BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., Vol. **11**(2)(2021), 395-402
DOI: 10.7251/BIMVI2102395P

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

A GENERAL COINCIDENCE AND FIXED POINT THEOREM FOR TWO PAIRS OF SELF MAPPINGS SATISFYING A COMMON COINCIDENCE RANGE PROPERTY IN PARTIAL METRIC SPACES

Valeriu Popa and Dan Popa

Abstract. In this paper a new type of common coincidence range property in partial metric space is introduced and a general fixed point theorem is proved. As applications, new results for the mappings satisfying a contractive condition of integral type and for the mappings satisfying a $\phi - contractive$ condition are obtained.

1. Introduction and Preliminaries

In 1994, Matthews [**16**] introduced the concept of partial metric spaces as a part of study of denotational semantics of dataflow networks and proved the Banach contraction principle in such spaces. Many authors have studied some contractive conditions in complete partial metric spaces in [**1, 4, 6, 13**] and other papers like [**2, 3**].

Definition 1.1. Let X be a nonempty set. A function $p : X \times X \to \Re_+$ is said to be a partial metric on X, if for any $x, y, z \in X$ the following conditions hold:
$(P1) : p(x, x) = p(y, y) = p(x, y)$ if and only if $x = y$;
$(P2) : p(x, x) \leqslant p(x, y)$;
$(P3) : p(x, y) = p(y, x)$;
$(P4) : p(x, z) \leqslant p(x, y) + p(y, z) - p(y, y)$.

2010 *Mathematics Subject Classification.* Primary 54H25; Secondary 47H10.
Key words and phrases. Coincidence point, fixed point, common coincidence range property, partial metric spaces, implicit relation.
Communicated by Stojan Radenović.

The pair,(X, p) is called a partial metric space ([**16**]).

If $p(x, y) = 0$, then (P1) and (P2) imply $x = y$, but the converse does not always hold.

In 2011, Sintunavarat and Kumam [**25**] introduced the notion of common limit range property for a pair of mappings. Also, Imdad et all [**11**], introduced the notion of joint common limit range property for two pairs of mappings. Other results for pairs of mappings satisfying common limit range property are obtained in [**9, 10, 22**] and other papers.

In all these papers and others on this topic, there exists some convergent sequences in X. We will introduce a new type of range property **without sequences**.

DEFINITION 1.2. Let (X, p) be a partial metric space and A, S, T be self mappings on (X, p). A pair (A, S) is said to have a coincidence range property with respect to T, denoted $CRP_{(A,S)T}$ -property, if there exists $z = Ax = Sx$ for some $x \in X$, with $z \in T(X)$ and $p(z, z) = 0$.

EXAMPLE 1.1. Let $X = [0, \infty)$ be a partial metric space with

$$p(x, y) = \max \{x, y\} \text{ and } Ax = 0,\ Sx = \frac{x}{x+1},\ Tx = x.$$

If $Ax = Sx$ then $x = 0$ and $z = 0 \in T(X) = [0, \infty)$ and $p(z, z) = p(0, 0) = 0$ Hence, (A, S) and T satisfy the $CRP_{(A,S)T}$-property.

DEFINITION 1.3. An altering distance, [**14**] is a mapping $\psi : [0, \infty) \to [0, \infty)$ which satisfies
$(\psi 1) : \psi$ is increasing and continuous,
$(\psi 2) : \psi(t) = 0$ if and only if $t = 0$.

Fixed point theorems involvings alterings distance have been studied in [**20, 25**] and other papers.

DEFINITION 1.4. A weak altering distance is a mapping $\psi : [0, \infty) \to [0, \infty)$ which satisfies
$(\psi 1) : \psi$ is increasing,
$(\psi 2) : \psi(t) = 0$ if and only if $t = 0$.

EXAMPLE 1.2. $\psi(t) = \begin{cases} t \text{ if } t \in [0, 1), \\ e^t \text{ if } t \in [1, \infty), \end{cases}$ is a weak altering distance and is not an altering distance.

