BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., Vol. **11**(1)(2021), 91-98
DOI: 10.7251/BIMVI2101091S

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

C-BOCHNER CURVATURE TENSOR UNDER D-HOMOTHETIC DEFORMATION IN LP-SASAKIAN MANIFOLD

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ABSTRACT. The object of the present paper is to study the C-Bochner curvature tensor under D-homothetic deformation in LP-Sasakian manifolds. Also, we have proved some important results.

1. Introduction

The notion of Lorentzian para-Sasakian manifold was introduced by Matsumoto [**7**] in 1989. Then Mihai and Rosca [**9**] defined the same notion independently and they obtained several results on this manifold. LP-Sasakian manifolds have also been studied by Matsumoto and Mihai [**8**], De and Shaikh [**5**], Ozgur [**10**] and others.

Let $M(\phi, \xi, \eta, g)$ be a almost contact metric manifold with dimension

$$M = m = 2n + 1.$$

The equation $\eta = 0$ defines an (m-1)-dimensional distribution D on M [**11**]. By an $(m-1)$ homothetic deformation or D-Homothetic deformation [**12**], we mean change of structure tensors of the form:

$$\bar{\eta} = a\eta, \;\; \bar{\xi} = \frac{1}{a}\xi, \;\; \bar{\phi} = \phi, \;\; \bar{g} = ag + a(a-1)\eta \otimes \eta, \tag{1.1}$$

where a is a positive constant. If $M(\phi, \xi, \eta, g)$ is an almost constant metric structure with contact form η, then $M(\bar{\phi}, \bar{\xi}, \bar{\eta}, \bar{g})$ is also an almost contact metric structure

2010 *Mathematics Subject Classification.* 53C15, 53C25.
Key words and phrases. LP-Sasakian manifold, D-Homothetic deformation, C-Bochner curvature tensor.
Communicated by Daniel A. Romano.

[**12**].

$$W(X_1, X_2) = (1-a)[\eta(X_2)\phi X_1 + \eta(X_1)\phi X_2] + \tag{1.2}$$

$$\frac{1}{2}\left(1-\frac{1}{a}\right)[(\nabla_{X_1}\eta)X_2 + (\nabla_{X_2}\eta)X_1]\xi \tag{1.3}$$

for all $X_1, X_2 \in \chi(M)$. If R and $\bar{R}$ denote respectively the curvature tensor of the manifold $M(\phi, \xi, \eta, g)$ and $M(\bar{\phi}, \bar{\xi}, \bar{\eta}, \bar{g})$, then from [**12**] we have:

$$\begin{aligned}\bar{R}(X_1, X_2)X_3 =& R(X_1, X_2)X_3 + (\nabla_{X_1}W)(X_3, X_2) - (\nabla_{X_2}W)(X_3, X_1) \\ &+ W(W(X_3, X_2), X_1) - W(W(X_3, X_1), X_2),\end{aligned} \tag{1.4}$$

for all $X_1, X_2, X_3 \in \chi(M)$.

A Riemannian manifold M is called locally symmetric if the curvature tensor satisfies $\nabla R = 0$, where ∇ denotes the Levi-Civita connection. A Riemannian manifold M is said to be semi-symmetric if its curvature tensor R satisfies

$$R(X_1, X_2).R = 0, \quad \forall\ X_1, X_2 \in T_p(M) \tag{1.5}$$

where $R(X_1, X_2)$ acts on R as a derivation. In [**13**], the author shown that a semi-symmetric K-contact manifold M is locally isometric to the unit sphere $S^n(1)$.

In [**1, 2**], Bochner introduced a Kähler analogue of the Weyl conformal curvatur tensor by purely formal considerations, which is now well known as the Bochner curvature tensor. A geometric meaning of the Bochner curvature tensor is given by Blair [**3**].

