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A CHARACTERIZATION OF SEMIHYPERGROUPS IN TERMS OF CUBIC INTERIOR HYPERIDEALS

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Abstract. The notion of cubic set was introduced by Jun. It is also called 3-dimensional fuzzy set. In this paper we introduce cartesian product of cubic sets in semihypergroups and also we prove that cartesian product of two cubic interior hyperideals is also a cubic interior hyperideal. We study some properties of cubic subsemihypergroup and cubic interior hyperideals in semihypergroups. we find that the intersection of cubic hyperideals and a cubic hyperideal is a cubic interior hyperideal. We characterize semihypergroups in terms of cubic interior hyperideals. Finally relations between cubic hyperideals and cubic interior hyperideals are also discussed.

1. Introduction

In 1934, Marty [**4**] initiated algebraic hyperstructures as a generalization of algebraic structures. The attraction of hyper-structure is its special property that the image of each pair of a cross product of two sets is lead to a set where in classical structures it is an element again. Fuzzy set was introduced by Zadeh [**5**] in 1965. Since then it was used in many applications. It is one of the important tool in science and engineering like control engineering, information sciences, etc. In 1975 Zadeh [**6**] extended the concept of fuzzy set to interval valued fuzzy set. By combining fuzzy set and interval valued fuzzy set, Jun et al. [**2**] initiated cubic set. In 2013, the concept of cubic ideals was introduced by Jun and Khan for semigroups.

In this research work we define cartesian product of cubic sets, cubic subsemihypergroup and cubic interior hyperideal in semihypergroups. We find the relation between cubic hyperideals and cubic interior hyperideals in semihypergroups. We

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also study that the intersection of a cubic interior hyperideals and cubic hyperideals is also a cubic hyperideals in semihypergroups.

2. Preliminaries

In this section we recall the definitions of hyperideals in hyperstructures such as hyperideal, subsemihypergroup and interior hyperideal.

DEFINITION 2.1. ([**3**]) Let $\mathscr{H}$ be a non-empty universe set and $\mathcal{F}(\mathscr{H})$ is the collection of all subsets of $\mathscr{H}$. The hyperoperation $\bullet$ on $\mathscr{H}$ is defined by

$$\bullet : \mathscr{H} \times \mathscr{H} \to \mathcal{F}(\mathscr{H}).$$

The set $\mathscr{H}$ with the hyperoperation $\bullet$ is called hypergroupoid(say $\mathscr{H}^{\bullet}$). The image of $(h_1, h_2) \in \mathscr{H} \times \mathscr{H}$ is denoted by $h_1 \bullet h_2$.

Let $\mathscr{H}_1$ and $\mathscr{H}_2$ be the subsets of $\mathcal{F}(\mathscr{H})$ then the hyperoperation($\star$) between $\mathscr{H}_1$ and $\mathscr{H}_2$ is defined by

$$\mathscr{H}_1 \star \mathscr{H}_2 = \bigcup_{(h_1,h_2)\in \mathscr{H}_1 \times \mathscr{H}_2} h_1 \bullet h_2$$

where

$$\star : \mathcal{F}(\mathscr{H}) \times \mathcal{F}(\mathscr{H}) \to \mathcal{F}(\mathscr{H}).$$

DEFINITION 2.2. ([**3**]) A hypergroupoid ($\mathscr{H}^{\bullet}$) is called a semihypergroup (say $\mathscr{H}^{\star}$) if

$$(\{u\} \star \{v\}) \star \{w\} = \{u\} \star (\{v\} \star \{w\})$$

for all $u, v, w \in \mathscr{H}$.

DEFINITION 2.3. ([**3**]) A subset A of $\mathscr{H}^{\star}$ is said to be a left (right) hyperideal if

$$\mathscr{H}^{\star} \star A \subseteq A (A \star \mathscr{H}^{\star} \subseteq A).$$

A subset A of $\mathscr{H}^{\star}$ is said to be hyperideal if it is both left and right hyperideal.

DEFINITION 2.4. ([**2**]) An interval number $\bar{y} = [y^-, y^+]$ on $[0,1]$ is a closed subinterval of $[0,1]$, where $0 \leqslant y^- \leqslant y^+ \leqslant 1$. We denote the family of all closed subintervals of $[0,1]$ as $D[0,1]$.

DEFINITION 2.5. ([**2**]) Let $\bar{y} = [y^-, y^+]$ and $\bar{z} = [z^-, z^+]$ are the two interval numbers in $D[0,1]$ then

(1) $\bar{y} \leqslant \bar{z}$ if and only if $y^- \leqslant z^-$ and $y^+ \leqslant z^+$.
(2) $\bar{y} \geqslant \bar{z}$ if and only if $y^- \geqslant z^-$ and $y^+ \geqslant z^+$.
(3) $\bar{y} = \bar{z}$ if and only if $y^- = z^-$ and $y^+ = z^+$.
(4) $r\min\{\bar{y}, \bar{z}\} = [\min\{y^-, z^-\}, \min\{y^+, z^+\}]$.
(5) $r\max\{\bar{y}, \bar{z}\} = [\max\{y^-, z^-\}, \max\{y^+, z^+\}]$.

DEFINITION 2.6. ([**6**]) Let U be an universe set. An interval valued fuzzy set I of an universe set U is an object of the form $I = \{x : \dfrac{x}{\tilde{\rho}_I(x)} for\ all\ x \in U\}$ where $\tilde{\rho} : U \to D[0,1]$.

3. Cartesian product of Cubic hyperideals in semihypergroups

In this section we introduce the notion of cartesian product of cubic sets in semihypergroups and study some properties.