Let X be a nonempty set and $A, S : X \to X$ two self mappings on X . A point $x \in$X is a coincidence point of A and S if $w = Ax = Sx$ for some $x \in X$. The set of all coincidence points of A and S is denoted by $C(A, S)$, and w is said to be a point of coincidence of A and S.

DEFINITION 1.5. Let X be a nonempty set and A and S be two self mappings on X. A and S are weakly compatible if $ASu = SAu$ for all $u \in C(A, S)$.

2. Implicit relations

Several fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit function in [**17, 18, 19**] and other papers.

Some fixed point theorems for pairs of mappings satisfying implicit relations in partial metric spaces are proved in [**7, 8, 21, 22, 26**] and other papers.

DEFINITION 2.1. We define F_{CP} being the set of all functions $F(t_1,..,t_6):\Re_+^6\to\Re$ satisfying the following conditions:
$(F1)$: F is non increasing in t_3,t_4,
$(F2)$: $F(t,0,0,t,t,0)>0,\forall t>0$,
$(F3)$: $F(t,t,0,t,t,t)>0,\forall t>0$,
$(F4)$: $F(t,t,t,0,t,t)>0,\forall t>0$.

EXAMPLE 2.1. $F(t_1,...,t_6)=t_1-k\ \max\{t_2,t_3,...,t_6\}$, where $k\in[0,1)$.

EXAMPLE 2.2. $F(t_1,...,t_6)=t_1-k\ \max\{t_2,t_3,t_4,\frac{t_5+t_6}{2}\}$, where $k\in[0,1)$.

EXAMPLE 2.3. $F(t_1,...,t_6)=t_1-k\ \max\{t_2,\frac{t_3+t_4}{2},\frac{t_5+t_6}{2}\}$, where $k\in[0,1)$.

EXAMPLE 2.4. $F(t_1,...,t_6)=t_1-a\,t_2-b\,t_3-c\,t_4-d\,t_5-e\,t_6$, where $a,b,c,d,e>0$ and $a+b+c+d+e<1$.

EXAMPLE 2.5. $F(t_1,...,t_6)=t_1-\max\{c\,t_2,c\,t_3,\ c\,t_4,a\,t_5+b\,t_6\}$, where $c\in(0,1)$. $a,b\geqslant 0$ and $a+b<1$.

EXAMPLE 2.6. $F(t_1,...,t_6)=t_1-a\,t_2-b\ \max\{t_3,t_4\}-c\ \max\{t_5,t_6\}$, where $a,b,c\geqslant 0$ and $a+b+c<1$.

EXAMPLE 2.7. $F(t_1,...,t_6)=t_1-\alpha\ \max\{t_2,t_3,t_4\}-(1-\alpha)(a\,t_5+b\,t_6)$ where $\alpha\in(0,1)$, $a,b\geqslant 0$ and $a+b<1$.

EXAMPLE 2.8. $F(t_1,...,t_6)=t_1^2-a\,t_2\,t_3-b\,t_3t_4-c\,t_4t_5-d\,t_5t_6$, where $a,b,c,d\geqslant 0$ and $a+b+c+d<1$.

The purpose of this paper is to prove a common fixed point theorem in partial metric spaces for two pairs (A,S) and (B,T) of self mapping satisfying some implicit relations having $CRP_{(A,S)T}$-property. As applications we obtain some results for mappings satisfying an integral condition and ϕ-contractive conditions.

3. Main results

THEOREM 3.1. *Let (X,p) be a partial metric space and A,B,S,T be self mappings on X such that for all $x,y\in X$*

$$F(\psi(p(Ax,By)),\psi(p(Sx,Ty)),\psi(p(Sx,Ax)),\psi(p(Ty,By)),\psi(p(Sx,By)),\psi(p(Ty,Ax))\leqslant 0 \tag{3.1}$$

for some $F\in F_{CP}$ and ψ is a weakly altering distance. If (A,S) and T satisfy $CRP_{(A,S)T}$-property, then $C(B,T)\neq\emptyset$. Moreover, if (A,S) and (B,T) are weakly compatibles then A,B,S,T have a unique common fixed point.

