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PYTHAGOREAN SEMI-OPEN SETS IN PYTHAGOREAN NEYTROSHOPIC PYTHAGOREAN SPACES

Carlos Granados

Abstract. In this paper, we introduce and study a new notion of Pythagorean neutrosophic set which is called Pythagorean neutrosophic semi-open sets. Besides, we define the concepts of Pythagorean neutosophic semi-open function, Pythagorean neutrosophic semi-continuous function and Pythagorean neutrosophic semi-homeomorphism. Moreover, some of their properties are shown.

1. Introduction and Preliminaries

The notion of fuzzy set was originally introduced by Zadeh [**13**], since then this notion has been studied by many authors in different fields of the general topology (see [**7, 4**]). In 1968, Chang [**5**] introduced the notion of fuzzy topological spaces, as well as, some basics concepts in general topology. Besides, Atanassov [**2, 3**] in 1983 defined the concept of intuitionistic fuzzy set. Furthermore, the notion of neutrosophic set was introduced by Smarandache [**10**] and so Wang et. al. [**14**] studied some of its properties on interval neutrosophic set. Moreover, the notion of neutrosophic topological space was also studied by Salama and Albowi [**9**]. By using the notions mentioned above, Yager [**12**] in 2013 introduced the concept of Pythagorean membership grades, later Yager, Zahand and Xu [**11**] proved some properties on Pythagorean fuzzy set. On the other hand, in 2017 Arockiarani [**1**] introduced and studied the notion of neutrosophic b-open set, Besides, Shena and Nirmala [**8**] introduced the notion of Pythagorean neutrosophic open sets and

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showed some properties on Pythagorean neutrosophic α-open set. Recently, Granados in 2020 [**6**], studied some properties of Pythagorean neutrosophic pre-open sets in Pythagorean neutrosophic spaces, as well as, some of their properties and characterizations on Pythagorean neutrosophic pre-continuous functions were studied by him.

In this paper, we used the notions of Pythagorean neutrosophic open set and the notions mentioned above for introducing and studying the concept of Pythagorean neutrosophic semi-open set. Besides, we show some of its properties. We also define the concepts of Pythagorean neutosophic pre-open function, Pythagorean neutrosophic pre-continuous function and Pythagorean neutrosophic pre-homeomorphism. Moreover, some of their properties are proved.

Throughout this paper, $(X, \mathfrak{T})$, $(Y, \mathfrak{Y})$ and $(Z, \mathfrak{Z})$ are topological spaces on which no separation axioms are assumed unless otherwise mentioned. Furthermore, we sometimes write X, Y or Z instead of $(X, \mathfrak{T})$, $(Y, \mathfrak{Y})$ or $(Z, \mathfrak{Z})$, respectively. Now, we show some Definitions which are useful for the developing of this paper.

Definition 1.1. ([**13**]) A fuzzy set $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$ is a universe of discourse X, which is characterized by a membership function μ_A as $\mu_A : X \to [0, 1]$.

Definition 1.2. ([**2, 3**]) Let X be a non-empty set. Then, A is said to be an intuitionistic fuzzy set of X if there is a $A = \{\langle x, \mu_A, \gamma_A\rangle : x \in X\}$ where the function $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership $\mu_A(x)$ and degree of non-membership γ_A of every element $x \in X$ to the set A and satisfies the condition

$$0 \leqslant \mu_A(x) + \gamma_A(x) \leqslant 2.$$

Definition 1.3. ([**10**]) Let X be a non-empty set. Then, A is said to be a neutrosophic set of X if there is a $A = \{\langle x, \mu_A, \sigma_A, \gamma_A\rangle : x \in X\}$ where the function $\mu_A : X \to [0, 1]$, $\sigma_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$), degree of indeterminacy (namely $\sigma_A(x)$) and degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A and satisfies the condition

$$0 \leqslant \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leqslant 3.$$

Definition 1.4. ([**12**]) Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs on X and it is defined by $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership respectively , of the element $x \in X$ to A, which is subsets in X and for every $x \in X : 0 \leqslant (\mu_A(x))^2 + (\gamma_A(x))^2 \leqslant 1$. Assuming that $0 \leqslant (\mu_A(x))^2 + (\gamma_A(x))^2 \leqslant 1$, there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{(\mu_A(x))^2 + (\gamma_A(x))^2}$ and $\pi_A(x) \in [0, 1]$.