DEFINITION 3.1. Let $\mathscr{H}^\star$ be a semihypergroup. A cubic set $\mathscr{C}$ of $\mathscr{H}^\star$ is defined by

$$\mathscr{C} = \left\{ h : \frac{h}{(\tilde{\rho}_{\mathscr{C}}(h), \rho_{\mathscr{C}}(h))} \ for \ all \ h \in \mathscr{H}^\star \right\}$$

where $\tilde{\rho}_{\mathscr{C}}$ is an interval valued fuzzy set in $\mathscr{H}^\star$ and $\rho_{\mathscr{C}}$ is a fuzzy set in $\mathscr{H}^\star$. The membership function $\tilde{\rho}_{\mathscr{C}}$ is a mapping from $\mathscr{H}^\star$ into $D[0,1]$ and the non-membership function $\rho_{\mathscr{C}}$ is a mapping from $\mathscr{H}^\star$ into $[0,1]$.

In the interest of clarity, we shall mention the sign $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$ for the cubic set $\mathscr{C} = \left\{ h : \frac{h}{(\tilde{\rho}_{\mathscr{C}}(h), \rho_{\mathscr{C}}(h))} \ for \ all \ h \in \mathscr{H}^\star \right\}$.

DEFINITION 3.2. Let $\mathscr{C}$ and $\mathscr{D}$ are two cubic sets of $\mathscr{H}^\star$. Then the following operations are defined:

(1) $\mathscr{C} \cup \mathscr{D} = \left\{ \frac{x}{(\tilde{\rho}_{\mathscr{C}}(x) \vee \tilde{\rho}_{\mathscr{D}}(x), \rho_{\mathscr{C}}(x) \wedge \rho_{\mathscr{D}}(x))} : x \in \mathscr{H}^\star \right\}$

(2) $\mathscr{C} \cap \mathscr{D} = \left\{ \frac{x}{(\tilde{\rho}_{\mathscr{C}}(x) \wedge \tilde{\rho}_{\mathscr{D}}(x), \rho_{\mathscr{C}}(x) \vee \rho_{\mathscr{D}}(x))} : x \in \mathscr{H}^\star \right\}$

DEFINITION 3.3. Let $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$. For any $\alpha = [\alpha_1, \alpha_2] \in D[0,1]$ and $\beta \in [0,1]$, the cubic level set of $\mathscr{C}(\mathscr{C}^{(\alpha,\beta)} = (\tilde{\rho}^{\alpha}_{\mathscr{C}}, \rho^{\beta}_{\mathscr{C}}))$ is defined by

$$\mathscr{C}^{(\alpha,\beta)} = \{x \in \mathscr{H}^\star : \tilde{\rho}_{\mathscr{C}}(x) \geqslant \alpha, \rho_{\mathscr{C}}(x) \leqslant \beta\}.$$

DEFINITION 3.4. A cubic set $\mathscr{C}$ of $\mathscr{H}^\star$ is called cubic left hyperideal if

(i) $\tilde{\rho}_{\mathscr{C}}(w) \leqslant \inf\limits_{h \in v \star w} \tilde{\rho}_{\mathscr{C}}(h)$ and

(ii) $\rho_{\mathscr{C}}(w) \geqslant \sup\limits_{h \in v \star w} \rho_{\mathscr{C}}(h) \ \forall \ v, w, h \in \mathscr{H}^\star$.

A cubic set $\mathscr{C}$ of $\mathscr{H}^\star$ is called cubic right hyperideal if

(i) $\tilde{\rho}_{\mathscr{C}}(v) \leqslant \inf\limits_{h \in v \star w} \tilde{\rho}_{\mathscr{C}}(h)$ and

(ii) $\rho_{\mathscr{C}}(v)\} \geqslant \sup\limits_{h \in v \star w} \rho_{\mathscr{C}}(h) \ \forall \ v, w, h \in \mathscr{H}^\star$.

A cubic set $\mathscr{C}$ of $\mathscr{H}^\star$ is called cubic hyperideal if

(i) $\max\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\} \leqslant \inf\limits_{h \in v \star w} \tilde{\rho}_{\mathscr{C}}(h)$ and

(ii) $\min\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\} \geqslant \sup\limits_{h \in v \star w} \rho_{\mathscr{C}}(h) \ \forall \ v, w, h \in \mathscr{H}^\star$.

EXAMPLE 3.1. Let $\mathscr{H}^\star = \{p, q, r, s\}$ be a semihypergroup with the hyperoperation $(\bullet)$:

$\bullet$	p	q	r	s
p	$\{p\}$	$\{p\}$	$\{p\}$	$\{p\}$
q	$\{p\}$	$\{p\}$	$\{p\}$	$\{p\}$
r	$\{p\}$	$\{p\}$	$\{p,q\}$	$\{p,q\}$
s	$\{p\}$	$\{p\}$	$\{p,q\}$	$\{p\}$

Define a cubic set $\mathscr{C}$ as

$$\mathscr{C} = \left\{ \frac{p}{([0.5,0.6],0.6)}, \frac{q}{([0.5,0.6],0.7)}, \frac{r}{([0.3,0.4],1)}, \frac{a_4}{([0.2,0.4],0.9)} \right\}.$$

By routine calculation we can say that $\mathscr{C}$ is a cubic hyperideal of $\mathscr{H}^\star$.

DEFINITION 3.5. Let $\mathscr{H}^\star$ be a semihypergroup. The cartesian product of cubic sets $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$ and $\mathscr{D} = (\tilde{\rho}_{\mathscr{D}}, \rho_{\mathscr{D}})$ is denoted by $\mathscr{C} \times \mathscr{D}$ and is defined by

$$\mathscr{C} \times \mathscr{D} = \left\{ (c,d) : \frac{(c,d)}{(\tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(c,d), \rho_{\mathscr{C}\times\mathscr{D}}(c,d))} \; for\ all\ (c,d) \in \mathscr{H}^\star \times \mathscr{H}^\star \right\}$$

where $\tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(c,d) = \min\{\tilde{\rho}_{\mathscr{C}}(c), \rho_{\mathscr{D}}(d)\}$ and $\rho_{\mathscr{C}\times\mathscr{D}}(c,d) = \max\{\rho_{\mathscr{C}}(c), \rho_{\mathscr{D}}(d)\}$.