PROOF. Since (A, S) and T satisfies $CR_{(A,S)T}$ -property, then there exists $z = Av = Sv$ fo some $v \in X$, $z \in T(X)$ and $p(z, z) = 0$. Since $z \in T(X)$ there exists $u \in X$ such that $z = Tu$. Then, by (3.1)

$F(\psi(p(Av, Bu)), \psi(p(Sv, Tu)), \psi(p(Sv, Av)), \psi(p(Tu, Bu)), \psi(p(Sv, Bu)), \psi(p(Tu, Av))) \leqslant 0$

$F(\psi(p(z, Bu)), 0, 0, \psi(p(z, Bu)), \psi(p(z, Bu)), 0) \leqslant 0$ contradiction with (F_2) if $\psi(p(z, Bu)) > 0$. Hence, $\psi(p(z, Bu)) = 0$ which implies $z = Bu = Tu$ and $C(T, B) \neq \emptyset$.

Therefore, $z = Sv = Av = Tu = Bu$. Moreover, if (A, S) and (B, T) are weakly compatible, then $Sz = SAv = ASv = Az$ and $Tz = TBu = BTu = Bz$. By $(3, 1)$ we obtain

$F(\psi(p(Av, Bz)), \psi(p(Sv, Tz)), \psi(p(Sv, Av)), \psi(p(Tz, Bz)), \psi(p(Sv, Bz)), \psi(p(Tz, Av))) \leqslant 0,$

$F(\psi(p(z, Bz)), \psi(p(z, Bz)), 0, \psi(p(Bz, Bz)), \psi(p(z, Bz)), \psi(p(z, Bz)) \leqslant 0.$

By (F_1) we have

$F(\psi(p(z, Bz)), \psi(p(z, Bz)), 0, \psi(p(z, Bz)), \psi(p(z, Bz)), \psi(p(z, Bz)) \leqslant 0,$

a contradiction with (F_3) if $\psi(p(z, Bz)) > 0$. Hence, $\psi(p(z, Bz)) = 0$ which implies $z = Bz = Tz$ and z is a common fixed point of T and B.

Similarly, by (3.1) we obtain

$F(\psi(p(Az, Bu)), \psi(p(Sz, Tu)), \psi(p(Az, Sz)), \psi(p(Tu, Bu)), \psi(p(Sz, Bu)), \psi(p(Tu, Az)) \leqslant 0.$

$F(\psi(p(Az, z)), \psi(p(Az, z)), \psi(p(Az, Az)), 0, \psi(p(z, Az)), \psi(p(z, Az)) \leqslant 0.$

By (P_2), $p(Az, Az) \leqslant p(z, Az)$. By (F_1) we obtain

$F(\psi(p(Az, z)), \psi(p(Az, z)), \psi(p(Az, z)), 0, \psi(p(Az, z)), \psi(p(Az, z))) \leqslant 0$, a contradiction with (F_4) if $\psi(p(z, Az)) > 0$, hence, $\psi(p(z, Az)) = 0$. Which implies $z = Az = Sz = Bz = Tz$. Therefore, z is a common fixed point of A, S, B and T.

Suppose that $w \neq z$ is another common fixed point of A, B, S and T. Then by (3.1) we obtain

$F(\psi(p(Az, Bw)), \psi(p(Sz, Tw)), \psi(p(Sz, Az)), \psi(p(Tw, Bw)), \psi(p(Sz, Bw)), \psi(p(Az, Tw))) \leqslant 0.$

$F(\psi(p(z, w)), \psi(p(z, w)), 0, \psi(p(w, w)), \psi(p(z, w)), \psi(p(z, w))) \leqslant 0.$

By (P_2) $p(w, w) \leqslant p(w, z)$ which implies $\psi(p(w, w)) \leqslant \psi(p(w, z))$.

