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THE COMPLEMENT CONNECTED EDGE GEODETIC NUMBER OF A GRAPH

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ABSTRACT. An edge geodetic set S of a connected graph G is said to be a complement connected edge geodetic set of G if $S = V$ or $G[V - S]$ is connected. The minimum cardinality of a complement connected edge geodetic set is the complement connected edge geodetic number and is denoted by $g_{cce}(G)$. Some general properties satisfied by this concept are studied Connected graphs of order $p \geqslant 3$ with $g_{cce}(G)$ to be $p - 1$ is given. Connected graphs of order p with $g_{cce}(G)$ to be 2 or p are characterized. It is shown that for every pair a and b of integers $2 \leqslant a \leqslant b$, there exists a connected graph G with $g_e(G) = a$ and $g_{cce}(G) = b$, where $g_e(G)$ is the edge geodetic number of G.

1. Introduction

By a graph $G = (V, E)$, we mean a finite,undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by p and q respectively. For basic graph theoretic terminology, we refer to [1]. $N(v) = \{u \in V(G) : uv \in E(G)\}$ is called the *neighborhood* of the vertex v in G. The *degree* of a vertex of a graph is the number of edges that are incident to the vertex and is denoted $deg(v)$. The *maximum degree* of a graph G, denoted by $\triangle(G)$, and the *minimum degree* of a graph denoted by $\delta(G)$ are the maximum and minimum degree of its

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vertices. Let $S \subset V$ be any subset of vertices of G. Then the *induced subgraph* $G[S]$ is the graph whose vertex set is S and whose edge set consists of all of the edges in E that have both endpoints in S. A vertex v is called an *extreme vertex* if $G[N(v)]$ is complete. A vertex $v \in V(G)$ in a connected graph G is said to be *semi-extreme vertex* of G if $\triangle(G[N(v)]) = |N(v)| - 1$. A graph G is said to be *semi-complete graph* if every vertex of G ia a semi-extreme vertex of G.

The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest u-v path in G. An *u-v path* of length $d(u, v)$ is called an *u-v geodesic*. A vertex x is said to lie on an u-v geodesic P if x is a vertex of P including the vertices u and v. The *eccentricity* $e(v)$ of a vertex v in G is the maximum distance from v and a vertex of G,(i.e) $e(v) = max\{d(v, u) : u \in V\}$. The minimum eccentricity among the vertices of G is the *radius*, *radG* and the maximum eccentricity is its *diameter*, *diamG*. We denote $rad(G)$ by r and $diamG$ by d. Two vertices u and v of G are *antipodal vertex* if $d(u, v) = d$. A vertex v is called a *peripheral vertex* of G, if $e(v) = d$. For two vertices u and v, the *closed interval* $I[u, v]$ consists of u and v together with all edges lying in an u-v geodesic. If u and v are adjacent, then $I[u, v] = \{u, v\}$. For a set S of vertices, let $I[S] = \cup_{u,v\in S} I[u, v]$. Then certainly $S \subseteq I[S]$. A set $S \subseteq V(G)$ is called a *geodetic set* of G if $I[S] = V$. The *geodetic number* $g(G)$ of G is the minimum order of its geodetic sets and any geodetic set of order $g(G)$ is a *geodetic basis* or a *g-set* of G. The geodetic number of a graph was studied in [**1, 3, 4, 6, 7**] . For two vertices u and v, the closed interval $I_e[u, v]$ consists of all edges lying in a u-v geodesic. If u and v are adjacent, then $I_e[u, v] = \{uv\}$. For a set S of vertices, let $I_e[S] = \cup_{u,v\in S} I_e[u, v]$. A set $S \subseteq V(G)$ is called an edge geodetic set of G if $I_e[S] = E$. The *edge geodetic number* $g_e(G)$ of G is the minimum order of its edge geodetic sets and any edge geodetic set of order $g_e(G)$ is an *edge geodetic basis* or a *g_e-set* of G. The edge geodetic number was studied in [**2, 5, 6, 9, 10, 11, 12, 13, 14, 15**]. A graph G is said to be *geodetic graph* if there exists exactly one geodesic between every pair of vertices.

