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REMODELLED SIGMOID FUNCTION IN THE SPACE OF UNIVALENT FUNCTIONS

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Abstract. In this paper remodelled sigmoid function in the mirror of univalent functions was investigated using the well-known differential operator. The growth and distortion theorems were obtained. Also, application of fractional calculus to this class of functions was established.

1. Introduction

In recent time, it was established in [**6**] that the sigmoid function has a lot of roles to play in geometric functions theory. Out of the three known activation functions, sigmoid function is mostly engaged in Artificial Neural Network because of its gradient descendent learning algorithm. The function is differentiable, increases monotonically, maps a very large input domain to a small range of outputs, never loses information and also output real numbers between 0 and 1. The function has different ways of evaluation but the most effective one is the truncated series expansion. For more details see [**8, 10**].

Let the class of Caratheodory function $\mathcal{P}$ be of the form

$$p(z) = 1 + d_1 z + d_2 z^2 + d_3 z^3 + ...$$

having positive real part and analytic in $\mathbb{U} = \{z : |z| < 1\}$. Then the following sharp estimate holds

$$|d_n| \leqslant 2 \qquad (n = 1, 2, 3, ...) .$$

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Represent A by the class of functions $f(z)$ of the form

$$f\left(z\right)=z+\sum_{t=2}^{\infty}a_{t}z^{t} \tag{1.1}$$

which are analytic in the open disk $\mathbb{U}$ satisfying the conditions $f(0)=f^{'}(0)-1=0$. Recall that, S denote the family of all functions in A which are univalent in $\mathbb{U}$. Many authors such as in [**3, 5, 11, 14**] and other literature established that the subclasses of A are convex and starlike functions.

Let f and g be two analytic functions in $\mathbb{U}$, the function $f(z)$ is subordinate to $g(z)$ in $\mathbb{U}$, written as

$$f(z)\prec g(z)$$

($z\in\mathbb{U}$) if there exists a Schwartz function $w(z)$ analytic in $\mathbb{U}$ with $w(0)=0$ and $|w(z)|<1$ ($z\in\mathbb{U}$) such that

$$f(z)=g(w(z)).$$

Particularly, if the function g is univalent in $\mathbb{U}$, the above subordination is equivalent to $f(0)=g(0)$ and $f(\mathbb{U})\subset g(\mathbb{U})$. ([**9**]).

The differential operator $D^{n}f, n\in N_{0}=0,1,2,3,\ldots$ for functions $f(z)$ belonging to class A of analytic functions in the unit disk $\mathbb{U}$:

$$D^{n}f(z)=z+\sum_{t=2}^{\infty}t^{n}a_{t}z^{t};\quad n\in N_{0}$$

see details in [**13**].

The harmonic function which satisfies

$$Re\left(\frac{D^{n+1}f(z)^{\phi}}{D^{n}f(z)^{\phi}}\right)>\alpha$$

was introduced by [**2**] where $\phi\geqslant 1$ and $0\leqslant\alpha<1$. Specializing the parameters involved, different classes of functions which have been studied repeatedly by many scholars would be obtained. See [**1, 4, 13**] for details.

Researchers in [**7**] extended Sălăgean differential operator by the product of two functions (Convolution) to obtain

$$D^{n}f_{\gamma}(z)=\gamma^{n}(s)z+\sum_{t=2}^{\infty}\gamma^{n+1}(s)t^{n}a_{t}z^{t},$$

where

$$\gamma(s)=\frac{2}{1+e^{-s}}=1+\frac{1}{2}s-\frac{1}{24}s^{3}+\frac{1}{240}s^{5}\ldots$$

and

$$f_{\gamma}(z)=z+\sum_{t=2}^{\infty}\gamma(s)a_{t}z^{t} \tag{1.2}$$

which is referred to as the modified Sălăgean differential operator involving sigmoid function.

REMARK 1.1. The function $f_\gamma \in A_\gamma$ we have $\lim_{s\to 0}\gamma(s) = 1$. If $\gamma = 1$, then $A_\gamma = A_1 = A$.

Motivated by the earlier work done in [**7**] and [**12**], remodelled sigmoid function in the space of univalent functions was critically studied in this work. The authors were able to investigate the growth and distortion theorems. Moreso, the application of fractional calculus to the class of functions was considered.

For the purpose of this work, the definition below is very important.