By (F_1) we obtain

$F(\psi(p(z, w)), \psi(p(z, w)), 0, \psi(p(z, w)), \psi(p(z, w)), \psi(p(z, w))) \leqslant 0$, a contradiction with (F_3) if $\psi(p(z, w)) > 0$, which implies $\psi(p(z, w)) = 0$ hence, $z = w$. Therefore, z is the unique common fixed point of A, B, S, T. □

If $\psi(t) = t$ by Theorem 3.1 we obtain

THEOREM 3.2. *Let (X, p) be a partial metric space and A, B, S, T be four self-mappings on X such that for for all $x, y \in X$*

$$F(p(Ax, By), p(Sz, Ty), p(Sx, Ax), p(Ty, By), p(Sx, By), p(Ax, Ty)) \leqslant 0, \tag{3.2}$$

for some $F \in F_{CP}$. If (A, S) and T satisfy $CRP_{(A,S)T}$-property then $C(B, T) \neq \emptyset$. Moreover, if A, S and B, T are weakly compatible, then A, S, B, T, have a unique common fixed point.

In order to apply this theorem we have to do the followings steps:

Step 1. Solve the equation $Sx = Ax$ on X and establish $C(A,S)$. If $C(A,S) = \emptyset$ the theorem is not applicable.

Step 2. If $C(A,S) \neq \emptyset$ we have to select z from $C(A,S)$ such that $z \in T(X)$ As a consequence, A, S, T satisfy the $CRP_{(A,S)T}$ property.

Step 3. Verify if the pairs (A,S) and (B,T) are weakly compatible. i.e., if one of those pairs are not weakly compatible, the theorem can not be applied. Stop.

Step 4. If the Relation 3.1 is satisfied then, by Theorem 3.1, A, S, B, T have a unique fixed point: z.

EXAMPLE 3.1. Let $X = [0, \infty)$ and $p(x,y) = \max\{x,y\}$, then (X,p) is a p-metric space. Let A, B, S, T be four self mappings on X: $Ax = 0, Sx = \frac{x}{x+2}, Bx = \frac{x}{3}, Tx = x$. If $Ax = Sx = z$ then $x = 0$ and $C(A,S) = \{0\}$. Then, $z = 0$, $z \in T(X) = X$ with $p(0,0) = 0$. Hence,(A,S) and T satisfy $CRP_{(A,S)T}$-property.

Moreover, $AS0 = SA0 = 0$, and $BT0 = TB0 = 0$ hence, (A,S) and (B,T) are weakly compatible. On the other hand, $p(Ax, By) = \max\{0, \frac{y}{3}\} = \frac{y}{3}$, $p(Ty, By) = \max\{y, \frac{y}{3}\} = y$, which implies, $p(Ax, By) \leqslant k\, p(Ty, By)$. The fact that $k \in [\frac{1}{3}, 1)$ implies $p(Ax, By) \leqslant k \max\{p(Sx,Ty), p(Sx,Ax), p(Ty,By), p(Sx,By), p(Ax,Ty)\}$, with $k \in [\frac{1}{3}, 1)$.

By Theorem 3.2, and Example 2.1, A, B, S and T have a unique common fixed point $z = 0$ with $p(z,z) = p(0,0) = 0$.

By Theorem 3.2 and Examples 2.2-2.8 we can obtain new particular results.

4. Applications

4.1. Coincidence and common fixed point for the mappings satisfying contractive conditions of integral type. In [**5**], Branciari established the folowing theorem which opened the way of the study of fixed point for the mappings satisfying a contractive condition of integral type.