In an application point of view, a computer network is a collection of computers where each one acts as a message sender and receiver in the network. In a computer network, leading computers are a set of computers, which cover the entire edge of the network with the property of message passing through the shortest communication path. Even though these set of leading computers fail, in order to make the network fault-tolerant, the rest of the computers are still able to communicate with each other. Consider a computer network as a graph model and each computer as a vertex. Then the minimum cardinality of a set of leading computers is a minimum complement connected edge geodetic set for the graph representing computer network. The following theorems are used in the sequel.

THEOREM 1.1 ([**8**]). *Each semi extreme vertex of a connected graph belongs to every edge geodetic set of G.*

THEOREM 1.2 ([**8**]). *For a non-trivial tree T,*

$$g_e(T) = \text{ number of end vertices of } T.$$

THEOREM 1.3 ([**8**]). *For the complete graph $G = K_p$ $(p \geqslant 3)$,*

$$g_e(G) = p.$$

2. The complement connected edge geodetic number of a graph

DEFINITION 2.1. An edge geodetic set S of a connected graph G is said to be a complement connected edge geodetic set of G if $S = V$ or $G[V - S]$ is connected. The minimum cardinality of a complement connected edge geodetic set is the *complement connected edge geodetic number* of G is denoted by $g_{cce}(G)$. A complement connected edge geodetic set of minimum cardinality is called the g_{cce}-set of G.

EXAMPLE 2.1. For the graph G given in Figure 1, $S = \{v_1, v_3, v_4, v_6\}$ is a g_e-set of G and so $g_e(G) = 4$. Since the subgraph $G[V - S]$ is disconnected, S is not a complement connected edge geodetic set of G and so $g_{cce}(G) \geqslant 5$. Let $S_1 = \{v_1, v_3, v_4, v_5, v_6\}$. Then S is a complement connected edge geodetic set of G so that $g_{cce}(G) = 5$.

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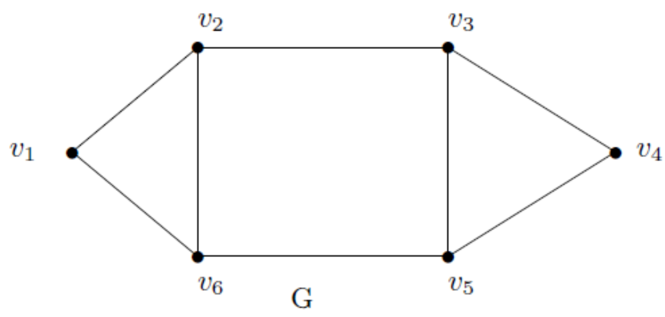

FIGURE 1. Graph 1

THEOREM 2.1. *For a connected graph G of order $p \geqslant 2$,*

$$2 \leqslant g_e(G) \leqslant g_{cce}(G) \leqslant p.$$

PROOF. Any edge geodetic set needs at least two vertices and so $g_e(G) \geqslant 2$. Since every complement connected edge geodetic set is also an edge geodetic set, $g_e(G) \leqslant g_{cce}(G)$. Also, since $V(G)$ is a complement connected edge geodetic set, it is clear that $g_{cce}(G) \leqslant p$. Thus

$$2 \leqslant g_e(G) \leqslant g_{cce}(G) \leqslant p.$$

□

REMARK 2.1. The bounds in Theorem 2.1 are sharp. For any non-trivial path P_p, the set of two end vertices is the unique edge geodetic set so that $g_e(P_p) = 2$. For any non-trivial tree T, the set of all end vertices is the unique geodetic set as well as unique edge complement connected edge geodetic set and so $g_e(T) = g_{cce}(T)$. For the complete graph $G = K_p$, $g_{cce}(K_p) = p$. Also the inequalities in the Theorem 2.1 are strict. For the graph G given in Figure 1,

$$g_e(G) = 4 \text{ and } g_{cce}(G) = 5 \text{ and } p = 6.$$

Thus $2 < g_e(G) < g_{cce}(G) < p$.