THEOREM 3.1. *If cubic sets $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$ and $\mathscr{D} = (\tilde{\rho}_{\mathscr{D}}, \rho_{\mathscr{D}})$ are cubic hyperideals of $\mathscr{H}^\star$ then $\mathscr{C} \times \mathscr{D}$ is also a cubic hyperideal of $\mathscr{H}^\star \times \mathscr{H}^\star$.*

PROOF. Let $\mathscr{C}$ and $\mathscr{D}$ are cubic hyperideals of $\mathbb{X}^\star$. Consider for

$$(c_1,d_1),(c_2,d_2),(h_1,h_2) \in \mathscr{H}^\star \times \mathscr{H}^\star,$$

$$\begin{aligned}
&\max\{\tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(c_1,d_1), \tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(c_2,d_2)\} \\
&\qquad = \max\left\{\min\{\tilde{\rho}_{\mathscr{C}}(c_1), \tilde{\rho}_{\mathscr{D}}(d_1)\}, \min\{\tilde{\rho}_{\mathscr{C}}(c_2), \tilde{\rho}_{\mathscr{D}}(d_2)\}\right\} \\
&\qquad \leqslant \min\left\{\max\{\tilde{\rho}_{\mathscr{C}}(c_1), \tilde{\rho}_{\mathscr{C}}(c_2)\}, \max\{\tilde{\rho}_{\mathscr{D}}(d_1), \tilde{\rho}_{\mathscr{D}}(d_2)\}\right\} \\
&\qquad \leqslant \min\left\{ \inf_{h_1 \in c_1c_2} \tilde{\rho}_{\mathscr{C}}(h_1), \inf_{h_2 \in d_1d_2} \tilde{\rho}_{\mathscr{D}}(h_2) \right\} \\
&\qquad \leqslant \inf_{\substack{h_1 \in c_1c_2,\\ h_2 \in d_1d_2}} \{\min\{\tilde{\rho}_{\mathscr{C}}(h_1), \tilde{\rho}_{\mathscr{D}}(h_2)\}\} \\
&\qquad \leqslant \inf_{(h_1,h_2) \in (c_1c_2, d_1d_2)} \tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(h_1,h_2) \\
&\qquad \leqslant \inf_{(h_1,h_2) \in (c_1,d_1)(c_2,d_2)} \tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(h_1,h_2)
\end{aligned}$$

and

$$\begin{aligned}
&\min\{\rho_{\mathscr{C}\times\mathscr{D}}(c_1,d_1), \rho_{\mathscr{C}\times\mathscr{D}}(c_2,d_2)\} \\
&\qquad = \min\left\{\max\{\rho_{\mathscr{C}}(c_1), \rho_{\mathscr{D}}(d_1)\}, \max\{\rho_{\mathscr{C}}(c_2), \rho_{\mathscr{D}}(d_2)\}\right\} \\
&\qquad \geqslant \max\left\{\min\{\rho_{\mathscr{C}}(c_1), \rho_{\mathscr{C}}(c_2)\}, \min\{\rho_{\mathscr{D}}(d_1), \rho_{\mathscr{D}}(d_2)\}\right\} \\
&\qquad \geqslant \max\left\{ \sup_{h_1 \in c_1c_2} \rho_{\mathscr{C}}(h_1), \sup_{h_2 \in d_1d_2} \rho_{\mathscr{C}}(h_2) \right\} \\
&\qquad \geqslant \sup_{\substack{h_1 \in c_1c_2,\\ h_2 \in d_1d_2}} \{\max\{\rho_{\mathscr{C}}(h_1), \rho_{\mathscr{D}}(h_2)\}\} \\
&\qquad \geqslant \sup_{(h_1,h_2) \in (c_1c_2, d_1d_2)} \rho_{\mathscr{C}\times\mathscr{D}}(h_1,h_2) \\
&\qquad \geqslant \sup_{(h_1,h_2) \in (c_1,d_1)(c_2,d_2)} \rho_{\mathscr{C}\times\mathscr{D}}(h_1,h_2)
\end{aligned}$$

Thus $\mathscr{C} \times \mathscr{D}$ is a cubic hyperideal of $\mathscr{H}^\star \times \mathscr{H}^\star$. □

Definition 3.6. Let $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$ be a cubic set in $\mathscr{H}^\star$. For any $\alpha \in D[0,1]$ and $\beta \in [0,1]$, we define (α,β)-level set of $\mathscr{C} \times \mathscr{D}$ is as

$$(\mathscr{C} \times \mathscr{D})^{(\alpha,\beta)} = (\tilde{\rho}^{\alpha}_{\mathscr{C}\times\mathscr{D}}, \rho^{\beta}_{\mathscr{C}\times\mathscr{D}})$$

where

$$\tilde{\rho}^{\alpha}_{\mathscr{C}\times\mathscr{D}} = \{(a,b) \in \mathscr{H}^\star \times \mathscr{H}^\star / \tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(a,b) \geqslant \alpha\},$$

and

$$\rho^{\beta}_{\mathscr{C}\times\mathscr{D}} = \{(a,b) \in \mathscr{H}^\star \times \mathscr{H}^\star / \rho_{\mathscr{C}\times\mathscr{D}}(a,b) \leqslant \beta\} \text{ for all } a,b \in \mathscr{H}^\star.$$

Theorem 3.2. *Let $\mathscr{H}^\star$ be a semihypergroup. If the cubic sets $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$ and $\mathscr{D} = (\tilde{\rho}_{\mathscr{D}}, \rho_{\mathscr{D}})$ are cubic right hyperideals of $\mathscr{H}^\star$ then for $\alpha \in D[0,1]$ and $\beta \in [0,1]$ the (α,β)-level set of $\mathscr{C} \times \mathscr{D}$ is a right hyperideal of $\mathscr{H}^\star \times \mathscr{H}^\star$.*