THEOREM 4.1. *Let (X,d) be a complete metric space, $c \in (0,1)$, and $f : X \to X$ a mapping such that for all $x, y \in X$*

$$\int_0^{d(fx,fy)} h(t)dt \leqslant c \int_0^{d(x,y)} h(t)dt \tag{4.1}$$

where $h : [0,\infty) \to [0,\infty)$ is a Lebesgue measurable mapping which is summable, (i.e. with finite integral) on each compact subset of $[0,\infty)$ such that for $\epsilon > 0$, $\int_0^\epsilon h(t)dt > 0$. Then, f has a unique fixed point such that for all $x \in X$,

$$z = \lim_{n\to\infty} f^n(x) \tag{4.2}$$

LEMMA 4.1. *Let $h : [0,\infty) \to [0,\infty)$ as in Theorem 4.1 then $\psi(t) = \int_0^t h(x)dx$ is a weakly altering distance.*

PROOF. The proof follows by Lemma 2.5 from [**20**]. $\square$

THEOREM 4.2. *Let (X, p) be a partial metric space and A, B, S and T be self mappings on X such that for all $x, y \in X$*

$$F(\int_0^{p(Ax,By)} h(t)dt,\ \int_0^{p(Sx,Ty)} h(t)dt,\ \int_0^{p(Sx,Ax)} h(t)dt,\ \int_0^{p(Ty,By)} h(t)dt,\ \int_0^{p(Sx,By)} h(t)dt,\ \int_0^{p(Ty,Ax)} h(t)dt) \leqslant 0$$

where $h(t)$ is as in Theorem (4.1), for some $F \in F_{CP}$. If (A, S) and T satisfy $CRP_{(A,S)T}$-property, then $C(B, T) \neq \emptyset$. Moreover, if (A, S) and (B, T) are weakly compatible, then, A, B, S, T have a unique common fixed point.

PROOF. Let $\psi(t)$ be as in Lemma 4.1, then

$\psi(p(Ax, By)) = \int_0^{p(Ax,By)} h(t)dt, \qquad \psi(p(Sx, Ty)) = \int_0^{p(Sx,Ty)} h(t)dt,$

$\psi(p(Sx, Ax)) = \int_0^{p(Sx,Ax)} h(t)dt, \qquad \psi(p(By, Ty)) = \int_0^{p(By,Ty)} h(t)dt,$

$\psi(p(Sx, By)) = \int_0^{p(Sx,By)} h(t)dt, \qquad \psi(p(Ax, Ty)) = \int_0^{p(Ax,Ty)} h(t)dt.$

Then we obtain

$$F(\psi(p(Ax, By)),\ \psi(p(Sx, Ty)),\ \psi(p(Sx, Ax),\ \psi(p(Ty, By)),\ \psi(p(Sx, By),\ \psi(p(Ax, Ty))) \leqslant 0$$

which is Inequality (3.1) hence, the conditions of Theorem 3.1 are satisfied and Theorem 4.2 follows by Theorem 3.1. □

By Theorem 4.2 and Example 2.1 we obtain

THEOREM 4.3. *Let (X, p) be a partial metric space, A, B, S, T self mapping on X and $h(t)$ as in Theorem 4.1 such that for all $x, y \in X$*

$\int_0^{p(Ax,By)} \leqslant$

$k\ max\ \{ \int_0^{p(Sx,Ty)} h(t)dt, \int_0^{p(Sx,Ax)} h(t)dt, \int_0^{p(By,Ty)} h(t)dt, \int_0^{p(Sx,By)} h(t)dt, \int_0^{p(Ax,Ty)} h(t)dt\}.$

If (A, S) and T satisfy $CRP_{(A,S)T}$-property, then $C(B, T) \neq \emptyset$. Moreover, if (A, S) and (B, T) are weakly compatible, then, A, B, S, T have a unique common fixed point.

By Theorem 4.2 and Examples 2.2-2.8 we obtain many new particular results.

4.2. Coincidence and common fixed point for the mappings satisfying φ -contractive condition. As in [**15**], let Φ be the set of nondecreasing continuous function $\varphi : [0, \infty) \to [0, \infty)$ with $\lim_{n\to\infty} \varphi^n(t) = 0$ for $t \in [0, \infty)$. If $\varphi \in \Phi$ then

$(\Phi 1) : \varphi(t) < t$ for $t > 0$,

$(\Phi 2) : \varphi(0) = 0$.

In the following we denote by Φ_C the set of all nondecreasing functions satisfying condition $(\Phi 1)$ and $(\Phi 2)$.