THEOREM 2.2. *Each semi extreme vertex of a graph G belongs to every complement connected edge geodetic set of G.*

PROOF. Since every complement connected edge geodetic set is an edge geodetic set, the result follows from Theorem 1.1. $\square$

OBSERVATION 2.1. (i). *For the complete graph K_p $(p \geqslant 2)$, $g_{cce}(K_p) = p$.*
(ii). *For a semi-complete graph G, $g_{cce}(K_p) = p$.*

THEOREM 2.3. *For any tree T with k end vertices, $g_{cce}(T) = k$.*

PROOF. Let S be the set of all end vertices of T. By Theorem 2.2, $g_{cce}(T) \geqslant \mid S \mid$. Since the subgraph $G[V - S]$ is connected, S is the unique complement connected edge geodetic set of T. Therefore $g_{cce}(T) =\mid S \mid= k$. $\square$

THEOREM 2.4. *For the cycle $G = C_p$ $(p \geqslant 3)$,*

$$g_{cce}(C_p) = \begin{cases} \dfrac{p}{2} + 1 & \textit{if } p \textit{ is even} \\ \left\lceil \dfrac{p}{2} \right\rceil + 1 & \textit{if } p \textit{ is odd} \end{cases}$$

PROOF. Let us assume that p is even. Let $p = 2k$. Let $C_{2k} = v_1, v_2, ...v_{2k}, v_1$. Let $S = \{x, y\}$ be a set of antipodal vertices of C_p. Since $I_e(S) = E(C_p)$, S is an edge geodetic set of G. Since $G[V - S]$ is not connected, S is not a complement connected edge geodetic set of C_p. Let $S_1 = \{v_1, v_2, ..., v_{k+1}\}$. Then $G[V - S_1]$ is complement connected edge geodetic set of C_p and so $g_{cce}(G) \leqslant k + 1$. We prove that $g_{cce}(C_p) = k + 1$. On the contrary suppose that $g_{cce}(C_p) \leqslant k$. Then there exists a g_{cce}-set $S^{'}$ such that $|S^{'}| \leqslant k$. Since $|S^{'}| \leqslant k$, $S^{'}$ contains no antipodal vertices of C_p. Therefore $I_e[S^{'}] \neq E(C_p)$. Hence it follows that $S^{'}$ is not a g_{cce}-set of G, which is a contradiction. Therefore

$$g_{cce}(G) = k + 1 = \frac{p}{2} + 1.$$

Let us assume that p is odd. Let $p = 2k + 1$. Let $C_{2k+1} = v_1, v_2, ..., v_{2k+1}, v_1$. Let x be a vertex of C_p and y, z be the two antipodal vertices of x. Then $M = \{x, y, z\}$ is an edge geodetic set of G. $G[V - M]$ is not connected, M is not a complement connected edge geodetic set of C_p. Let $M_1 = \{v_1, v_2, ..., v_{k+1}, v_{k+2}\}$. Then $G[V - M_1]$ is complement connected edge geodetic set of C_p and so $g_{cce}(C_p) < k + 2$. We prove that $g_{cce}(C_p) = k + 2$. Suppose that $g_{cce}(C_p) \leqslant k + 2$. Then there exists an edge geodetic set $M^{'}$ such that $|M^{'}| < k + 1$. Let $x^{'} \in M^{'}$ and $y^{'}, z^{'}$ be the antipodal vertices of $x^{'}$. Since $|M^{'}| \leqslant k + 1$, $\{y^{'}, z^{'}\} \nsubseteq M^{'}$. Therefore $I_e[M^{'}] \neq E(C_p)$. Hence it follows that $M^{'}$ is not a complement connected edge geodetic set of C_p, which is a contradiction. Therefore