Proof. Let $\mathscr{C} \times \mathscr{D}$ be a cubic hyperideal of $\mathscr{H}^\star$. Let $(c_1,d_1),(c_2,d_2) \in \mathscr{H}^\star \times \mathscr{H}^\star$ such that $(c_1,d_1) \in (\mathscr{C} \times \mathscr{D})^{(\alpha,\beta)}$. i.e.,$(c_1,d_1) \in \tilde{\rho}^{\alpha}_{\mathscr{C}\times\mathscr{D}}$ and $(c_1,d_1) \in \rho^{\beta}_{\mathscr{C}\times\mathscr{D}}$ which implies that

$$\tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(c_1,d_1) \geqslant \alpha \quad \text{and} \quad \rho_{\mathscr{C}\times\mathscr{D}}(c_1,d_1) \leqslant \beta.$$

Since

$$\tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(c_1,d_1) \leqslant \inf_{(h_1,h_2)\in(c_1,d_1)\star(c_2,d_2)} \tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(h_1,h_2)$$

and

$$\rho_{\mathscr{C}\times\mathscr{D}}(c_1,d_1) \geqslant \sup_{(h_1,h_2)\in(c_1,d_1)\star(c_2,d_2)} \rho_{\mathscr{C}\times\mathscr{D}}(h_1,h_2)$$

we have

$$\inf_{(h_1,h_2)\in(c_1,d_1)\star(c_2,d_2)} \tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(h_1,h_2) \geqslant \alpha$$

and

$$\sup_{(h_1,h_2)\in(c_1,d_1)\star(c_2,d_2)} \rho_{\mathscr{C}\times\mathscr{D}}(h_1,h_2) \leqslant \beta.$$

It follows from here $\tilde{\rho}_{\mathscr{C}\times\mathscr{D}}(h_1,h_2) \geqslant \alpha$ and $\rho_{\mathscr{C}\times\mathscr{D}}(h_1,h_2) \leqslant \beta$ for $(h_1,h_2) \in (c_1,d_1)\star(c_2,d_2)$. Further on, we have $(h_1,h_2) \in \tilde{\rho}^{\alpha}_{\mathscr{C}\times\mathscr{D}}$ and $(h_1,h_2) \in \tilde{\rho}^{\beta}_{\mathscr{C}\times\mathscr{D}}$ for $(h_1,h_2) \in (c_1,d_1)\star(c_2,d_2)$ and $(c_1,d_1)\star(c_2,d_2) \in \tilde{\rho}^{\alpha}_{\mathscr{C}\times\mathscr{D}}$ and $(c_1,d_1)\star(c_2,d_2) \in \rho^{\beta}_{\mathscr{C}\times\mathscr{D}}$. Thus $\mathscr{C}_{(\alpha,\beta)}$ is a right hyperideal of $\mathscr{H}^\star$. □

Corollary 3.1. *Let $\mathscr{H}^\star$ be a semihypergroup. If the cubic sets $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$ and $\mathscr{D} = (\tilde{\rho}_{\mathscr{D}}, \rho_{\mathscr{D}})$ are cubic left hyperideal of $\mathscr{H}^\star$ then the (α,β)-level set of $\mathscr{C} \times \mathscr{D}$ is a left hyperideal of $\mathscr{H}^\star \times \mathscr{H}^\star$.*

Proof. The proof is straight forward. □

Corollary 3.2. *Let $\mathscr{H}^\star$ be a semihypergroup. If the cubic sets $\mathscr{C} = (\tilde{\rho}_{\mathscr{C}}, \rho_{\mathscr{C}})$ and $\mathscr{D} = (\tilde{\rho}_{\mathscr{D}}, \rho_{\mathscr{D}})$ are cubic hyperideal of $\mathscr{H}^\star$ then the (α,β)-level set of $\mathscr{C} \times \mathscr{D}$ is a hyperideal of $\mathscr{H}^\star \times \mathscr{H}^\star$.*

Proof. The proof is straight forward. □

4. Cubic interior hyperideals in semihypergroups

In this section we introduce the concept of cubic subsemiypergroup and cubic interior hyperideals in semihypergroups. We also discuss the relation between cubic hyperideals and cubic interior hyperideals.

DEFINITION 4.1. A subset A of $\mathscr{H}^\star$ is said to be a subsemihypergroup if $A\star A \subseteq A$.

A subsemihypergroup A of $\mathscr{H}^\star$ is said to be an interior hyperideal if $\mathscr{H}^\star \star A \star \mathscr{H}^\star \subseteq A$.

DEFINITION 4.2. A cubic set $\mathscr{C}$ of $\mathscr{H}^\star$ is called cubic subsemihypergroup if
(i) $\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\} \leqslant \inf\limits_{h\in v\star w} \tilde{\rho}_{\mathscr{C}}(h)$ and
(ii) $\max\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\} \geqslant \sup\limits_{h\in v\star w} \rho_{\mathscr{C}}(h)\ \forall\ h, v, w \in \mathscr{H}^\star$.

DEFINITION 4.3. A cubic subsemihypergroup $\mathscr{C}$ of $\mathscr{H}^\star$ is called cubic interior hyperideal if
(i) $\tilde{\rho}_{\mathscr{C}}(z) \leqslant \inf\limits_{h\in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h)$
(ii) $\rho_{\mathscr{C}}(z) \geqslant \sup\limits_{h\in v\star z\star w} \rho_{\mathscr{C}}(h)\ \forall\ h, v, w, z \in \mathscr{H}^\star$.

EXAMPLE 4.1. Let us consider a semihypergroup $\mathscr{H}^\star = \{u, v, w, x\}$ with the hyperoperation $(\bullet)$:

$\bullet$	u	v	w	x
u	$\{u\}$	$\{u\}$	$\{u\}$	$\{u\}$
v	$\{u\}$	$\{u, v\}$	$\{u, w\}$	$\{u\}$
w	$\{u\}$	$\{u\}$	$\{u\}$	$\{u\}$
x	$\{u\}$	$\{u, x\}$	$\{u\}$	$\{u\}$

Let

$$\mathscr{C} = \left\{ \frac{u}{([1,1], 0.2)}, \frac{q}{([0.2, 0.5], 1)}, \frac{r}{([0.2, 0.5], 0.9)}, \frac{a_4}{([0.7, 0.8], 0.7)} \right\}$$

be a cubic set in $\mathscr{H}^\star$. Then by calculation one can easily verify that $\mathscr{C}$ is a cubic interior hyperideal of $\mathscr{H}^\star$.