The following function $F(t_1, ..., t_6) : \Re_+ \to \Re$ satisfying condition $(F_1) - (F_4)$.

EXAMPLE 4.1. $F(t_1, ..., t_6) = t_1 - \varphi(\max \{t_2, t_3, t_4, t_5, t_6)\})$.

EXAMPLE 4.2. $F(t_1, ..., t_6) = t_1 - \varphi(\max \{t_2, t_3, t_4, \frac{t_5+t_6}{2})\})$.

Example 4.3. $F(t_1, ..., t_6) = t_1 - \varphi(\max\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2})\})$.

Example 4.4. $F(t_1, ..., t_6) = t_1 - \varphi(\max\{t_2, \sqrt{t_3t_4}, \sqrt{t_3t_5}, \sqrt{t_4t_6}, \sqrt{t_5t_6})\})$.

Example 4.5. $F(t_1, ..., t_6) = t_1 - \varphi(at_2 + bt_3 + ct_4 + dt_5 + et_6)$ where $a, b, c, d, e \geqslant 0$ and $a + b + c + d + e \leqslant 1$.

Example 4.6. $F(t_1, ..., t_6) = t_1 - \varphi(at_2 + b\max\{t_3, t_4\} + c\max\{t_5, t_6\})$ where $a, b, c \geqslant 0$ and $a + b + c \leqslant 1$.

By Theorem 3.2 and Example 4.1 we obtain:

Theorem 4.4. *Let A, B, S, T be self mapping on a partial metric space such that for all $x, y \in X$*
$p(Ax, By) \leqslant \varphi(max\{p(Sx, Ty), p(Sx, Ax), p(Ty, By), p(Sx, By), p(Ax, Ty)\}\ )$ *where $\varphi \in \Phi_C$. If (A, S) and T satisfy $CRP_{(A,S)T}$-property, then $C(B, T) \neq \emptyset$. Moreover, if (A, S) and (B, T) are weakly compatible, then, A, B, S, T have a unique common fixed point.*

By Theorem 3.2 and Examples 4.2 - 4.6 we obtain new particular results.

Acknowledgments The authors are grateful to the referee for his/her valuable comments and suggestions for the paper.

References

[1] T. Abdeljawad, E. Karapinar and K. Ta. Existence and uniqueness of common fixed point on a partial metric space. *Appl. Math. Lett.*, **24**(11)(2011), 1900–1904.

[2] A. G. B. Ahmad, Z. M. Fadail, V. Ćojbaši- Rajić and S. Radenović. Nonlinear contractions in 0-complete partial metric spaces. *Abstr. Appl. Anal.*, Volume **2012**, Article ID 451239.

[3] A. G. B. Ahmad, Z. M. Fadail, H. K. Nashine, Z. Kadelburg and S. Radenović. Some new common fixed point results through generalized altering distances on partial metric spaces. *Fixed Point Theory Appl* Volume **2012**, ID Article 120.

[4] I. Altun, F. Sola and H. Simsek. Generalized contractions in partial metric space. *Topology Appl.*, **157**(18)(2010), 2778–2785.

[5] A. Branciari. A fixed point theorem for mappings satisfying a general contractive condition of integral type. *Int. J. Math. Math. Sci.*, **29**(9)(2002), 531-536.

[6] R. Chi, R. Karapinar and T. Thanah. K. P. Chi, E. Karapnar and T. D. Thanha. A generalized contraction principle in partial metric spaces. *Math. Comput. Modelling*, **55**(5-6)(2012), 1673–1681.

[7] S. Güliaz and E. Karapinar. Coupled fixed point in partial metric spaces through implicit function. *Hacet. J. Math. Stat.*, **42**(4)(2013), 347–357.

[8] S. Güliaz, E. Karapinar and I. Yüce. A coupled common point theorem in partial metric spaces with an implicit relation. *Fixed Point Theory Appl.*, Volume **2013**;Article 38.

[9] M. Imdad and S. Chauhan. Employing common limit range property to prove unified metrical fixed point theorem. *Int. J. Anal.*, Volume **2013**, Article ID 763261.