Definition 1.5. ([**8**]) Let X be a non-empty set. Then, A is said to be a Pythagorean neutrosophic set (or simply, PN) of X if there is a

$$A = \{\langle x, \mu_A, \sigma_A, \gamma_A\rangle : x \in X\}$$

where the function $\mu_A : X \to [0,1]$, $\sigma_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$), degree of indeterminacy (namely $\sigma_A(x)$) and degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A and satisfies the condition

$$0 \leqslant \mu_A(x)^2 + \sigma_A(x)^2 + \gamma_A(x)^2 \leqslant 2.$$

Definition 1.6. ([**8**]) A Pythagorean neutrosophic topology (or simply, PNT) on a non-empty set X is a family of $\mathfrak{T}$ of Pythagorean neutrosophic sets in X satisfying the following conditions:

(1) $0, 1 \in \mathfrak{T}$.
(2) $\mathfrak{G}_1 \cap \mathfrak{G}_2 \in \mathfrak{T}$, for any $\mathfrak{G}_1, \mathfrak{G}_2 \in \mathfrak{T}$.
(3) $\bigcup \mathfrak{G}_i \in \mathfrak{T}$, for any arbitrary family $\{\mathfrak{G}_i : \mathfrak{G}_i \in \mathfrak{T}, i \in I\}$.

In this case, the pair $(X, \mathfrak{T})$ is said to be a Pythagorean neutrosophic topological spaces, besides any Pythagorean neutrosophic set in $\mathfrak{T}$ is known as Pythagorean neutrosophic neutrosophic open set in X.

Definition 1.7. For a Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is said to be Pythagorean neutrosophic α-open set [**8**] if $A \subseteq PNInt((PNCl(PNInt(A)))$.

Theorem 1.8 ([**8**]). *Every Pythagorean neutrosophic open set is Pythagorean neutrosophic α-open set.*

Definition 1.9. ([**8**]) Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic continuous function if $f^{-1}(V)$ is a Pythagorean neutrosophic in X for every Pythagorean neutrosophic open set V in Y.

Definition 1.10. ([**8**]) Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic α-continuous if $f^{-1}(V)$ is a Pythagorean neutrosophic α-open in X for every Pythagorean neutrosophic open set V in Y.

Definition 1.11. ([**8**]) Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic α-open if $f(A)$ is Pythagorean neutrosophic α-open set in Y for every Pythagorean neutrosophic open set A in X.

2. Pythagorean neutrosophic semi-open sets

In this section we introduce and study the notion of Pythagorean neutrosophic semi-open set. Furthermore, we show some of its properties.

Definition 2.1. Let X be a non-empty set. If a, b, c are real standard or non standard subsets of $]0^-, 1^+[$, then the Pythagorean neutrosophic set $x_{a,b,c}$ is said to be Pythagorean neutrosophic point (or simply, PNP) in X and it is given by:

$$x_{a,b,c}(x_p) = \begin{cases} (a, b, c) & \text{if } x = x_p \\ (0, 0, 1) & \text{if } x \neq x_p \end{cases}$$

For each $x_p \in X$ is said to be the support of $x_{a,b,c}$, where a denotes the degree of membership value, b denotes the degree of indeterminacy and c is the degree of non-membership value of $x_{a,b,c}$.

Definition 2.2. For a Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is said to be Pythagorean neutrosophic semi-open set (or simply, $PNsOS$) if $A \subseteq PNCl((PNInt(A))$. The complement of a Pythagorean neutrosophic semi-open set is called Pythagorean neutrosophic semi-closed set.

Remark 2.3. The collection of all Pythagorean neutrosophic semi-open sets and Pythagorean neutrosophic semi-closed sets are denoted by $PNsOS(X, \mathfrak{T})$ and $PNsCS(X, \mathfrak{T})$, respectively.

Proposition 2.4. *Let $(X, \mathfrak{T})$ be a Pythagorean neutrosophic topological space and $A \subseteq X$. Then, If A is a Pythagorean neutrosophic α-open set, then A is Pythagorean neutrosophic semi-open set.*

Proof. Let A be a Pythagorean neutrosophic α-open, then by the Definition 1.7, $A \subseteq PNInt(PNCl(PNInt(A)))$, since $Int(Cl(A)) \subseteq A$, this implies that $A \subset PNCl(PNInt(A))$. Therefore, A is Pythagorean neutrosophic semi-open. □

Definition 2.5. A Pythagorean neutrosophic set V in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is said to be Pythagorean neutrosophic semi-closed (or simply, $PNsCS$)) if $V \supseteq PNInt(PNCl(V))$.