THEOREM 4.1. *If $\mathscr{C}$ and $\mathscr{D}$ are cubic interior hyperideals of $\mathscr{H}^\star$ then $\mathscr{C} \cap \mathscr{D}$ is also a cubic interior hyperideal of $\mathscr{H}^\star$.*

PROOF. Consider for $h \in \mathscr{H}^\star$,

$$\begin{aligned}
\tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(h) &= \tilde{\rho}_{\mathscr{C}}(h) \wedge \tilde{\rho}_{\mathscr{D}}(h) \\
&\leqslant \inf_{h\in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h) \wedge \inf_{h\in v\star z\star w} \tilde{\rho}_{\mathscr{D}}(h) \\
&\leqslant \inf_{h\in v\star z\star w} \{\tilde{\rho}_{\mathscr{C}}(h) \wedge \tilde{\rho}_{\mathscr{D}}(h)\} \\
&\leqslant \inf_{h\in v\star z\star w} \tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(h)\ \forall\ v, w, z \in \mathscr{H}^\star.
\end{aligned}$$

Now,

$$\begin{aligned}\rho_{\mathscr{C}\cap\mathscr{D}}(h) &= \rho_{\mathscr{C}}(h) \vee \rho_{\mathscr{D}}(h)\\ &\geqslant \sup_{h\in v\star w} \rho_{\mathscr{C}}(h) \vee \sup_{h\in v\star w} \rho_{\mathscr{D}}(h)\\ &\geqslant \sup_{h\in v\star w} \{\rho_{\mathscr{C}}(h) \vee \rho_{\mathscr{D}}(h)\}\\ &\geqslant \sup_{h\in v\star w} \rho_{\mathscr{C}\cap\mathscr{D}}(h) \ \forall\ v, w \in \mathscr{H}^{\star}.\end{aligned}$$

Thus $\mathscr{C} \cap \mathscr{D}$ is a cubic interior hyperideal of $\mathscr{H}^{\star}$. □

Lemma 4.1. *Let $\mathscr{H}^{\star}$ be a semihypergroup. A cubic set $\mathscr{C}$ of $\mathscr{H}^{\star}$ is a cubic subsemihypergroup if and only if for $\alpha \in D[0,1]$ and $\beta \in [0,1]$, $\mathscr{C}^{(\alpha,\beta)}$ is a subsemihypergroup of $\mathscr{H}^{\star}$.*

Proof. Let $\mathscr{C}$ be a cubic subsemihypergroup of $\mathscr{H}^{\star}$. Let us show that $\mathscr{C}^{(\alpha,\beta)}$ is a subsemihypergroup.

Let us consider that $v, w \in \mathscr{C}^{(\alpha,\beta)}$. Then we have $\tilde{\rho}_{\mathscr{C}}(v) \geqslant \alpha$, $\rho_{\mathscr{C}}(v) \leqslant \beta$ and $\tilde{\rho}_{\mathscr{C}}(w) \geqslant \alpha$, $\rho_{\mathscr{C}}(w) \leqslant \beta$. Thus, we have

$$\inf_{h\in v\star w} \tilde{\rho}_{\mathscr{C}}(h) \geqslant \min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\},\ \sup_{h\in v\star w} \rho_{\mathscr{C}}(h) \leqslant \min\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\}$$
$$\Longrightarrow \inf_{h\in v\star w} \tilde{\rho}_{\mathscr{C}}(h) \geqslant \min\{\alpha, \alpha\},\ \sup_{h\in v\star w} \rho_{\mathscr{C}}(h) \leqslant \min\{\beta, \beta\}$$
$$\Longrightarrow \inf_{h\in v\star w} \tilde{\rho}_{\mathscr{C}}(h) \geqslant \alpha,\ \sup_{h\in v\star w} \rho_{\mathscr{C}}(h) \leqslant \beta$$
$$\Longrightarrow \tilde{\rho}_{\mathscr{C}}(h) \geqslant \alpha,\ \rho_{\mathscr{C}}(h) \leqslant \beta \text{ for } h \in v \star w$$
$$\Longrightarrow h \in \mathscr{C}^{(\alpha,\beta)} \text{ for } h \in v \star w$$
$$\Longrightarrow v \star w \in \mathscr{C}^{(\alpha,\beta)}.$$

Thus $\mathscr{C}^{(\alpha,\beta)}$ is a subsemihypergroup of $\mathscr{H}^{\star}$.

Conversely let $\mathscr{C}^{(\alpha,\beta)}$ be a subsemihypergroup of $\mathscr{H}^{\star}$. Suppose if not we have

$$\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\} > \inf_{h\in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h) \ \text{ and } \ \min\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\} < \sup_{h\in v\star z\star w} \rho_{\mathscr{C}}(h)$$

then there exists $\alpha_0 \in D[0,1]$ and $\beta_0 \in [0,1]$ such that

$$\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\} > \alpha_0 > \inf_{h\in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h)$$

and

$$\min\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\} < \beta_0 < \sup_{h\in v\star z\star w} \rho_{\mathscr{C}}(h).$$

Then

$\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\} > \alpha_0$, $\inf_{h\in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h) < \alpha_0$ and

$$\min\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\} < \beta_0,\ \sup_{h\in v\star z\star w} \rho_{\mathscr{C}}(h) > \beta_0.$$

This implies

$\tilde{\rho}_{\mathscr{C}}(v) > \alpha_0$ or $\tilde{\rho}_{\mathscr{C}}(w) > \alpha_0$, $v \star z \star w \notin \tilde{\rho}_{\mathscr{C}}^{\alpha_0}$ and

$$\tilde{\rho}_{\mathscr{C}}(v) < \beta_0 \text{ or } \tilde{\rho}_{\mathscr{C}}(w) < \beta_0,\ v \star z \star w \notin \rho_{\mathscr{C}}^{\beta_0}.$$