[10] M. Imdad, A. S. Sharma and S. Chauhan. Unifying a multitude of metrical fixed point theorem in symmetryc spaces. *Filomat*, **28**(6)(2014), 1113–1132.

[11] M. Imdad, S. Chauhan and P. Kumam. Fixed point theorem for two pairs of nonself mappings under joint common limit range property in metric spaces. *J. Nonlinear Convex Anal.*, **16**(2)(2015), 243–254.

[12] G. Jungck. Common fixed points for noncommuting nonself mappings on nonnumeric spaces. *Far East J. Math. Sci.*, **4**(2)(1996), 195–215.

[13] Z. Kadelburg, H. K. Nashine and S. Radenović. Fixed point rezults under various contractive conditions in partial metric spaces. *Rev. R. Acad. Cienc. Exactas Fs. Nat., Ser. A Mat., RACSAM* **107**(2)(2013), 241–256.

[14] M. S. Khan, M. Swaleh and R. Kumar. Fixed point theorem by altering distance between points. Bull. Austral. Math. Soc. **30**(1)(1984), 1–9.

[15] J. Matkowski. Fix point theorem for mappings with a contractive iterate point. *Proc. Am. Math. Soc.*, **62**(2)(1977), 344–348.

[16] S. G. Matthews. Partial metric topology. In S. Andima et.al.)Eds.). *Papers on General Topology and Applications, Eighth Summer Conference at Queens College*. Annals of the New York Academy of Sciences, (vol. 728, pp. 183–197). New York Acad. Sci. 1994.

[17] V. Popa. Fixed point theorem for implicit contractive mapping. *Stud. Cerc. Sti. Ser. Mat. Univ. Bacău*, **7**(1997), 127–134.

[18] V. Popa. Some fixed point theorems for compatible mappings satisfying an implicit relation. *Demonstr. Math.*, **32**(1)(1999), 157–163.

[19] V. Popa. Common fixed point theorem for compatible mapping of type (A) satisfying an implicit relation. *Stud. Cerc. Sti. Ser. Mat. Univ. Bacău*, **9**(1999), 165–163.

[20] V. Popa and M. Mocanu. Altering distance and common fixed point under implicit relation. *Hacet. J. Math. Stat.* **38**(3)(2009), 329–332.

[21] V. Popa and A.-M. Patriciu. A general fixed point theorem for a pair of mapping in partial metric spaces. *Acta Univ. Apulensis, Math. Inform.*, **43**(2015), 93–103.

[22] V. Popa and A.-M. Patriciu. A general fixed point theorem for a pair of self mappings with a common limit range property in partial metric spaces. *Bul. Inst. Politeh. Iai, Sec. I, Mat. Mec. Teor. Fiz.*, **61**(65)(3)(2015), 85–99.

[23] B. E. Rhoades. Two fixed point theorems for mappings satisfying a general contractive condition of integral type. *Int. J. Math. Math. Sci.*, **63**(2003), 4007–4013.

[24] P. K. Sastri and G. V. R. Babu. Fixed point theorem in metric space by altering distances. *Bull. Calcutta Math. Soc.*, **90**(3)(1998), 175–182.

[25] W. Sintunavarat and P. Kumam. Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces. *J. Appl. Math.*, Volume **2011**, Article ID 637958.

[26] C. Vetro and F. Vetro. Common fixed point of mappings satisfying implicit relation in partial metric spaces. *J. Nonlinear Sci. Appl.*, **6**(3)(2013), 156–161.

Received by editors 03.02.2021; Revised version 12.02.2021; Available online 01.03.2021.

Department of Mathematics, Vasile Alecsandri University of Bacău,157 Calea Mărăşeşti, Bacău, 600115, România,,

E-mail address: vpopa@ub.ro

Department of Mathematics, Vasile Alecsandri University of Bacău,157 Calea Mărăşeşti, Bacău, 600115, România,,

E-mail address: danvpopa@ub.ro