Definition 2.6. Let $(X, \mathfrak{T})$ be a Pythagorean neutrosophic topological space and V be a Pythagorean neutrosophic set on X. Then we define the Pythagorean neutrosophic semi-interior and Pythagorean neutrosophic semi-closure of V as:

(1) Pythagorean neutrosophic semi-interior of V (or simply, $PNSINT(V)$) as the union of all Pythagorean neutrosophic semi-open sets of X contained in V. It means that $PNSINT(V) = \bigcup\{A : A$ is a $PNsOS$ in X and $A \subseteq V\}$.
(2) Pythagorean neutrosophic semi-closure of V (or simply, $PNSCL(V)$) as the intersection of all Pythagorean neutrosophic semi-closed set of X containing V. It means that $PNSCL(V) = \bigcap\{B : B$ is a $PNsCS$ in X and $V \subseteq B\}$.

Remark 2.7. By the Definition 2.6, we can see that $PNSCL(V)$ is the smallest Pythagorean neutrosophic semi-closed set of X which contains V. Besides, $PNSINT(V)$ is the largest Pythagorean neutrosophic semi-open set of X which is contained in V.

Proposition 2.8. *Let V be a Pythagorean neutrosophic set in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. Then, the following statements hold:*

(1) *If V is Pythagorean neutrosophic semi-open set, then $Cl(V)$ is is a Pythagorean neutrosophic semi-closed set.*
(2) *If V is Pythagorean neutrosophic semi-closed set, then $Cl(V)$ is is a Pythagorean neutrosophic semi-open set.*

PROOF. The proof is followed by the Definitions 2.2, 2.5 and 2.6. □

THEOREM 2.9. *Let V be a Pythagorean neutrosophic set in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. Then, the following statements hold:*

(1) $Cl(PNSINT(V)) = PNSCL(Cl(V))$.
(2) $Cl(PNSCL(V)) = PNSINT(Cl(V))$.

PROOF. We begin proving (1): Let V be a Pythagorean neutrosophic set. Now, by the Definition 2.6 part (1), $PNSINT(V) = \bigcup\{A : A$ is a $PNsOS$ in X and $A \subseteq V\}$, this implies that $Cl(PNSINT(V)) = Cl(\bigcup\{A : A$ is a $PNsOS$ in X and $A \subseteq V\})= \bigcap\{Cl(A) : Cl(A)$ is a $PNsCS$ in X and $Cl(V) \subseteq Cl(A)\}$. Now, we will replace $Cl(A)$ by B, then we have that $Cl(PNSINT(V) = \bigcap\{B : B$ is a $PNsCS$ in X and $Cl(V) \subseteq B\}$, and so $Cl(PNSINT(V)) = PNSCL(Cl(V))$.
The proof of (2) is similar to (1). □

THEOREM 2.10. *For a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ and $A, B \subseteq X$. The following statements hold:*

(1) *Every Pythagorean neutrosophic set is Pythagorean neutrosophic semi-open set.*
(2) $PNSINT(PNSINT(A)) = PNSINT(A)$.
(3) $PNSCL(PNSCL((A)) = PNSCL(A)$.
(4) *Let A, B be two Pythagorean neutrosophic semi-open sets, then*
$$PNsOS(A) \cup PNsOS(B) = PNsOS(A \cup B).$$
(5) *Let A, B be two Pythagorean neutrosophic semi-closed sets, then*
$$PNsCS(A) \cap PNsCS(B) = PNsCS(A \cap B).$$
(6) *For any two sets A, B, $PNSINT(A) \cap PNSINT(B) = PNSInt(A \cap B)$.*
(7) *For any two sets A, B, $PNSCL(A) \cup PNSCL(B) = PNSCL(A \cup B)$.*
(8) *If A is $PNsOS(X, \tau)$, then $A = PNSINT(A)$.*
(9) *If $A \subseteq B$, then $PNSINT(A) \subseteq PNSINT(B)$.*
(10) *For any two sets A, B, $PNSINT(A) \cup PNSINT(B) \subseteq PNSINT(A \cup B)$.*
(11) *If A is $PNsCS(X, \tau)$, then $A = PNSCL(A)$.*
(12) *If $A \subseteq B$, then $PNSCL(A) \subseteq PNSCL(B)$.*
(13) *For any two sets A, B, $PNSCL(A \cap B) \subseteq PNSCL(A) \cap PNSCL(B)$.*

PROOF. The proofs of (1), (2), (3), (4), (5), (9), (11) and (12) are followed by the Definitions 2.2 and 2.5. The proofs of (6), (7) and (8) are followed by the Definition 2.6 and the proofs of (10) and (13) are followed by the Definition 2.6 and parts (9) and (12) of this Theorem, □

The following example shows that the intersection of two Pythagorean neutrosophic pre-open sets need not be a Pythagorean neutrosophic pre-open set.