$$g_{cce}(G) = k + 2 = \left\lceil \frac{p}{2} \right\rceil + 1.$$

$\square$

THEOREM 2.5. *For the complete bipartite graph $G = K_{m,n}$ $(2 \leqslant m \leqslant n)$,*

$$g_{cce}(G) = m + n.$$

PROOF. Let $X = \{x_1, x_2, .., x_m\}$, and $Y = \{y_1, y_2, .., y_n\}$ be the bipartition of G. Then $S = X$ and $S_1 = Y$ are the edge geodetic sets of G. Since $G[V - S]$ and $G[V - S_1]$ are not connected, S and S_1 are not connected edge geodetic sets of G. Hence it follows that $V(G)$ is the unique complement connected edge geodetic set of G so that $g_{cce}(G) = m + n$. □

THEOREM 2.6. *For any graph G, no cut vertex of G belongs to any minimum complement connected edge geodetic set of G.*

PROOF. Let S be g_{cce}-set of G and $v \in S$ be a vertex of G. We have to prove that v is not a cut vertex of G. On the contrary suppose that v is a cut vertex of G. Let $G_1, G_2, ..., G_r$ $(r \geqslant 2)$ be the components of $G - v$. Then v is a adjacent to at least one element of each G_i $(1 \leqslant i \leqslant r)$. Let $S^{'} = S - \{v\}$ and uw be an edge of G. Since S and $S^{'}$ are complement connected edge geodetic sets of G, $uw \in I_e[x, v]$ where $x \in S$. Without loss of generality, let $x \in G_1$. Let us assume the $x - v$ geodesic be P. Let us assume that $y \in G_k$ $(k \neq 1)$. Since S is a g_{cce}-set of G, $vy \in I_e[v, z]$ where $z \in S$. Hence it follows that $z \neq v$. Since v is a cut vertex of G, $P \cup Q$ is a $x - z$ geodesic of G. Thus the edge $uw \in I_e[x, z]$, where $x, z \in S^{'}$. Therefore $S^{'}$ is a complement connected edge geodetic set of G with $|S^{'}| = |S| - 1$, which is a contradiction to S is a g_{cce}-set of G. Therefore $v \in S$. Hence no cut vertex of G belongs to any edge g_{cce}-set of G. □

THEOREM 2.7. *For a connected graph G, $g_{cce}(G) = 2$ if and only if there exists peripheral vertices u and v such that every edge of G is on a diametral path joining u and v and $S = \{u, v\}$ is not a cut-set of G.*

PROOF. Let u and v be peripheral vertices of G such that every edge of G is on a diametral path joining u and v and $S = \{u, v\}$ is not a cut-set of G. Then S is an edge geodetic set of G and $G[V - S]$ is connected. Therefore S is a complement connected edge geodetic set of G so that $g_{cce}(G) = 2$. Conversely let $g_{cce}(G) = 2$. Let $S = \{u, v\}$ be a g_{cce}-set of G. Then $G[V - S]$ is connected. Therefore $S = \{u, v\}$ is not a cut-set of G. Since S is a complement connected edge geodetic set of G, $e \in I_e[S]$ for every $e \in E(G)$. Hence it follows that $v \in I_e[S]$ for every $v \in V$. We show that $d(u, v) = d(G)$. If $d(u, v) < d(G)$, then let x and y be two vertices of G such that $d(x, y) = d(G)$. Now, it follows that x and y lie on distinct geodesics joining u and v. Hence

$$d(u, v) = d(u, x) + d(x, v) \ ... \ (1) \quad \text{and} \quad d(u, v) = d(u, y) + d(y, v) ...(2).$$