Definition A. A function $f_\gamma \in A_\gamma$, defined by (1.2), belongs to the class

$$M_\gamma(n,\lambda,\phi,\alpha,\mu,\delta,\psi)$$

where $0 \leqslant \lambda \leqslant 1$, $\phi > 0$, $n \in N_0$, $0 \leqslant \alpha < 1$, $\mu \geqslant 1$, $\frac{1}{2} \leqslant \delta \leqslant 1$ and $0 < \psi \leqslant 1$ if

$$Re\left(\frac{\frac{(1-\lambda)D^{n+1}f_\gamma(z)^\phi+\lambda D^{n+2}f_\gamma(z)^\phi}{\lambda D^{n+1}f_\gamma(z)^\phi+(1-\lambda)D^nf_\gamma(z)^\phi}-\mu}{\left(\frac{(1-\lambda)D^{n+1}f_\gamma(z)^\phi+\lambda D^{n+2}f_\gamma(z)^\phi}{\lambda D^{n+1}f_\gamma(z)^\phi+(1-\lambda)D^nf_\gamma(z)^\phi}+\psi\right)-2\delta\left(\frac{(1-\lambda)D^{n+1}f_\gamma(z)^\phi+\lambda D^{n+2}f_\gamma(z)^\phi}{\lambda D^{n+1}f_\gamma(z)^\phi+(1-\lambda)D^nf_\gamma(z)^\phi}-\mu\right)}\right) > \alpha \tag{1.3}$$

2. Main Results

2.1. Growth and Distortion theorem for the class $M_\gamma(n,\lambda,1,\alpha,\mu,\delta,\psi)$.

THEOREM 2.1. *Let $f_\gamma \in M_\gamma(n,\lambda,1,\alpha,\mu,\delta,\psi)$ where $0 \leqslant \lambda \leqslant 1$, $\phi > 0$, $n \in N_0$, $0 \leqslant \alpha < 1$, $\mu \geqslant 1$, $\frac{1}{2} \leqslant \delta \leqslant 1$ and $0 < \psi \leqslant 1$. Then*

$$1-\gamma(s)\frac{(1+\lambda(\gamma(s)-1))[\gamma(s)-\mu-\alpha((1-2\delta)\gamma(s)+\psi+2\delta\mu)]}{\gamma(s)(1+\lambda(2\gamma(s)-1))[\alpha((1-2\delta)(2\gamma(s)+\psi+2\delta\mu]-2\gamma(s)+\mu)}2^{1-n}r$$
$$\leqslant |f_\gamma'(z)| \leqslant 1+\gamma(s)\frac{(1+\lambda(\gamma(s)-1))[\gamma(s)-\mu-\alpha((1-2\delta)\gamma(s)+\psi+2\delta\mu)]}{\gamma(s)(1+\lambda(2\gamma(s)-1))[\alpha((1-2\delta)(2\gamma(s)+\psi+2\delta\mu]-2\gamma(s)+\mu)}2^{1-n}r.$$

PROOF. Suppose f_γ belongs to the class $M_\gamma(n,\lambda,\phi,\alpha,\mu,\delta,\psi)$, then by (1.3)

$$\left|\frac{\frac{(1-\lambda)D^{n+1}f_\gamma(z)^\phi+\lambda D^{n+2}f_\gamma(z)^\phi}{\lambda D^{n+1}f_\gamma(z)^\phi+(1-\lambda)D^nf_\gamma(z)^\phi}-\mu}{\left(\frac{(1-\lambda)D^{n+1}f_\gamma(z)^\phi+\lambda D^{n+2}f_\gamma(z)^\phi}{\lambda D^{n+1}f_\gamma(z)^\phi+(1-\lambda)D^nf_\gamma(z)^\phi}+\psi\right)-2\delta\left(\frac{(1-\lambda)D^{n+1}f_\gamma(z)^\phi+\lambda D^{n+2}f_\gamma(z)^\phi}{\lambda D^{n+1}f_\gamma(z)^\phi+(1-\lambda)D^nf_\gamma(z)^\phi}-\mu\right)}\right| > \alpha \tag{2.1}$$