EXAMPLE 2.11. Let $X = \{q, w\}$ with
$\mathfrak{A} = \langle(0.3, 0.4, 0.1), (0.6, 0.3, 0.1)\rangle$, $\mathfrak{B} = \langle(0.1, 0.1, 0.5), (0.2, 0.6, 0.5)\rangle$,
$\mathfrak{C} = \langle(0.6, 0.8, 0.1), (0.6, 0.3, 0.3)\rangle$ and $\mathfrak{D} = \langle(0.1, 0.5, 0.1), (0.7, 0.5, 0.7)\rangle$.

Then, $\mathfrak{T}$ is a Pythagorean neutrosophic topological space. Now, choose

$$\mathfrak{A}_1 = \langle (0.4, 0.5, 0.4), (1.0, 0.1, 1.0) \rangle \text{ and } \mathfrak{A}_2 = \langle (1.0, 1.0, 0.3), (0.5, 0.7, 0.6) \rangle.$$

We can see that $\mathfrak{A}_1 \cap \mathfrak{A}_2$ is not a Pythagorean neutrosophic semi-open set of $(X, \mathfrak{T})$.

The following example shows that the union of two Pythagorean neutrosophic semi-closed sets need not be a Pythagorean neutrosophic pre-closed set.

EXAMPLE 2.12. By the example 2.11, we can imply that $\mathfrak{A}_1^c \cup \mathfrak{A}_2^c$ is not a Pythagorean neutrosophic semi-closed set of $(X, \mathfrak{T})$.

PROPOSITION 2.13. *Let A be a Pythagorean neutrosophic set in Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. If B is a Pythagorean neutrosophic semi-open set and $B \subseteq A \subseteq PNCl(PNInt(A))$, then A is a Pythagorean neutrosophic semi-open set.*

PROOF. Let B be a Pythagorean neutrosophic semi-open set, then by the Definition 2.2, $B \subseteq PNCl(PNInt(B))$, and so $B \subseteq A \subseteq PNCl(PNInt(B)) \subseteq PNCl(PNInt(A))$. In consequence, A is a Pythagorean neutrosophic semi-open set □

THEOREM 2.14. *Arbitrary union of Pythagorean neutrosophic semi-open sets is a Pythagorean neutrosophic semi-open set.*

PROOF. Let $A_1, A_2, ...A_n$ be a collection of Pythagorean neutrosophic pre-open sets. Then by the Definition 2.2,

$$A_1 \subseteq PNCl(PNInt(A_1)),\ A_2 \subseteq PNCl(PNInt(A_2)),\ ...,\\ A_n \subseteq PNCl(PNInt(A_n)).$$

Now, $A_1 \cup A_2 \cup ... \cup A_n \subseteq$

$$PNCl(PNInt(A_1)) \cup PNCl(PNInt(A_2)) \cup ... \cup PNCl(PNInt(A_n)),$$

by the Theorem 2.10 parts (7) and (10),

$$A_1 \cup A_2 \cup ... \cup A_n \subseteq PNCl(PNInt(A_1 \cup A_2 \cup ... \cup A_n)).$$

This proofs that $A_1 \cup A_2 \cup ... \cup A_n$ is a Pythagorean neutrosophic semi-open set. □

REMARK 2.15. By the Example 2.11, the arbitrary intersection of Pythagorean neotrosophic semi-open sets need not be a Pythagorean neotrosophic semi-open set.

PROPOSITION 2.16. *Arbitrary intersection of Pythagorean neutrosophic semi-closed sets is a Pythagorean neutrosophic semi-closed set.*

PROOF. The proof is followed by the Theorem 2.14 and parts (6) and (13) of the Theorem 2.10. □

REMARK 2.17. By the Example 2.12, the arbitrary union of Pythagorean neotrosophic semi-closed sets need not be a Pythagorean neotrosophic semi-closed set.

Theorem 2.18. *A Pythagorean neutrosophic set A in a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$ is Pythagorean neutrosophic semi-open if and only for every Pythagorean neutrosophic point $x_{a,b,c} \in A$ there exits a Pythagorean neutrosophic semi-open $B_{x_{a,b,c}}$ such that $x_{a,b,c} \in B_{x_{a,b,c}} \subseteq A$.*

Proof. Necessary: Let A be a Pythagorean neutrosophic semi-open set. Then, we have that $B_{x_{a,b,c}} = A$ for each $x_{a,b,c}$.