By the triangle inequality, $d(x, y) \leqslant d(x, u) + d(u, y)$ (3). Since $d(u, v) < d(x, y)$,(3) becomes $d(u, v) < d(x, u) + d(u, y)$ ... (4). Using (4) in (1), we get $d(x, v) < d(x, u) + d(u, y) - d(u, x) = d(u, y)$. Thus $d(x, v) < d(u, y)$ ... (5). Also, by triangle inequality, we have $d(x, y) \leqslant d(x, v) + d(v, y)$ ... (6). Now, using (5) and (2),(6) becomes $d(x, y) < d(u, y) + d(v, y) = d(u, v)$. Thus, $d(G) < d(u, v)$, which is a contradiction. Hence $d(u, v) = d(G)$ and since $S = \{u, v\}$ is an edge geodetic minimum for G, It follows that each edge of G is on diametral path joining u and v. □

THEOREM 2.8. *Let G be a connected graph with $\triangle(G) = p-1$. If G contains only one universal vertex say v. Then $N(v) \subseteq S$ where S is a complement connected edge geodetic set of G.*

PROOF. Let $N(v) = \{v_1, v_2, ..., v_{p-1}\}$. We prove that $N(v) \subseteq S$. Suppose this is not the case. Then there exists at least one $v_l \in N(v)$ such that $v_l \notin S$. Then $vv_l \notin I_e[S]$. Hence S is not a complement connected edge geodetic set of G, which is a contradiction. Therefore $N(v) \subseteq S$. □

THEOREM 2.9. *Let G be a connected graph with $\triangle(G) = p-1$. If G contains only one universal vertex. Then $g_{cce}(G) = p-1$.*

PROOF. Let v be the only one universal vertex of G. Then by Theorem 2.8, $g_{cce}(G) \geqslant p-1$. Let S be a complement connected edge geodetic set of G. If v is a cut vertex of G. Then $v \notin S$. Then $S = N(v)$ is the unique minimum complement connected edge geodetic set of G so that $g_{cce}(G) = p-1$. If v is not a cut vertex of G, then let $S = N(v)$. Let $x \in N(v)$. Then $xv \in I_e[S]$. Let $x, y \in N(v)$ such that $xy \in E(G)$. Then $xy \in I_e[x, y]$ with $x, y \in S$. Therefore S is an edge geodetic set of G, Since $G[S-V]$ is connected, S is the unique minimum complement connected edge geodetic set of G so that $g_{cce}(G) = p-1$. □

REMARK 2.2. The converse of the Theorem 2.9 need not be true. For the graph G given in Figure 2, $g_{cce}(G) = 4 = p-1$. But G contains no universal vertices.

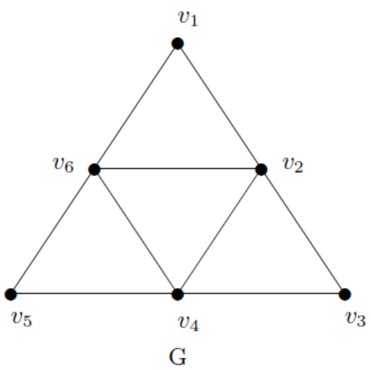

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FIGURE 2. Graph 2

COROLLARY 2.1. For the graph

$$G = K_1 + (m_1K_1 \cup m_2K_2 \cup ... \cup m_kK_k),$$

where $m_1 + m_2 + ... + m_k \geqslant 2$, $g_{cce}(G) = p-1$.

PROOF. This follows from Theorem 2.9. □

Corollary 2.2. For the wheel $W_p = K_1 + C_{p-1}$ $(p \geqslant 3)$, $g_{cce}(W_p) = p - 1$.

Proof. This follows from Theorem 2.9. □

Corollary 2.3. For the Fan graph $F_p = K_1 + P_{p-1}$, $g_{cce}(G) = p - 1$.