Then, equation (2.1) gives

$$\left|\frac{(\gamma(s)\phi-\mu)\,z^\phi+\sum_{t=\phi+1}^\infty\frac{\gamma(s)t^n(\gamma(s)k-\mu)(1+\lambda(\gamma(s)t-1))}{\phi^n(1+\lambda(\gamma(s)\phi-1))}a_tz^t}{((1-2\delta)\gamma(s)\phi+\psi+2\delta\mu)\,z^\phi+\sum_{t=\phi+1}^\infty\frac{\gamma(s)t^n((1-2\delta)\gamma(s)t+\psi+2\delta\mu)(1+\lambda(\gamma(s)t-1))}{\phi^n(1+\lambda(\gamma(s)\phi-1))}a_tz^t}\right| > \alpha \tag{2.2}$$

Further simplification of (2.2) yields

$$((1-2\delta)\gamma(s)\phi+\psi+2\delta\mu)\,|z^\phi|$$
$$+\sum_{t=\phi+1}^\infty\frac{\gamma(s)t^n((1-2\delta)\gamma(s)t+\psi+2\delta\mu)(1+\lambda(\gamma(s)t-1))}{\phi^n(1+\lambda(\gamma(s)\phi-1))}|a_t||z^t|$$
$$<\alpha\left\{(\gamma(s)\phi-\mu)\,|z^\phi|+\sum_{k=\phi+1}^\infty\frac{\gamma(s)t^n(\gamma(s)t-\mu)(1+\lambda(\gamma(s)t-1))}{\phi^n(1+\lambda(\gamma(s)\phi-1))}|a_t||z^t|\right\}.$$

which leads to

$$\frac{\sum_{t=\phi+1}^\infty\gamma(s)t^n(1+\lambda(\gamma(s)t-1))[\alpha((1-2\delta)(\gamma(s)t+\psi+2\delta\mu]-\gamma(s)k+\mu)}{\phi^n(1+\lambda(\gamma(s)\phi-1))}|a_t||z^t|$$
$$<\left[\gamma(s)\,\phi-\mu-\alpha\left((1-2\delta)\gamma(s)\,\phi+\psi+2\delta\mu\right)|z^\phi|\right]. \tag{2.3}$$

Since $|z| = r < 1$, as $|z| \to 1$, (2.3) gives
(2.4)
$$\sum_{t=\phi+1}^{\infty} \gamma(s)t^n\left(1+\lambda(\gamma(s)t-1)\right)\left[\alpha\left((1-2\delta)(\gamma(s)t+\psi+2\delta\mu\right]-\gamma(s)t+\mu\right)|a_t| \\ < \phi^n\left(1+\lambda(\gamma(s)\phi-1)\right)\left[\gamma(s)\phi-\mu-\alpha\left((1-2\delta)\gamma(s)\phi+\psi+2\delta\mu\right)\right]$$

From (2.4), we can deduce that

$$|a_t| < \frac{\phi^n(1+\lambda(\gamma(s)\phi-1))[\gamma(s)\phi-\mu-\alpha((1-2\delta)\gamma(s)\phi+\psi+2\delta\mu)]}{\gamma(s)t^n(1+\lambda(\gamma(s)t-1))[\alpha((1-2\delta)(\gamma(s)t+\psi+2\delta\mu]-\gamma(s)t+\mu)} \tag{2.5}$$

which equality holds for the function
(2.6)
$$f_\gamma(z)^\phi = z^\phi + \frac{\phi^n\left(1+\lambda(\gamma(s)\phi-1)\right)\left[\gamma(s)\phi-\mu-\alpha\left((1-2\delta)\gamma(s)\phi+\psi+2\delta\mu\right)\right]}{\gamma(s)t^n\left(1+\lambda(\gamma(s)t-1)\right)\left[\alpha\left((1-2\delta)(\gamma(s)t+\psi+2\delta\mu\right]-\gamma(s)t+\mu\right)} z^t.$$

If $f_\gamma \in A_\gamma$ belongs to the class $M_\gamma(n, \lambda, 1, \alpha, \mu, \delta, \psi)$ $0 \leqslant \lambda \leqslant 1$, $\phi > 0$, $n \in N_0$, $0 \leqslant \alpha < 1$, $\mu \geqslant 1$, $\frac{1}{2} \leqslant \delta \leqslant 1$ and $0 < \psi \leqslant 1$ then,

$$|a_t| < \frac{(1+\lambda(\gamma(s)-1))[\gamma(s)-\mu-\alpha((1-2\delta)\gamma(s)+\psi+2\delta\mu)]}{\gamma(s)t^n(1+\lambda(\gamma(s)t-1))[\alpha((1-2\delta)(\gamma(s)t+\psi+2\delta\mu]-\gamma(s)t+\mu)} \tag{2.7}$$

equality holds for the function
(2.8)
$$f_\gamma(z) = z + \frac{\left(1+\lambda(\gamma(s)-1)\right)\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{\gamma(s)t^n\left(1+\lambda(\gamma(s)t-1)\right)\left[\alpha\left((1-2\delta)(\gamma(s)t+\psi+2\delta\mu\right]-\gamma(s)t+\mu\right)} z^t.$$