Sufficiency: Suppose that for every Pythagorean neutrosophic point $x_{a,b,c} \in A$, tehre exits a neutrosophic semi-open set $B_{x_{a,b,c}}$ such that $x_{a,b,c} \in B_{x_{a,b,c}} \subseteq A$. Thus, $A = \bigcup\{x_{a,b,c} : x_{a,b,c} \in A\} \subseteq \{B_{x_{a,b,c}} : x_{a,b,c} \in A\} \subseteq A$ and then, $A = \bigcup\{B_{x_{a,b,c}} : x_{a,b,c} \in A\}$. Therefore, by the Theorem 2.14, it is a Pythagorean neutrosophic semi-open set □

Definition 2.19. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic semi-open if $f(A)$ is Pythagorean neutrosophic semi-open set in Y for every Pythagorean neutrosophic open set A in X.

Proposition 2.20. *Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. If f is Pythagorean neutrosophic α-open, then f is Pythagorean neutrosophic semi-open.*

Proof. Let f be a Pythagorean neutrosophic α-open and A be a Pythagorean neutrosophic open set in X. Then, by hypothesis $f(A)$ is a Pythagorean neutrosophic α-open set in Y, by the Proposition 2.4, $f(A)$ is a Pythagorean neutrosophic semi-open set in X. Therefore, f is a Pythagorean neutrosophic semi-open function. □

3. Pythagorean neutrosophic semi-continuous functions

In this section, we used the notion of Pythagorean neutrosophic semi-open set to introduce and study the concepts of Pythagorean neutrosophic pre-continuous function and Pythagorean neutrosophic pre-homeomorphism, we also show some of their properties are shown.

Definition 3.1. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic semi-continuous if $f^{-1}(V)$ is a Pythagorean neutrosophic semi-open set in X for every Pythagorean neutrosophic open set V in Y.

Proposition 3.2. *Every Pythagorean neutrosophic continuous function is Pythagorean neutrosophic semi-continuous function.*

Proof. The proof is followed by the Definition 1.8 and Proposition 2.4. □

Proposition 3.3. *For a function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$, the following statements are equivalent:*

(1) *f is Pythagorean neutrosophic semi-continuous.*
(2) *$f^{-1}(V)$ is Pythagorean neutrosophic semi-closed set in X for every Pythagorean neutrosophic closed set V in Y.*

(3) $PNCl(PNInt(f^{-1}(V))) \subseteq f^{-1}(PNCl(V))$ *for every Pythagorean neutrosophic set* V *in* Y.

Proof. (1) $\Rightarrow$ (2): It is followed by the Definition 3.1.

(2) $\Rightarrow$ (3): Let A be a Pythagorean neutrosophic set in Y. Then, $PNCl(A)$ is a Pythagorean neutrosophic closed set. Now, by hypothesis, $f^{-1}(PNCl(A))$ is a Pythagorean neutrosophic semi-closed set in X and so $PNCl(PNInt(f^{-1}(A))) \subseteq PNCl(PNInt(f^{-1}(PNCl(A))))) \subseteq f^{-1}(PNCl(A))$.

(3) $\Rightarrow$ (1): Let A be a Pythagorean neutrosophic open set of Y. Then, $Cl(A)$ is a Pythagorean neutrosophic closed set of Y. Thus,

$$PNCl(PNInt(f^{-1}(Cl(A)))) \subseteq f^{-1}(PNCl(Cl(A))) = f^{-1}(A).$$

Indeed, $Cl(PNInt(PNCl(f^{-1}(A)))) = f^{-1}(A)$ and hence

$$\begin{aligned} Cl(PNInt(PNCl(f^{-1}(A)))) = PNCl(PNInt(f^{-1}(Cl(A)))) \subseteq f^{-1}(Cl(A)) \\ = Cl(f^{-1}(A)), \end{aligned}$$

this implies that $f^{-1}(A) \subseteq PNCl(PNInt(f^{-1}(A)))$. Therefore, $f^{-1}(A)$ is a Pythagorean neutrosophic semi-open set of X and by the Definition 3.1, f is a Pythagorean neutrosophic semi-continuous function. □

