closed sets called nano g_s-closed sets are introduced in nano spaces. The properties of the nano g_s-closed sets are investigated and they are compared with the obtainable related nano generalized closed sets and continuous function between nano topological spaces are also defined and their properties are investigated.

Recently, Lellis Thivagar *et. al* (2013) introduced the notions of weaker form of nano semi-open sets, nano regular-open sets and nano pre-open sets, K. Bhuvaneswari *et al* (2014, 2015, 2016) introduced and studied the concept of nano

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generalized closed sets, nano generalized semi closed sets and nano generalized continuous, nano generalized semi continuous, Nagaveni *et. al* (2016, 2017) introduced nano weakly generalized closed sets and nano weakly generalized continuous were studied in detail.

In the rest of the paper, we denote a nano topological space by $(U,\mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by n-$int(A)$ and n-$cl(A)$, respectively.

2. On ng_s-closed sets

DEFINITION 2.1. Let $(U,\mathcal{N})$ be a nanotopological space.

(1) Nano regular weakly generalized closed set (briefly, $nrwg$-closed) if

$$ncl(nint(Q)) \subseteq G$$

whenever $Q \subseteq G$ and G is nr-open.

(2) A subset Q of U is said to be a ng_s-closed set if Q is both ns-open and ng-closed.

PROPOSITION 2.1. *In a $(U,\mathcal{N})$, every ng_s-closed set is ng-closed.*

REMARK 2.1. The following example shows that the Proposition 2.1 of converse is not true.

EXAMPLE 2.1. Let $U = \{a, b, c, d\}$ and $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ with $X = \{a, d\}$. Then $\mathcal{N} = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$. In the space U, then $\{c\}$ is ng-closed set but not ng_s-closed.

THEOREM 2.1. *In a $(U,\mathcal{N})$, the union of two ng_s-closed sets is also a ng_s-closed.*

PROOF. Let P and Q be two ng_s-closed sets. Then P and Q are ng-closed sets and ns-open sets. Therefore, $P \cup Q$ is ng-closed set and ns-open set and hence it is a ng_s-closed. □

REMARK 2.2. The following example shows that the intersection of two ng_s-closed sets is not a ng_s-closed in $(U,\mathcal{N})$.

EXAMPLE 2.2. Let $U = \{a, b, c\}$ and $U/R = \{\{a\}, \{b, c\}\}$ with $X = \{a\}$. Then $\mathcal{N} = \{\phi, \{a\}, U\}$. We have $A = \{a, b\}$ and $B = \{a, c\}$ are ng_s-closed sets. But their intersection of U as $A \cap B = \{a\}$ is not ng_s-closed.

THEOREM 2.2. *In a $(U,\mathcal{N})$, if P and Q are both n-closed and n-open subsets then $P \cap Q$ is a ng_s-closed.*

PROOF. Suppose that P and Q are both n-closed and n-open subsets. Then, $P \cap Q$ is a n-closed set and so is a ng-closed set. Also, $P \cap Q$ is a n-open set and so is a ns-open set. Hence $P \cap Q$ is a ng_s-closed. □

THEOREM 2.3. *In a $(U,\mathcal{N})$, if P is ng_s-closed and Q is n-open such that $Q \subseteq P$, then Q is a ng_s-closed set.*

PROOF. Suppose that P is a ng_s-closed set. Then, n-$cl(P) \subseteq G$ whenever $P \subseteq G$, G is n-open and $P \subseteq n$-$cl(n$-$int(P))$. We now suppose that $Q \subseteq P$, Q is n-open and $P \subseteq G$, G is n-open. Then, we get n-$cl(Q) \subseteq G$ whenever $Q \subseteq G$, G is n-open. Thus, Q is a ng-closed set. As n-$cl(Q) \subseteq n$-$cl(n$-$int(Q))$, we also get $Q \subseteq n$-$cl(n$-$int(Q))$. Thus, Q is ns-open set. Hence Q is a ng_s-closed. □

THEOREM 2.4. *In a $(U,\mathcal{N})$, if Q is a n-clopen set then Q is ng_s-closed.*

PROOF. Suppose that Q is clopen set. Then, Q is ng-closed set and ns-open set. Hence Q is ng_s-closed. □

THEOREM 2.5. *In a $(U,\mathcal{N})$, if a ng_s-closed set Q is np-closed, then Q is nr-closed.*

PROOF. Suppose that the ng_s-closed set Q is np-closed set. Then, we get $Q \subseteq n$-$cl(n$-$int(Q))$ and n-$cl(n$-$int(Q)) \subseteq Q$. Thus $Q = n$-$cl(n$-$int(Q))$. Hence Q is nr-closed set. □

THEOREM 2.6. *In a $(U,\mathcal{N})$, every ng_s-closed set is nwg-closed.*

PROOF. Let Q be ng_s-closed. Then, n-$cl(Q) \subseteq G$ whenever $Q \subseteq G$ and G is n-open and $Q \subseteq n$-$cl(n$-$int(Q))$. Let $Q \subseteq G$, G is n-open. Then, n-$cl(n$-$int(Q)) \subseteq n$-$cl(Q) \subseteq G$. Hence Q is nwg-closed. □