Proof. This follows from Theorem 2.9. □

Theorem 2.10. *For a connected graph G, $g_{cce}(G) = p$ if and only if G is semi complete.*

Proof. Let G be semi-complete. Then by Observation 2.1(ii), $g_{cce}(G) = p$.

Conversely, let $g_{cce}(G) = p$. We show that G is semi-complete. Suppose this is not the case. Then there exists $u \in V$ such that v is not a semi extreme vertex of G. Then for each $u \in N(v)$, there exists $x_u \in N(v)\backslash u$ such that $ux_u \notin E(G)$. Let $S = V - \{v\}$. Consider the edge uv. Since $u, x_u \in S$, the edge uv lies on the geodesic u, v, x_u. Therefore S is an edge geodetic set of G. Since $G[V - S]$ is connected, S is a complement connected edge geodetic set of G so that $g_{cce}(G) \leqslant p - 1$, which is a contradiction. Therefore, G is a semi complete. □

Theorem 2.11. *Let G be a connected graph with at least two universal vertices. Then $g_{cce}(G) = p$.*

Proof. Since G contains at least two universal vertices, G is semi complete. Then the result follows from Theorem 2.10. □

Corollary 2.4. For the graph $G = K_p - \{e\}$, $p \geqslant 4$, $g_{cce}(G) = p$.

Proof. Since G contains at least two universal vertices, the result follows from Theorem 2.11. □

Theorem 2.12. *Let G be a geodetic graph with diameter d. Then*

$$g_{cce}(G) \leqslant p - d + 1.$$

Proof. If $G = K_p$ $(p \geqslant 2)$, then $g_{cce}(G) = p$ so the result is trivial. So assume that $G \neq K_p$. Let u and v be two antipodal vertices of G. Then $d = d(u, v) \geqslant 2$. Let $P : u = u_0, u_1, u_2, ..., u_{d-1}, u_d = v$ be a diametral path of G. Let $S = V - \{u_1, u_2, ..., u_{d-1}\}$. We prove that S is a complement connected edge geodetic set of G. Let $e \in E$. If $e \in E(P)$, then $e \in I_e[u, v]$. If e is not incident with any vertex of P, then $e \in I_e[S]$. Now let $e = v_i x$ $(1 \leqslant i \leqslant d-1)$ such that $x \notin P$. We claim that either $u = u_0, u_1, ..., u_{i-1}, u_i x$ or $x u_i, u_{i+1}, ..., u_{d-1}, u_d = v$ is geodesic. Suppose that both of them are not geodesics. Let $P_1 : u = u_0, v_1, v_2, ..., v_l = x$ and $P_2 : u = v_d, w_1, w_2, ..., w_k = x$ be two geodesics. Then P_1 does not contain at least one edge from P. Similarly P_2 does not contain at least one edge from P. Also we have $l \leqslant i$ and $k \leqslant d - i$. Now $P_1 \cup P_2$ is an u-v walk of length $\leqslant l + k \leqslant d$ such that does not contain at least one edge from P. Hence it follows that $P_1 \cup P_2$ contains an u-v path $P^{'}$ of length $\leqslant d$ such that $P^{'} \neq P$. If $\mid P^{'} \mid < d$, then it is a contradiction to $d(u, v) = d$. If $\mid P^{'} \mid = d$, then since $P^{'} \neq P$, we have two different

u-v geodesics, which is a contradiction to G a geodetic graph. Thus we see that either $u = u_0, u_1, ..., u_{i-1}, u_i x$ or $x u_i, u_{i+1}, ..., u_{d-1}, u_d = v$ is the geodesic. Hence it follows that either $e \in I_e[u, x]$ or $e \in I_e[x, v]$. Thus S is an edge geodetic set of G. Since $G[V - S] = G[P]$, which is a connected. Therefore S is a complement connected edge geodetic set of G so that $g_{cce}(G) \leqslant p - d + 1$. □

Example 2.2. The bounds in Theorem 2.12 is sharp. For the geodetic graph given in Figure 3 as the 'Graph 3',