Definition 3.4. Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of a Pythagorean neutrosophic topological space $(X, \mathfrak{T})$. A Pythagorean neutrosophic set D of X is said to be Pythagorean neutrosophic neighbourhood of $x_{a,b,c}$ if there exits a Pythagorean neutrosophic open set V in X such that $x_{a,b,c} \in V \subseteq D$

Proposition 3.5. *Let* $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ *be a function where* $(X, \mathfrak{T})$ *and* $(Y, \mathfrak{Y})$ *are Pythagorean neutrosophic topological spaces. Then, the following statements are equivalent:*

(1) f *is a Pythagorean neutrosophic semi-continuous function.*
(2) *For each Pythagorean neutrosophic point* $x_{a,b,c}$ *and every Pythagorean neutrosophic* A *of* $f(x_{a,b,c})$, *there exits a Pythagorean neutrosophic semi-open set* B *of* X *such that* $x_{a,b,c} \in B \subseteq f^{-1}(A)$.
(3) *For each Pythagorean neutrosophic point* $x_{a,b,c} \in X$ *and every Pythagorean neutrosophic neighbourhood* A *of* $f(x_{a,b,c})$, *there exits a Pythagorean neutrosophic semi-open set* B *of* X *such that* $x_{a,b,c} \in B$ *and* $f(B) \subseteq A$.

Proof. (1) $\Rightarrow$ (2): Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of X and let A be a Pythagorean neutrosophic neighbourhood of $f(x_{a,b,c})$. Then, there exits a Pythagorean neutrosophic open set B of Y such that $f(x_{a,b,c}) \in B \subseteq A$. Now, since f is a Pythagorean neutrosophic semi-continuous function, we have that $f^{-1}(B)$ is a Pythagorean neutrosophic semi-open set of X and $x_{a,b,c} \in f^{-1}(f(x_{a,b,c})) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$ and this ends the proof.

(2) $\Rightarrow$ (3): Let $x_{a,b,c}$ be a Pythagorean neutrosophic point of X and let A be a Pythagorean neutrosophic neighbourhood of f($x_{a,b,c}$). By hypothesis, there exits a Pythagorean neutrosophic semi-open set B of X such that $x_{a,b,c} \in B \subseteq f^{-1}(A)$ and then $x_{a,b,c} \in B$ of X such that $f(B) \subseteq f(f^{-1}(A)) \subseteq A$ and this ends the proof.

(3) $\Rightarrow$ (1): Let B be a Pythagorean neutrosophic open set of Y and let $x_{a,b,c} \in f^{-1}(B)$ and so $f(x_{a,b,c}) \in B$ and then B is a Pythagorean neutrosophic neighbourhood of $f(x_{a,b,c})$. Now, since B is a Pythagorean neutrosophic open set and by hypothesis, there exits a Pythagorean neutrosophic semi-open set A of X such that $x_{a,b,c} \in A$ and $f(A) \subseteq B$. Indeed, $x_{a,b,c} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$ and this implies that $f^{-1}(B)$ is a Pythagorean neutrosophic semi-open set of X. Therefore, f is a Pythagorean neutrosophic semi-open continuous function. □

PROPOSITION 3.6. *Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. If f is a Pythagorean neutrosophic α-continuous function, then f is a Pythagorean neutrosophic semi-open function.*

PROOF. The proof is followed by the Definitions 1.11, 3.1 and Proposition 2.4. □

DEFINITION 3.7. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean nuetrosophic semi-homeomorphism if f and f^{-1} are Pythagorean neutrosophic semi-continuous functions.

EXAMPLE 3.8. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\mathfrak{T} = \{0_N, \mathfrak{U}_1, \mathfrak{U}_2, 1_N\}$ and $\mathfrak{Y} = \{0_N, \mathfrak{V}, 1_N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where $\mathfrak{U}_1 = \langle x, (0.2, 0.5, 0.1), (0.4, 0.2, 0.3)\rangle$, $\mathfrak{U}_2 = \langle x, (0.7, 0.6, 0.1), (0.5, 0.2, 0.8)\rangle$ and $\mathfrak{V} = \langle y, (0.5, 0.1, 0.7), (0.4, 0.4, 0.6)\rangle$. Then, we define the function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ as $f(q) = e$ and $f(w) = w$. We can see that f and f^{-1} are Pythagorean neutrosophic semi-continuous and then f is Pythagorean neutrosophic semi-homeomorphism.

DEFINITION 3.9. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean nuetrosophic homeomorphism if f and f^{-1} are Pythagorean neutrosophic continuous functions.