Thus

$$v, w \in \tilde{\rho}_{\mathscr{C}}^{\alpha_0} \text{ but } v \star z \star w \notin \rho_{\mathscr{C}}^{\alpha_0} \text{ and } v, w \in \rho_{\mathscr{C}}^{\beta_0} \text{ but } v \star z \star w \notin \rho_{\mathscr{C}}^{\beta_0}.$$

This contradicts the fact that $\mathscr{C}^{(\alpha,\beta)}$ is a subsemihypergroup of $\mathscr{H}^\star$. Therefore $\mathscr{C}$ is a cubic subsemihypergroup of $\mathscr{H}^\star$. □

Theorem 4.2. *Let $\mathscr{H}^\star$ be a semihypergroup. A cubic set $\mathscr{C}$ of $\mathscr{H}^\star$ is a cubic interior hyperideal if and only if for $\alpha \in D[0,1]$ and $\beta \in [0,1]$, $\mathscr{C}^{(\alpha,\beta)}$ is an interior hyperideal of $\mathscr{H}^\star$.*

Proof. Let $\mathscr{C}$ be a cubic interior hyperideal of $\mathscr{H}^\star$. By Lemma 4.1, we know that $\mathscr{C}^{(\alpha,\beta)}$ is a subsemihypergroup of $\mathscr{H}^\star$. Let $h, v, w, z \in \mathscr{H}^\star$ such that $z \in \mathscr{C}^{(\alpha,\beta)}$. Which implies that

$$\tilde{\rho}_{\mathscr{C}}(z) \geqslant \alpha, \quad \rho_{\mathscr{C}}(z) \leqslant \beta.$$

Since $\tilde{\rho}_{\mathscr{C}}(z) \leqslant \inf_{h \in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h)$ and $\rho_{\mathscr{C}}(z) \geqslant \sup_{h \in v\star z\star w} \rho_{\mathscr{C}}(h)$, we have

$$\begin{aligned}
&\inf_{h \in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h) \geqslant \alpha \text{ and } \sup_{h \in v\star z\star w} \rho_{\mathscr{C}}(h) \leqslant \beta\\
&\Longrightarrow h \in \tilde{\rho}_{\mathscr{C}}^{\alpha} \text{ and } h \in \rho_{\mathscr{C}}^{\beta}\\
&\Longrightarrow v \star z \star w \in \tilde{\rho}_{\mathscr{C}}^{\alpha} \text{ and } v \star z \star w \in \rho_{\mathscr{C}}^{\beta}\\
&\Longrightarrow v \star z \star w \in \mathscr{C}^{(\alpha,\beta)} \text{ for } z \in \mathscr{C}^{(\alpha,\beta)}.
\end{aligned}$$

Thus $\mathscr{C}^{(\alpha,\beta)}$ is an interior ideal of $\mathscr{C}^{(\alpha,\beta)}$.

Conversely, let us assume that $\mathscr{C}$ is not a cubic interior hyperideal. We have

$$\tilde{\rho}_{\mathscr{C}}(z) > \inf_{h \in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h) \text{ and } \rho_{\mathscr{C}}(z) < \sup_{h \in v\star z\star w} \rho_{\mathscr{C}}(h).$$

Then there exists $\alpha_1 \in D[0,1]$ and $\beta_1 \in [0,1]$ such that

$$\begin{aligned}
&\tilde{\rho}_{\mathscr{C}}(z) > \alpha_1 > \inf_{h \in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h) \text{ and } \rho_{\mathscr{C}}(z) < \beta_1 < \sup_{h \in v\star z\star w} \rho_{\mathscr{C}}(h)\\
&\Longrightarrow \tilde{\rho}_{\mathscr{C}}(z) > \alpha_1,\ \inf_{h \in v\star z\star w} \tilde{\rho}_{\mathscr{C}}(h) < \alpha_1 \text{ and } \rho_{\mathscr{C}}(z) < \beta_1,\ \sup_{h \in v\star z\star w} \rho_{\mathscr{C}}(h) > \beta_1\\
&\Longrightarrow z \in \tilde{\rho}_{\mathscr{C}}^{\alpha_1} \text{ and } z \in \rho_{\mathscr{C}}^{\beta_1} \text{ but } h \notin \tilde{\rho}_{\mathscr{C}}^{\alpha_1} \text{ and } h \notin \rho_{\mathscr{C}}^{\alpha_1} \text{ for } h \in v \star z \star w\\
&\Longrightarrow z \in \tilde{\rho}_{\mathscr{C}}^{\alpha_1} \text{ and } z \in \rho_{\mathscr{C}}^{\beta_1} \text{ but } v \star z \star w \notin \tilde{\rho}_{\mathscr{C}}^{\alpha_1} \text{ and } v \star z \star w \notin \rho_{\mathscr{C}}^{\beta_1}\\
&\Longrightarrow z \in \mathscr{C}^{(\alpha,\beta)} \text{ and } v \star z \star w \notin \mathscr{C}^{(\alpha,\beta)} \text{ for } h \in v \star z \star w.
\end{aligned}$$

This is a contradiction to the fact that $\mathscr{C}$ is a cubic interior hyperideal. Hence $\mathscr{C}$ is a cubic interior hyperideal. □

Proposition 4.1. *Intersection of a cubic interior hyperideal and a cubic hyperideal of $\mathscr{H}^\star$ is always a cubic interior hyperideal.*

Proof. Let $\mathscr{C}$ be a cubic interior hyperideal and $\mathscr{D}$ be a cubic hyperideal. For $h, v, w, z \in \mathscr{H}^\star$.