Now, differentiating (1.2) to obtain

$$f'_\gamma(z) = 1 + \sum_{t=2}^{\infty} \gamma(s)t a_t z^{t-1} \tag{2.9}$$

which implies that

$$|f'_\gamma(z)| \leqslant 1 + \sum_{t=2}^{\infty} \gamma(s)t|a_t||z|^{t-1} \tag{2.10}$$

that is

$$|f'_\gamma(z)| \leqslant 1 + r\sum_{t=2}^{\infty} \gamma(s)t|a_t|. \tag{2.11}$$

From (2.7) and (2.11), we get

$$|f'_\gamma(z)| \leqslant 1 + \gamma(s)\frac{2\left(1+\lambda(\gamma(s)-1)\right)\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{2^n\gamma(s)\left(1+\lambda(2\gamma(s)-1)\right)\left[\alpha\left((1-2\delta)(2\gamma(s)+\psi+2\delta\mu\right]-2\gamma(s)+\mu\right)} r$$

and

$$|f'_\gamma(z)| \geqslant 1 - \gamma(s)\frac{2\left(1+\lambda(\gamma(s)-1)\right)\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{2^n\gamma(s)\left(1+\lambda(2\gamma(s)-1)\right)\left[\alpha\left((1-2\delta)(2\gamma(s)+\psi+2\delta\mu\right]-2\gamma(s)+\mu\right)} r$$

Hence,

$$1 - \gamma(s)\frac{(1+\lambda(\gamma(s)-1))[\gamma(s)-\mu-\alpha((1-2\delta)\gamma(s)+\psi+2\delta\mu)]}{\gamma(s)(1+\lambda(2\gamma(s)-1))[\alpha((1-2\delta)(2\gamma(s)+\psi+2\delta\mu]-2\gamma(s)+\mu)} 2^{1-n} r \\ \leqslant |f'_\gamma(z)| \leqslant 1 + \gamma(s)\frac{(1+\lambda(\gamma(s)-1))[\gamma(s)-\mu-\alpha((1-2\delta)\gamma(s)+\psi+2\delta\mu)]}{2^n\gamma(s)(1+\lambda(2\gamma(s)-1))[\alpha((1-2\delta)(2\gamma(s)+\psi+2\delta\mu]-2\gamma(s)+\mu)} 2^{1-n} r. \qquad \square$$

2.2. Application of Fractional Calculus. In other to work on the application of fractional calculus with the class $M_\gamma(n, \lambda, 1, \alpha, \mu, \delta, \psi)$, the following definitions need to be stated.

Definition B. ([**4**]) The fractional integral of order v is defined by

$$D_z^{-v} f(z) = \frac{1}{\Gamma(v)} \int_0^z \frac{f(u)}{(z-u)^{1-v}} du$$

where $v > 0$. $f(z)$ is an analytic function in a simply connected region of $z-$ plane containing the origin and multiplicity of $(z-u)^{v-1}$ is removed by requiring $log(z-u)$ to be real when $(z-u) > 0$.

Definition C. ([**4**]) The fractional derivative of order v is defined by

$$D_z^{v} f(z) = \frac{1}{\Gamma(1-v)} \frac{d}{dz} \int_0^z \frac{f(u)}{(z-u)^{v}} du$$

where $0 \leqslant v < 1$, $f(z)$ is analytic in a simply connected region of $z-$ plane containing the origin and multiplicity of $(z-u)^{-v}$ is removed as in definition B.

Definition D. ([**4**]) Under the conditions of definition C, the fractional derivative of order $n+v$ is defined by

$$D_z^{n+v} f(z) = \frac{d^n}{dz^n} D_z^v f(z),$$

where $0 \leqslant v < 1$ and $n = 1, 2, \ldots$.