THEOREM 3.10. *Each Pythagorean neutrosophic homeomorphism is Pythagorean neustrosophic semi-homeomorphism.*

PROOF. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection and Pythagorean neutrosophic homeomorphism function in which f and f^{-1} are Pythagorean neutrosophic continuous functions. Since that every Pythagorean neutrosophic continuous function is Pythagorean neutrosophic semi-continuous, this implies that f and f^{-1} are Pythagorean neutrosophic semi-continuous functions. Therefore, f is a Pythagorean neutrosophic semi-homeomorphism. □

The following example shows that the converse of the above Theorem need not be true.

EXAMPLE 3.11. Let $X = \{q, w\}$ and $Y = \{e, r\}$. Then, $\mathfrak{T} = \{0_N, \mathfrak{U}_1, \mathfrak{U}_2, 1_N\}$ and $\mathfrak{Y} = \{0_N, \mathfrak{V}, 1_N\}$ are Pythagorean neutrosophic topological spaces on X and

Y respectively, where $\mathfrak{U}_1 = \langle x, (0.1, 0.7, 0.4), (0.2, 0.5, 0.3)\rangle$, $\mathfrak{U}_2 = \langle x, (0.1, 0.6, 0.9), (0.4, 0.1, 0.4)\rangle$ and $\mathfrak{V} = \langle y, (0.2, 0.7, 0.6), (0.1, 0.1, 0.3)\rangle$. Then, we define the function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ as $f(q) = e$ and $f(w) = w$. We can see that f is a Pythagorean neutrosophic semi-homeomorphism, but it is not a Pythagorean neutrosophic homeomorphism.

THEOREM 3.12. *Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a bijection function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, the following statements are equivalent:*

(1) *f is Pythagorean neutrosophic semi-closed.*
(2) *f is Pythagorean neutrosophic semi-open.*
(3) *f is Pythagorean neutrosophic semi-homeomorphism.*

PROOF. (1) $\Rightarrow$ (2) : Let f be a bijection Pythagorean neutrosophic semi-closed function. Then, f^{-1} is Pythagorean neutrosophic semi-continuous function. Now, since every Pythagorean neutrosophic open set of $(X, \mathfrak{T})$ is a Pythagorean neutrosophic semi-open set of $(X, \mathfrak{T})$, this implies that f is a Pythagorean neutrosophic semi-open function.

(2) $\Rightarrow$ (3) : Let f be a bijective Pythagorean neutrosophic semi-open function. Then, f^{-1} is a Pythagorean neutrosophic semi-continuous function. Indeed, f and f^{-1} are Pythagorean neutrosophic semi-continuous functions. Therefore, f is a Pythagorean neutrosophic semi-homeomorphism.

(3) $\Rightarrow$ (1) : Let f be a Pythagorean neutrosophic semi-homeomorphism. Then, f and f^{-1} are Pythagorean neutrosophic semi-continuous functions. Since every Pythagorean neutrosophic closed set of $(X, \mathfrak{T})$ is a Pythagorean neutrosophic semi-closed set of $(X, \mathfrak{T})$, this implies that f is a Pythagorean neutrosophic semi-closed function. □

The following example shows that the composition of two Pythagorean neutrosophic semi-homeomorphisms need not be a Pythagorean neutrosophic semi-homeomorphism.

EXAMPLE 3.13. Let $X = \{q, w\}$, $Y = \{e, r\}$ and $Z = \{t, y\}$. Then, $\mathfrak{T} = \{0_N, \mathfrak{U}, 1_N\}$, $\mathfrak{Y} = \{0_N, \mathfrak{V}, 1_N\}$ and $\mathfrak{Z} = \{0_N, \mathfrak{W}, 1_N\}$ are Pythagorean neutrosophic topological spaces on X, Y and Z respectively, where

$$\mathfrak{U} = \langle x, (0.1, 0.4, 0.1),\ (0.3, 0.7, 0.1)\rangle,\ \mathfrak{V} = \langle y, (0.5, 0.1, 0.3), (0.1, 0.6, 0.3)\rangle \text{ and}$$
$$\mathfrak{W} = \langle z, (0.6, 0.4, 0.1), (0.7, 0.5, 0.1)\rangle.$$

We define the function $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ as $f(q) = e$ and $f(w) = r$. Besides, we define the function $g : (Y, \mathfrak{Y}) \to (, \mathfrak{Z})$ as $g(e) = t$ and $g(r) = y$. We can see that f and g are Pythagorean neutrosophic semi-homeomorphism, but $g \circ f$ is not a Pythagorean neutrosophic semi-homeomorphism.