$$\begin{aligned}
\min\{\tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(v), \tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(w)\} &= \min\{\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{D}}(v)\}, \min\{\tilde{\rho}_{\mathscr{C}}(w), \tilde{\rho}_{\mathscr{D}}(w)\}\}\\
&= \min\{\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\}, \min\{\tilde{\rho}_{\mathscr{D}}(v), \tilde{\rho}_{\mathscr{D}}(w)\}\}
\end{aligned}$$

$$\leqslant \min\{\inf_{h\in vw}\tilde{\rho}_{\mathscr{C}}(h), \min\{\inf_{h\in vw}\tilde{\rho}_{\mathscr{D}}(h), \inf_{h\in vw}\tilde{\rho}_{\mathscr{D}}(h)\}\}$$

(*Since* $\mathscr{C}$ *is a cubic subsemihypergroup and* $\mathscr{D}$ *is a cubic left and right hyperideal.*)

$$\leqslant \min\{\inf_{h\in vw}\tilde{\rho}_{\mathscr{C}}(h), \inf_{h\in vw}\tilde{\rho}_{\mathscr{D}}(h)\}$$
$$\leqslant \inf_{h\in vw}\{\min\{\tilde{\rho}_{\mathscr{C}}(h), \tilde{\rho}_{\mathscr{D}}(h)\}\}$$
$$\leqslant \inf_{h\in vw}\tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(h)$$

Now,

$$\begin{aligned}\max\{\rho_{\mathscr{C}\cap\mathscr{D}}(v), \rho_{\mathscr{C}\cap\mathscr{D}}(w)\} &= \max\{\max\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{D}}(v)\}, \max\{\rho_{\mathscr{C}}(w), \rho_{\mathscr{D}}(w)\}\}\\ &= \max\{\max\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\}, \max\{\rho_{\mathscr{D}}(v), \rho_{\mathscr{D}}(w)\}\}\\ &= \max\{\min\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\}, \max\{\rho_{\mathscr{D}}(v), \rho_{\mathscr{D}}(w)\}\}\\ &\geqslant \max\{\sup_{h\in vw}\rho_{\mathscr{C}}(h), \max\{\sup_{h\in vw}\rho_{\mathscr{D}}(h), \sup_{h\in vw}\rho_{\mathscr{D}}(h)\}\}\end{aligned}$$

(*Since* $\mathscr{C}$ *is a cubic subsemihypergroup and* $\mathscr{D}$ *is a cubic left and right hyperideal.*)

$$\geqslant \max\{\sup_{h\in vw}\rho_{\mathscr{C}}(h), \sup_{h\in vw}\rho_{\mathscr{D}}(h)\}$$
$$\geqslant \sup_{h\in vw}\{\max\{\rho_{\mathscr{C}}(h), \rho_{\mathscr{D}}(h)\}\}$$
$$\geqslant \sup_{h\in vw}\rho_{\mathscr{C}\cap\mathscr{D}}(h)$$

and also

$$\begin{aligned}\inf_{h\in v\star z\star w}\tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(h) &= \inf_{h\in v\star z\star w}\{\min\{\tilde{\rho}_{\mathscr{C}}(h), \tilde{\rho}_{\mathscr{D}}(h)\}\}\\ &= \min\{\inf_{h\in v\star z\star w}\tilde{\rho}_{\mathscr{C}}(h), \inf_{h\in v\star z\star w}\tilde{\rho}_{\mathscr{D}}(h)\}\\ &\geqslant \min\{\inf_{h\in v\star z\star w}\tilde{\rho}_{\mathscr{C}}(h), \inf_{h\in v\star z}\tilde{\rho}_{\mathscr{D}}(h)\}\end{aligned}$$

(*Since* $\mathscr{D}$ *is a cubic right hyperideal*)

$$\geqslant \min\{\tilde{\rho}_{\mathscr{C}}(z), \tilde{\rho}_{\mathscr{D}}(z)\}$$

(*Since* $\mathscr{D}$ *is a cubic left hyperideal.*)

$$\geqslant \tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(z).$$

Now,

$$\begin{aligned}\sup_{h\in v\star z\star w}\rho_{\mathscr{C}\cap\mathscr{D}}(h) &= \sup_{h\in v\star z\star w}\{\rho_{\mathscr{C}}(h)\vee\rho_{\mathscr{D}}(h)\}\\ &= \max\{\sup_{h\in v\star z\star w}\rho_{\mathscr{C}}(h), \sup_{h\in v\star z\star w}\rho_{\mathscr{D}}(h)\}\\ &\leqslant \max\{\sup_{h\in v\star z\star w}\rho_{\mathscr{C}}(h), \sup_{h\in v\star z}\rho_{\mathscr{D}}(h)\}\end{aligned}$$

(*Since* $\mathscr{D}$ *is a cubic right hyperideal.*)

$$\leqslant \max\{\rho_{\mathscr{C}}(z), \rho_{\mathscr{D}}(z)\}$$

(*Since* $\mathscr{D}$ *is a cubic left hyperideal.*)

$$\leqslant \rho_{\mathscr{C}\cap\mathscr{D}}(z).$$ □

Lemma 4.2. *Every cubic hyperideal of* $\mathscr{H}^\star$ *is a cubic subsemihypergroup.*

Proof. Let $\mathscr{C}$ be a cubic hyperideal. For $h, v, h, w, z \in \mathscr{H}^\star$,

$$\begin{aligned}\min\{\tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(v), \tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(w)\} &= \min\{\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{D}}(v)\}, \min\{\tilde{\rho}_{\mathscr{C}}(w), \tilde{\rho}_{\mathscr{D}}(w)\}\}\\ &= \min\{\min\{\tilde{\rho}_{\mathscr{C}}(v), \tilde{\rho}_{\mathscr{C}}(w)\}, \min\{\tilde{\rho}_{\mathscr{D}}(v), \tilde{\rho}_{\mathscr{D}}(w)\}\}\\ &\leqslant \min\{\min\{\inf_{h\in vw}\tilde{\rho}_{\mathscr{C}}(h), \inf_{h\in vw}\tilde{\rho}_{\mathscr{C}}(h)\},\end{aligned}$$

$$
\begin{aligned}
&\min\{\inf_{h\in vw}\tilde{\rho}_{\mathscr{D}}(h), \inf_{h\in vw}\tilde{\rho}_{\mathscr{D}}(h)\}\}\\
&(Since\ \mathscr{D}\ is\ a\ cubic\ left\ and\ right\ hyperideal.)\\
&\leqslant \min\{\inf_{h\in vw}\tilde{\rho}_{\mathscr{C}}(h), \inf_{h\in vw}\tilde{\rho}_{\mathscr{D}}(h)\}\\
&\leqslant \inf_{h\in vw}\{\min\{\tilde{\rho}_{\mathscr{C}}(h), \tilde{\rho}_{\mathscr{D}}(h)\}\}\\
&\leqslant \inf_{h\in vw}\tilde{\rho}_{\mathscr{C}\cap\mathscr{D}}(h)
\end{aligned}
$$