THEOREM 2.2. *Let the function* $f(z)$, *defined by* (1.1), *belongs to the class* $M_\gamma(n, \lambda, 1, \alpha, \mu, \delta, \psi)$ *where* $0 \leqslant \lambda \leqslant 1$, $\phi > 0$, $n \in N_0$, $0 \leqslant \alpha < 1$, $\mu \geqslant 1$, $\frac{1}{2} \leqslant \delta \leqslant 1$ *and* $0 < \psi \leqslant 1$. *Then*

$$\left|D_z^{-v} f(z)\right| \leqslant \frac{|z|^{1+v}}{\Gamma(2+v)} \left(1 + \frac{2\left(1 + \lambda(\gamma\left(s\right) - 1)\right)\left[\gamma\left(s\right) - \mu - \alpha\left((1-2\delta)\gamma\left(s\right) + \psi + 2\delta\mu\right)\right]}{(2+v)2^n \gamma(s)\left(1 + \lambda(2\gamma\left(s\right) - 1)\right)\left[\alpha\left((1-2\delta)(2\gamma\left(s\right) + \psi + 2\delta\mu\right] - 2\gamma\left(s\right) + \mu\right)} |z|\right)$$

PROOF.

$$\sum_{t=2}^{\infty} |a_t| \leqslant \frac{\phi^n \left(1 + \lambda(\gamma\left(s\right)\phi - 1)\right)\left[\gamma\left(s\right)\phi - \mu - \alpha\left((1-2\delta)\gamma\left(s\right)\phi + \psi + 2\delta\mu\right)\right]}{\gamma(s) t^n \left(1 + \lambda(\gamma\left(s\right) t - 1)\right)\left[\alpha\left((1-2\delta)(\gamma\left(s\right) t + \psi + 2\delta\mu\right] - \gamma\left(s\right) t + \mu\right)}$$

by definition B

$$D_z^{-v} f(z) = \frac{1}{\Gamma(2+v)} z^{1+v} + \sum_{k=2}^{\infty} \frac{\Gamma(k+1)}{\Gamma(k+v+1)} a_k z^{k+v}$$

and

$\Gamma(2+v) z^{-v} D_z^{-v} f(z) = \Gamma(2+v) z^{-v} \left[\frac{1}{\Gamma(2+v)} z^{1+v} + \sum_{t=2}^{\infty} \frac{\Gamma(k+1)}{\Gamma(t+v+1)} a_t z^{t+v}\right]$

$= z + \sum_{t=2}^{\infty} \frac{\Gamma(t+1)\Gamma(v+2)}{\Gamma(t+v+1)} a_t z^{t+v}$.

Let $\Psi(t) = \frac{\Gamma(t+1)\Gamma(v+2)}{\Gamma(t+v+1)}$, we get that $\Psi(t)$ is decreasing for a univalent function of k and $0 < \Psi(t) \leqslant \Psi(2)$ where,

$$\Psi(2) = \frac{\Gamma(3)\Gamma(v+2)}{\Gamma(2+v+1)} = \frac{2\Gamma(v+2)}{(2+v)\Gamma(v+2)} = \frac{2}{2+v}$$

so that
$|\Gamma(2+v)z^{-v}D_z^{-v}f(z)| = |z+\sum_{t=2}^{\infty}\Psi(t)a_tz^t| = |z+\Psi(2)z^2\sum_{t=2}^{\infty}a_t|$
$\leqslant |z|+\Psi(2)|z|^2\sum_{t=2}^{\infty}|a_t|\,|\Gamma(2+v)z^{-v}D_z^{-v}f(z)|$
$\leqslant |z|+\frac{2}{2+v}\frac{(1+\lambda(\gamma(s)-1))[\gamma(s)-\mu-\alpha((1-2\delta)\gamma(s)+\psi+2\delta\mu)]}{2^n\gamma(s)(1+\lambda(2\gamma(s)-1))[\alpha((1-2\delta)(2\gamma(s)+\psi+2\delta\mu]-2\gamma(s)+\mu)}|z|^2$
$= |z|+\frac{2(1+\lambda(\gamma(s)-1))[\gamma(s)-\mu-\alpha((1-2\delta)\gamma(s)+\psi+2\delta\mu)]}{(2+v)2^n\gamma(s)(1+\lambda(2\gamma(s)-1))[\alpha((1-2\delta)(2\gamma(s)+\psi+2\delta\mu]-2\gamma(s)+\mu)}|z|^2$
$|D_z^{-v}f(z)| \leqslant \frac{|z|^{1+v}}{\Gamma(2+v)}\left(1+\frac{2(1-\lambda+\lambda\gamma(s))(\gamma(s)-\alpha)}{(2+v)2^n\gamma(s)(2\lambda\gamma(s)+1-\lambda)(\alpha-2\gamma(s))}|z|\right).$
which completes the proof. □