THEOREM 2.7. *In a $(U,\mathcal{N})$, every ng_s-closed set is ngs-closed.*

PROOF. Let Q be a ng_s-closed . Then, n-$cl(Q) \subseteq G$ whenever $Q \subseteq G$ and G is n-open and $Q \subseteq n$-$cl(n$-$int(Q))$. Let $Q \subseteq G$, G is n-open. Since n-$scl(Q) \subseteq n$-$cl(Q)$, we get n-$scl(Q) \subseteq G$ whenever $Q \subseteq G$ and G is n-open. Hence Q is ngs-closed. □

THEOREM 2.8. *In a $(U,\mathcal{N})$, every ng_s-closed set is nrwg-closed.*

PROOF. Let Q be a ng_s-closed. Then, n-$cl(Q) \subseteq G$ whenever $Q \subseteq G$ and G is n-open and $Q \subseteq n$-$cl(n$-$int(Q))$. Let $Q \subseteq G$, Q is n-open. Then, n-$cl(n$-$int(Q)) \subseteq G$ whenever $Q \subseteq G$ and G is nr-open. Hence Q is $nrwg$-closed. □

REMARK 2.3. The following Diagram shows all the above discussions.

$$\begin{array}{ccccc} & & nwg\textbf{-closed} & & \\ & & \uparrow & & \\ ng\textbf{-closed} & \leftarrow & ng_s\textbf{-closed} & \rightarrow & nrwg\textbf{-closed} \\ & & \downarrow & & \\ & & ngs\textbf{-closed} & & \end{array}$$

3. On ng_s-continuous functions

DEFINITION 3.1. A function $f : U \to V$ is called:

(1) $nrwg$-continuous function if $f^{-1}(T)$ is a $nrwg$-closed set of U for every n-closed set T of V.

(2) ng_s-continuous function if $f^{-1}(T)$ is a ng_s-closed set of U for every n-closed set T of V.

REMARK 3.1. The following example shows that the composition of two ng_s-continuous functions but not ng_s-continuous.

EXAMPLE 3.1. Let $U = \{1, 2\}$ and $U' = \{1, 2, 3\} = U''$ and $U/R = \{\{1\}, \{2\}\}$, $U'/R = \{\{1\}, \{2, 3\}\} = U''/R$ with $X = \{1\}$ then $\mathcal{N} = \{\phi, \{1\}, U\}$. $Y = \{1, 2\}$ then $\mathcal{N}' = \{\phi, \{1, 2\}, U\}$. $Z = \{1, 3\}$ then $\mathcal{N}'' = \{\phi, \{1, 3\}, U\}$. Define $f : U \to U'$ by $f(1) = 1, f(2) = 3$ and $g : U' \to U''$ by $g(1) = 1, g(3) = 2$. Then, f and g are ng_s-continuous but the composition $g \circ f$ is not a ng_s-continuous.

THEOREM 3.1. *Let U, V and W be two nanotopological spaces. Let $f : U \to V$ be a ng_s-continuous and $g : V \to W$ be a continuous. Then, $g \circ f$ is ng_s-continuous.*

PROOF. Suppose that Q be a n-closed set in W. Then, $g^{-1}(Q)$ is a closed set in V and so $f^{-1}(g^{-1}(Q))$ is a ng_s-closed. That is, $(g \circ f)^{-1}(Q)$ is ng_s-closed. Hence $g \circ f$ is ng_s-continuous. □

THEOREM 3.2. *Every ng_s-continuous function is ng-continuous.*

PROOF. Suppose that $f : U \to V$ is a ng_s-continuous function and let Q be a n-closed set in V. Then, $f^{-1}(Q)$ is a ng_s-closed set in U. Then by the Definition of ng_s-closed set, $f^{-1}(Q)$ is ng-closed set in U. Hence f is a ng-continuous function. □

THEOREM 3.3. *Every ng_s-continuous function is nwg-continuous.*

PROOF. Suppose that $f : U \to V$ is a ng_s-continuous function and let Q be a n-closed set in V. Then, $f^{-1}(Q)$ is ng_s-closed set in U. Then by Theorem 2.6, $f^{-1}(Q)$ is nwg-closed set in U. Hence f is nwg-continuous function. □

THEOREM 3.4. *Every ng_s-continuous function is ngs-continuous.*

PROOF. Suppose that $f : U \to V$ is a ng_s-continuous function and let Q be a n-closed set in V. Then, $f^{-1}(Q)$ is ng_s-closed set in U. Then by Theorem 2.7, $f^{-1}(Q)$ is ngs-closed set in U. Hence f is ngs-continuous function. □

THEOREM 3.5. *Every ng_s-continuous function is $nrwg$-continuous.*

PROOF. Suppose that $f : U \to V$ is a ng_s-continuous function and let Q be a closed set in V. Then, $f^{-1}(Q)$ is ng_s-closed set in U. Then by Theorem 2.8, $f^{-1}(Q)$ is $nrwg$-closed set in U. Hence f is $nrwg$-continuous function. □

REMARK 3.2. The following Diagram shows all the above discussions.

$$\begin{array}{ccccc} & & nwg\textbf{-continuous} & & \\ & & \uparrow & & \\ ng\textbf{-continuous} & \leftarrow & ng_s\textbf{-continuous} & \rightarrow & nrwg\textbf{-continuous} \\ & & \downarrow & & \\ & & ngs\textbf{-continuous} & & \end{array}$$

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