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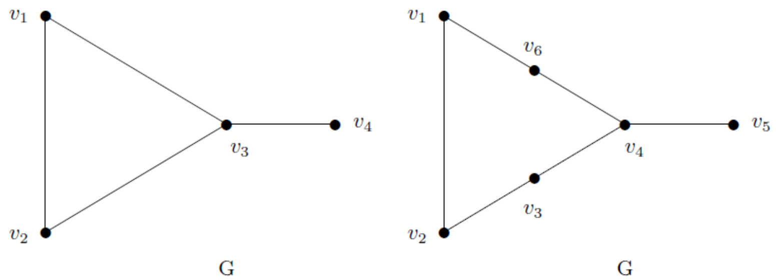

Figure 3. Graph 3 and Graph 4

$$d = 2, p = 4, p - d + 1 = 3 \text{ and } g_{cce}(G) = 3 \text{ so that } g_{cce}(G) = p - d + 1.$$

Also the bound in Theorem 2.12 can be strict. For the geodetic graph G given in Figure 3 as the 'Graph 4', $d = 3, p = 6, p - d + 1 = 4$ and $g_{cce}(G) = 3$. Thus $g_{cce}(G) < p - d + 1$.

Remark 2.3. The Theorem 2.12 is not true if G is not a geodetic graph. For the graph G given in Figure 4 as the 'Graph 5', $p = 6, d = 3, p - d + 1 = 4$ and $g_{cce}(G) = 6$ so that

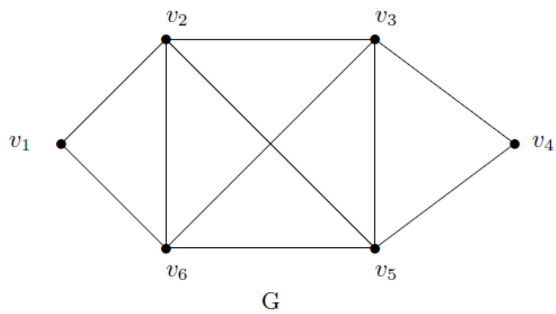

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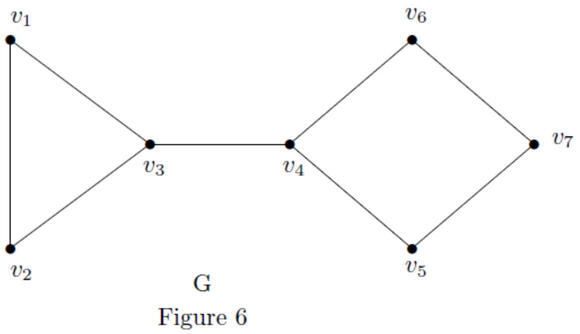
Figure 4. Graph 5

Remark 2.4. The converse of the Theorem 2.12 need not be true. For the graph G given in Figure 5 as the 'Graph 6', $p = 7, d = 4, g_{cce}(G) = 3$ and $p-d+1 = 4$. Thus $g_{cce}(G) \leqslant p - d + 1$. However G is not a geodetic graph.



6.eps 6.eps

Figure 5. Graph 6

Theorem 2.13. *Let G be a geodetic graph. Then $g_{cce}(G) = p$ if and only if $G = K_p$.*

Proof. If $G = K_p$, then by Observation 2.1 (i), $g_{cce}(G) = p$. Conversely let $g_{cce}(G) = p$. Suppose that $G \neq K_p$. Then $d \geqslant 2$. By Theorem 2.12, $g_{cce}(G) \leqslant p-1$, which is a contradiction. Therefore $G = K_p$. $\square$

In view of Theorem 2.1, We have the following realization result.