THEOREM 2.3. *Let the function $f(z)$ defined by (1.1) belongs to the class $M_\gamma(n,\lambda,\phi,\alpha)$, then*

$$|D_z^vf(z)| \leqslant \frac{|z|^{1-v}}{\Gamma(2-v)}\left(1+\frac{(1+\lambda(\gamma(s)-1))\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{(2-v)\gamma(s)t^n\left(1+\lambda(\gamma(s)t-1)\right)\left[\alpha\left((1-2\delta)(\gamma(s)t+\psi+2\delta\mu\right]-\gamma(s)t+\mu\right)}|z|\right).$$

PROOF. From (2.4) and subject to the condition of the parameters, we have

$$\sum_{t=2}^{\infty}|a_t| \leqslant \frac{(1+\lambda(\gamma(s)-1))\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{\gamma(s)t^n\left(1+\lambda(\gamma(s)t-1)\right)\left[\alpha\left((1-2\delta)(\gamma(s)t+\psi+2\delta\mu\right]-\gamma(s)t+\mu\right)}$$

$$\sum_{t=2}^{\infty}t|a_t| \leqslant \frac{t\left(1+\lambda(\gamma(s)-1)\right)\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{\gamma(s)t^n\left(1+\lambda(\gamma(s)t-1)\right)\left[\alpha\left((1-2\delta)(\gamma(s)t+\psi+2\delta\mu\right]-\gamma(s)t+\mu\right)}.$$

In particular,

$$2|a_2| \leqslant \frac{2\left(1+\lambda(\gamma(s)-1)\right)\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{2^n\gamma(s)\left(1+\lambda(2\gamma(s)-1)\right)\left[\alpha\left((1-2\delta)(2\gamma(s)+\psi+2\delta\mu\right]-2\gamma(s)+\mu\right)}.$$

Also, using definition C,

$$D_z^vf(z) = \frac{1}{\Gamma(1-v)}\frac{d}{dz}\int_0^z\frac{f(t)}{(z-t)^v}dt$$

then,

$$D_z^vf(z) = \frac{1}{\Gamma(2-v)}z^{1-v}+\sum_{t=2}^{\infty}\frac{\Gamma(t+1)}{\Gamma(t-v+1)}a_tz^{t-v}$$

$$z^vD_z^vf(z) = \frac{1}{\Gamma(2-v)}z+\sum_{t=2}^{\infty}\frac{\Gamma(t+1)}{\Gamma(t-v+1)}a_tz^t$$

$$\Gamma(2-v)z^vD_z^vf(z) = z+\sum_{t=2}^{\infty}\frac{\Gamma(t+1)\Gamma(2-v)}{\Gamma(t-v+1)}a_tz^t = z+\sum_{t=2}^{\infty}\frac{t\Gamma(t)\Gamma(2-v)}{\Gamma(t-v+1)}a_tz^t$$

$$= z+\sum_{t=2}^{\infty}\frac{\Gamma(k)\Gamma(2-v)}{\Gamma(t-v+1)}ta_tz^t = z+\sum_{t=2}^{\infty}\Phi(t)ta_tz^t$$

where, $\Phi(t) = \frac{\Gamma(t)\Gamma(2-v)}{\Gamma(t-v+1)}$. Thus,

$$\Phi(2) = \frac{\Gamma(2)\Gamma(2-v)}{\Gamma(2+1-v)} = \frac{\Gamma(2)\Gamma(2-v)}{(2-v)\Gamma(2-v)} = \frac{1}{2-v}.$$