2. Preliminaries

Let M^{2n+1} be a differentiable manifold endowed with a $(1,1)$ tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric field g which satisfies ([**7, 8**]):

$$\phi^2 = X_1 + \eta(X_1)\xi,\ \eta(\xi) = -1,\ \phi\xi = 0,\ \eta o\phi = 0, \tag{2.1}$$

$$g(X_1, \xi) = \eta(X_1),\ g(\phi X_1, \phi X_2) = g(X_1, X_2) + \eta(X_1)\eta(X_2),\ \nabla_{X_1}\xi = \phi X_1, \tag{2.2}$$

$$(\nabla_{X_1}\phi)X_2 = g(X_1, X_2)\xi + \eta(X_1)X_2 + 2\eta(X_1)\eta(X_2)\xi, \tag{2.3}$$

for all vector fields $X_1, X_2, X_3 \in T_pM$. Here ∇ denotes the operator of covariant differentiation with respect to Lorentzian metric g. Also in LP-Sasakian manifold, the following relations hold:

$$\begin{aligned}g(R(X_1, X_2)X_3, \xi) &= \eta(R(X_1, X_2)X_3) \\ &= g(X_2, X_3)\eta(X_1) - g(X_1, X_3)\eta(X_2),\end{aligned} \tag{2.4}$$

$$R(\xi, X_1)X_2 = g(X_1, X_2)\xi - \eta(X_2)X_1, \tag{2.5}$$

$$R(X_1, X_2)\xi = \eta(X_1)X_2 - \eta(X_2)X_1 \tag{2.6}$$

$$R(\xi, X_1)\xi = X_1 + \eta(X_1)\xi, \tag{2.7}$$

$$S(X_1, \xi) = 2n\eta(X_1), \tag{2.8}$$

for all vector fields X_1, X_2, X_3, where S is the Ricci tensor and R is the Riemannian curvature tensor. Also D-homothetic deformation of LP-Sasakian manifolds [**4**] for a dimension M^{2n+1} the following relations hold:

$$\begin{aligned}\bar{R}(X_1, X_2)X_3 = &\\ R(X_1, X_2)X_3 + (1-a)[g(X_1, X_3)\eta(X_2)\xi - g(X_2, X_3)\eta(X_1)\xi + &\\ 2\eta(X_2)\eta(X_3)X_1 - 2\eta(X_1)\eta(X_3)X_2 + g(\phi X_1, X_3)\phi X_2 - g(\phi X_2, X_3)\phi X_1] + &\\ \frac{a-1}{a}[g(\phi X_3, X_2)\phi X_1 - g(\phi X_3, X_1)\phi X_2] + &\\ (1-a)^2[\eta(X_1)\eta(X_3)X_2 - \eta(X_2)\eta(X_3)X_1] - &\\ \frac{(1-a)^2}{a}[g(\phi X_3, X_1)\phi X_2 - g(\phi X_3, X_2)\phi X_1], &\end{aligned} \tag{2.9}$$

$$\begin{aligned}\bar{S} = S(X_1, X_2) - [2(1-a) + (1-a)^2]g(X_1, X_2) - &\\ [2(1-a)(n-1) + (1-a)^2]\eta(X_1)\eta(X_2), &\end{aligned} \tag{2.10}$$

$$\begin{aligned}\bar{Q}X_1 = QX_1 - [2(1-a) + (1-a)^2]X_1 - &\\ [2(1-a)(n-1) + (1-a)^2]\eta(X_1)\xi. &\end{aligned} \tag{2.11}$$