Now,

$$
\begin{aligned}
\max\{\rho_{\mathscr{C}\cap\mathscr{D}}(v), \rho_{\mathscr{C}\cap\mathscr{D}}(w)\} &= \max\{\max\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{D}}(v)\}, \max\{\rho_{\mathscr{C}}(w), \rho_{\mathscr{D}}(w)\}\}\\
&= \max\{\max\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\}, \max\{\rho_{\mathscr{D}}(v), \rho_{\mathscr{D}}(w)\}\}\\
&= \max\{\max\{\rho_{\mathscr{C}}(v), \rho_{\mathscr{C}}(w)\}, \max\{\rho_{\mathscr{D}}(v), \rho_{\mathscr{D}}(w)\}\}\\
&\geqslant \max\{\max\{\sup_{h\in vw}\rho_{\mathscr{C}}(h), \sup_{h\in vw}\rho_{\mathscr{C}}(h)\},\\
&\qquad \max\{\sup_{h\in vw}\rho_{\mathscr{D}}(h), \sup_{h\in vw}\rho_{\mathscr{D}}(h)\}\}\\
&(Since\ \mathscr{D}\ is\ a\ cubic\ left\ and\ right\ hyperideal.)\\
&\geqslant \max\{\sup_{h\in vw}\rho_{\mathscr{C}}(h), \sup_{h\in vw}\rho_{\mathscr{D}}(h)\}\\
&\geqslant \sup_{h\in vw}\{\max\{\rho_{\mathscr{C}}(h), \rho_{\mathscr{D}}(h)\}\}\\
&\geqslant \sup_{h\in vw}\rho_{\mathscr{C}\cap\mathscr{D}}(h)
\end{aligned}
$$

Hence $\mathscr{C}$ is a cubic subsemihypergroup. □

PROPOSITION 4.2. *Every cubic hyperideal of $\mathscr{H}^\star$ is a cubic interior hyperideal.*

PROOF. Let $\mathscr{C}$ be a cubic hyperideal. For $h, v, h, w, z \in \mathscr{H}^\star$ we have

$$
\begin{aligned}
\inf_{h\in v\star z\star w}\tilde{\rho}_{\mathscr{C}}(h) &\geqslant \inf_{h\in (v\star z)\star w}\tilde{\rho}_{\mathscr{C}}(h)\\
&\geqslant \inf_{h\in v\star z}\tilde{\rho}_{\mathscr{C}}(h) \qquad (Since\ \mathscr{C}\ is\ a\ cubic\ right\ hyperideal.)\\
&\geqslant \tilde{\rho}_{\mathscr{C}}(z) \qquad (Since\ \mathscr{C}\ is\ a\ cubic\ left\ hyperideal.)
\end{aligned}
$$

and

$$
\begin{aligned}
\sup_{h\in v\star z\star w}\rho_{\mathscr{C}}(h) &\leqslant \sup_{h\in v\star z}\rho_{\mathscr{C}}(h)\\
&\leqslant \sup_{h\in v\star z}\rho_{\mathscr{C}}(h)\\
&\leqslant \rho_{\mathscr{C}}(z).
\end{aligned}
$$

Therefore we have $\mathscr{C}$ is a cubic interior hyperideal of $\mathscr{H}^\star$. □

The following example shows that every cubic interior hyperideal of $\mathscr{H}^\star$ need not be a cubic hyperideal of $\mathscr{H}^\star$.

EXAMPLE 4.2. Let $\mathscr{H}^\star = \{p, q, r, s\}$ be a semihypergroup with the hyperoperation $(\bullet)$ as follows:

$\bullet$	p	q	r	s
p	$\{q,s\}$	$\{r,s\}$	$\{s\}$	$\{s\}$
q	$\{r,s\}$	$\{q\}$	$\{s\}$	$\{s\}$
r	$\{s\}$	$\{s\}$	$\{s\}$	$\{s\}$
s	$\{s\}$	$\{s\}$	$\{s\}$	$\{s\}$

Define a cubic set $\mathscr{C}$ by

$$\mathscr{C} = \left\{ \frac{p}{([0.2,0.3],1)}, \frac{q}{([0.5,0.6],0.6)}, \frac{r}{([0.2,0.3],1)}, \frac{a_4}{([0.9,1],0.3)} \right\}.$$

The $(\alpha,\beta)-$level set of $\mathscr{C}$ is as follows:

$$\tilde{\rho}_{\mathscr{C}}^{[0.2,0.3]} = \rho_{\mathscr{C}}^{1} = \mathscr{H}^{\star}, \quad \tilde{\rho}_{\mathscr{C}}^{[0.5,0.6]} = \rho_{\mathscr{C}}^{0.6} = \{q,s\}, \quad \text{and } \tilde{\rho}_{\mathscr{C}}^{[0.9,1]} = \rho_{\mathscr{C}}^{0.3} = \{s\}$$

Since for $q \in \{q,s\}$ and $p \in \mathscr{H}^{\star}$ we have $q \bullet p = \{r,s\} \nsubseteq \{q,s\}$. Therefore $\{q,s\}$ is not a right hyperideal of $\mathscr{H}^{\star}$. Therefore $\{q,s\}$ is not a hyperideal of $\mathscr{H}^{\star}$. From the Theorem 4.2 it is clear that $\mathscr{C}$ is not a cubic hyperideal.

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