We know that $\Phi(t)$ is decreasing for a univalent function of k and $0 < \Phi(k) \leqslant \Phi(2) = \frac{1}{2-v}$. So that

$$|\Gamma(2-v)z^v D_z^v f(z)| = |z + \sum_{t=}^{\infty} \Phi(t) t a_t z^t| = |z + \Phi(2) z^2 2a_2|$$

$$\leqslant |z| + \Phi(2)2|a_2||z|^2 \leqslant |z| + \Phi(2) \frac{2\left(1+\lambda(\gamma(s)-1)\right)\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{2^n\gamma(s)\left(1+\lambda(2\gamma(s)-1)\right)\left[\alpha\left((1-2\delta)(2\gamma(s)+\psi+2\delta\mu\right]-2\gamma(s)+\mu\right)}|z|^2$$

$$|D_z^v f(z)| \leqslant \frac{|z|^{1-v}}{\Gamma(2-v)}\left(1+\frac{2\left(1+\lambda(\gamma(s)-1)\right)\left[\gamma(s)-\mu-\alpha\left((1-2\delta)\gamma(s)+\psi+2\delta\mu\right)\right]}{(2-v)2^n\gamma(s)\left(1+\lambda(2\gamma(s)-1)\right)\left[\alpha\left((1-2\delta)(2\gamma(s)+\psi+2\delta\mu\right]-2\gamma(s)+\mu\right)}|z|\right).$$

□

3. Conclusion

A new class of univalent function involving remodelled sigmoid function was defined. Furthermore, Salagean differential operator was used to establish growth, distortion theorems among other results. One can also investigate the class defined by using other differential opeartors and then compare the results.

References

[1] S. Abdul Halim. On a class of analytic functions involving Salagean differential operator. *Tamkang J. Math*,**23**(1)(1992), 51–58.

[2] K. Al-Shaqsi, M. Darus and O. A. Fadipe-Joseph. A new subclass of Salagean-type harmonic univalent functions. *Abstr. Appl. Anal.* **Volume 2010**. Article ID 821531

[3] S. Altinkaya and S. O. Olatunji. Generalized distribution for analytic function classes associated with error functions and Bell numbers. *Bol. Soc. Mat. Mex., III. Ser.*, **26**(2)(2020), 377–384.

[4] A. Amourah A. and M. Darus M. Some properties of a new class of inivalent functions involving a new generalized differential operator with negative coefficients. *Indian J. Technol.*, **9**(36)(2016), 1–7.

[5] A. Y. El-Ashwah, A. E. Lashin, A. E. and El-Shirbinyi. Subclasses of univalent functions with positive coefficients defined by Sălăgean operator. *International Journal of Open Problems in Complex Analysis*, **12**(1)(2020), 1–16.

[6] O. A. Fadipe-Joseph, A. T. Oladipo , (with A. U. Ezeafulukwe), Modified sigmoid function in univalent theory. *Int. J. Math. Sci. Eng. Appl.*, **7**(5)(2013), 313–317.

[7] O. A. Fadipe-Joseph, B. O. Moses and M. O. Olayemi. Certain new classes of analytic functions defined by using sigmoid function. *Adv. Math., Sci. J.*, **5**(1)(2016), 83–89.

[8] P. Goel and S. Kumar. Certain class of starlike functions associated with modified sigmoid function. *Bull. Malays. Math. Sci. Soc. (2)*, **43**(1)(2019), 957-991.

[9] A. W. Goodman. *Univalent Functions.* Vols. 1-2, Mariner, Tampa, Florida 1983.

[10] M. G. Khan, B. Ahmad, G. Murugusundaramoorthy, R. Chinram and W. K. Mashwani. Applications of modified sigmoid functions to a class of starlike functions. *J. Funct. Spaces*, **2020**(2020), Article ID 8844814.

[11] S. Li, H. Tang and E. Ao. Some further properties for analytic functions with varying argument defined by Hadamard products. *nt. Math. Forum*, textbf10(2)(2015), 75–93.

[12] M. Mahmoud, A. S. Juma and R.A.M. Al-Saphory. Certain classes of univalent functions with negative coefficients defined by general linear operator. *Tirkit Journal of Pure Science*, **24**(7)(2020), 129–134..

[13] G. S. Salagean G.S. Subclasses of univalent functions. In: Cazacu C.A., Boboc N., Jurchescu M., Suciu I. (eds). *Complex Analysis Fifth Romanian-Finnish Seminar.* Lecture Notes in Mathematics, vol 1013 (pp. 362-372). Springer, Berlin, Heidelberg. 1983.

[14] H. M. Srivastava, N. Khan M. Darus, S. Khan, Q.Z. Ahmad and S. Hussain. Fekete-Szegö type problems and their applications for a subclass of $q-$starlike functions with respect to symmerical points. *MDPI,$\sum$ Mathematics*, **8**(5)(2020), Article 842.

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