DEFINITION 3.14. Let $f : (X, \mathfrak{T}) \to (Y, \mathfrak{Y})$ be a function where $(X, \mathfrak{T})$ and $(Y, \mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic semi-irresolute if $f^{-1}(V)$ is a Pythagorean neutrosophic semi-open set in X for every Pythagorean neutrosophic semi-open set V in Y.

Definition 3.15. Let $f:(X,\mathfrak{T})\to(Y,\mathfrak{Y})$ be a bijection function where $(X,\mathfrak{T})$ and $(Y,\mathfrak{Y})$ are Pythagorean neutrosophic topological spaces. Then, f is said to be Pythagorean neutrosophic semii-homeomorphism if f and f^{-1} are Pythagorean neutrosophic semi-irresolute functions.

Theorem 3.16. *Every Pythagorean neutrosophic semii-homeomorphism is a Pythagorean neutrosophic semi-homeomorphism.*

Proof. Let $f:(X,\mathfrak{T})\to(Y,\mathfrak{Y})$ be a bijection and Pythagorean neutrosophic semii-homeomorphism function. Suppose that B is a Pythagorean neutrosophic closed set of $(Y,\mathfrak{Y})$, this implies that B is a Pythagorean neutrosophic semi-closed set of $(Y,\mathfrak{Y})$. Now, since f is Pythagorean neutrosophic semi-irresolute, $f^{-1}(B)$ is a Pythagorean neutrosophic semi-closed set of $(X,\mathfrak{T})$. Indeed, f is a Pythagorean neutrosophic semi-continuous function. therefore, f and f^{-1} are Pythagorean neutrosophi semi-continuous functions and then f is Pythagorean neutrosophic semi-homeomorphism. □

The following example shows that the converse of the above Theorem need not be true.

Example 3.17. Let $X=\{q,w\}$ and $Y=\{e,r\}$. Then, $\mathfrak{T}=\{0_N,\mathfrak{U}_1,\mathfrak{U}_2,1_N\}$ and $\mathfrak{Y}=\{0_N,\mathfrak{V},1_N\}$ are Pythagorean neutrosophic topological spaces on X and Y respectively, where $\mathfrak{U}_1=\langle x,(06,0.2,0.1),(0.5,0.3,0.1)\rangle$, $\mathfrak{U}_2=\langle x,(0.2,0.8,1.0),(0.1,0.3,0.3)\rangle$ and $\mathfrak{V}=\langle y,(0.4,0.9,0.9),(0.1,0.3,0.6)\rangle$. Then, we define the function $f:(X,\mathfrak{T})\to(Y,\mathfrak{Y})$ as $f(q)=e$ and $f(w)=w$. We can see that f is a Pythagorean neutrosophic semi-homeomorphism, but it is not a Pythagorean neutrosophic semii-homeomorphism.

Theorem 3.18. *If $f:(X,\mathfrak{T})\to(Y,\mathfrak{Y})$ and $g:(Y,\mathfrak{Y})\to(Z,\mathfrak{Z})$ are Pythagorean neutrosophic semii-homeomorphisms, then $g\circ f:(X,\mathfrak{T})\to(Z,\mathfrak{Z})$ is a Pythagorean neutrosophic semii-homeomorphism.*

Proof. Let f and g be two Pythagorean neutrosophic semi-homeomorphisms. Now, suppose that B is a Pythagorean neutrosophic semi-closed set of $(Z,\mathfrak{Z})$, then $g^{-1}(B)$ is a Pythagorean neutrosophic semi-closed set of $(Y,\mathfrak{Y})$. Then by hypothesis, $f^{-1}(g^{-1}(B))$ is a Pythagorean neutrosophic semi-closed set of $(X,\mathfrak{T})$. Therefore, $g\circ f$ is a Pythagorean neutrosophic semi-irresolute function Now, let β be a Pythagorean neutrosophic semi-closed set of $(X,\mathfrak{T})$. By assumption, $f(\beta)$ is a Pythagorean neutrosophic semi-closed set of $(Y,\mathfrak{Y})$. Then, by hypothesis, $g(f(\beta))$ is a Pythagorean neutrosophic semi-closed set of $(Z,\mathfrak{Z})$. This implies that $g\circ f$ is a Pythagorean neutrosophic semi-irresolute function and then $g\circ f$ is a Pythagorean neutrosophic semii-homeomorphism. □

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Master's degree in Mathematical Sciences, Universidad del Atlántico,
E-mail address: carosgranadosortiz@outlook.es