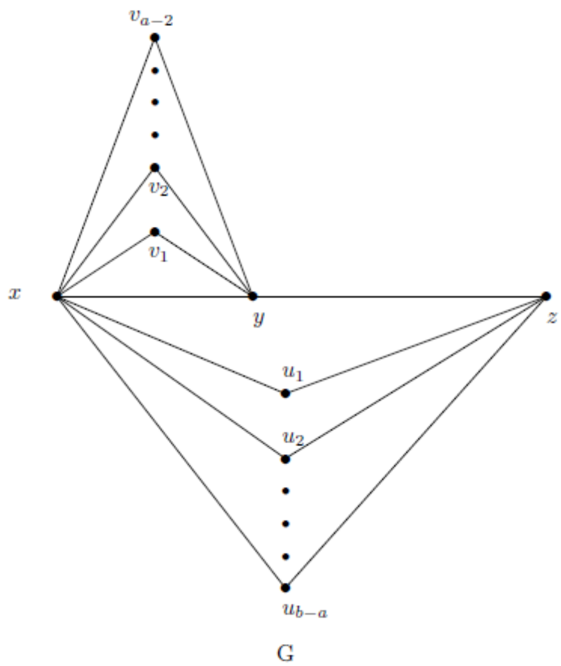
Theorem 2.14. *For every pair a and b of integers a and b with $2 \leqslant a \leqslant b$, there exists a connected graph G with $g_e(G) = a$ and $g_{cce}(G) = b$.*

Proof. **Case 1.** Let $a = b$, For $a = 2$, let $G = P_p$ $(p \geqslant 3)$. Then by Theorems 1.2 and 2.3, $g_e(G) = g_{cce}(G) = 2 = a$. For $3 \leqslant a = b$, let $G = K_a$. Then by Theorem 1.3 and Observation 2.1(i), $g_e(G) = g_{cce}(G) = a$.

Case 2. $3 \leqslant a < b$, Let $P : x, y, z$ be a path on three vertices. Let G be the graph given in Figure 6 as the 'Graph 7' obtained from P by adding the new vertices $v_1, v_2, ..., v_{a-2}$ and $u_1, u_2, ..., u_{b-a}$ and introducing the edges $u_i x, v_i y$ for all i $(1 \leqslant i \leqslant a-2)$ and xu_i, zv_i $(1 \leqslant i \leqslant b-a)$.

First we show that $g_e(G) = a$. Let $Z = \{v_1, v_2, ..., v_{a-2}\}$ be the set of all extreme vertices of G. Then by Theorem 1.1, Z is a subset of every edge geodetic set of G and so $g_e(G) \geqslant a - 2$. Since $I_e[Z] \neq E$, Z is not an edge geodetic set of G. It is easily verified that $Z \cup \{u\}$, where $u \notin Z$ is not an edge geodetic set of G and so $g_e(G) \geqslant a$. $Z' = Z \cup \{x, z\}$. Then $I_e[Z'] = E$ and so Z' is an edge geodetic set of G so that $g_e(G) = a$.

Next we prove that $g_{cce}(G) = b$. Since $G[V - Z']$ is disconnected, Z' is not a complement connected edge geodetic set of G. By Theorem 2.2, Z is a subset of every complement connected edge geodetic set of G. Also it is easily observed that every complement connected edge geodetic set of G contains each u_i $(1 \leqslant i \leqslant b-a)$ and so $g_{cce}(G) \geqslant a-2+b-a = b-2$. Let $S = Z \cup \{u_1, u_2, ..., u_{b-a}\}$. Then $I_e[Z] \neq E$, and so S is not a complement connected edge geodetic set of G. The edges xy and yz lies only on $x-z$ geodesic. $S^{'} = S \cup \{x, z\}$. Then $I_e[S^{'}] = E$ and so $S^{'}$ is an edge geodetic set of G. Since $G[V - S^{'}]$ is connected, $S^{'}$ is a complement connected edge geodetic set of G so that $g_{cce}(G) = b$. $\square$



7.eps 7.eps

Figure 6. Graph 7

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