In contact manifolds, the Bochner tensor was reinterpreted by Matsumoto and Chuman [**6**] as a C-Bochner curvature tensor in Sasakian manifolds. They showed that a Sasakian space form is a space with a vanishing C-Bochner curvature tensor. In [**6**], the C-Bochner curvature tensor is given by:

$$\begin{aligned} B(X_1, X_2)X_3 = &\\ &\bar{R}(X_1, X_2)X_3 + \frac{1}{2(n+2)}[S(X_1, X_3)X_2 - S(X_2, X_3)X_1\\ &+ g(X_1, X_3)QX_2 - g(X_2, X_3)QX_1 + S(\phi X_1, X_3)\phi X_2 -\\ &S(\phi X_2, X_3)\phi X_1 + g(\phi X_1, X_3)Q\phi X_2 - g(\phi X_2, X_3)Q\phi X_1\\ &+ 2S(\phi X_1, X_2)\phi X_3 + 2g(\phi X_1, X_2)Q\phi X_3 - S(X_1, X_3)\eta(X_2)\xi +\\ &S(X_2, X_3)\eta(X_1)\xi - \eta(X_1)\eta(X_3)QX_2 + \eta(X_2)\eta(X_3)QX_1] -\\ &\frac{\tau+2n}{2(n+2)}[g(\phi X_1, X_3)\phi X_2 - g(\phi X_2, X_3)\phi X_1 + 2g(\phi X_1, X_2)\phi X_3]\\ &- \frac{\tau-4}{2(n+2)}[g(X_1, X_3)X_2 - g(X_2, X_3)X_1]\\ &+ \frac{\tau}{2(n+2)}[g(X_1, X_3)\eta(X_2)\xi g(X_2, X_3)\eta(X_1)\xi + \eta(X_1)\eta(X_3)X_2\\ &- \eta(X_2)\eta(X_3)X_1], \end{aligned} \tag{2.12}$$

where $\tau = \frac{r+2n}{2(n+2)}$, Q is the Ricci operator i.e., $g(QX_1, X_2) = S(X_1, X_2)$ and r

is the scalar curvature of the manifold. Further, the C-Bochner curvature tensor on D-homothetic deormation of LP-Sasakian manifold also satisfies the following conditions:

$$B(X_1, X_2)\xi = A_1(\eta(X_2)X_1 - \eta(X_1)X_2), \tag{2.13}$$

$$B(\xi, X_2)X_3 = A_1(g(X_2, X_3)\xi - \eta(X_3)X_2), \tag{2.14}$$

$$B(X_1, \xi)X_3 = A_1(\eta(X_3)X_1 - g(X_1, X_3)\xi), \tag{2.15}$$

$$B(\xi, X_2)\xi = A_1(\eta(X_2)\xi + X_2), \tag{2.16}$$

$$B(\xi, \xi)\xi = 0, \tag{2.17}$$

where

$$\begin{aligned} A_1 =& 1 - 2(1-a) + (1-a)^2 - \frac{1}{2(n+2)}[2n - 2(1-a) - (1-a)^2 + 2(1-a)(n-1) \\ &+ (1-a)^2] - \frac{2n}{n+2} + \frac{2(1-a)+(1-a)^2}{n+2} + \frac{\tau - 4}{2(n+2)} + \frac{\tau}{2(n+2)}. \end{aligned}$$

3. C-Bochner semi-symmetric D-Homothetic Deformation of LP-Sasakian manifold

C-Bochner semi-symmetry in an M^{2n+1} dimensional D-homothetic deformation of LP-Sasakian contact metric manifold satisfying $(\bar{R}(X_1, X_2).B)(Y_1, Y_2)Y_3 = 0$. It implies,

$$\begin{aligned} &\bar{R}(X_1, X_2)B(Y_1, Y_2)Y_3 - B(\bar{R}(X_1, X_2)Y_1, Y_2)Y_3 - B(Y_1, \bar{R}(X_1, X_2)Y_2)Y_3 - \\ &B(Y_1, Y_2)\bar{R}(X_1, X_2)Y_3 = 0, \end{aligned} \tag{3.1}$$

substitituting $X_1 = \xi$ in the equation (3.1), we get

$$\begin{aligned} &\bar{R}(\xi, X_2)B(Y_1, Y_2)Y_3 - B(\bar{R}(\xi, X_2)Y_1, Y_2)Y_3 - B(Y_1, \bar{R}(\xi, X_2)Y_2)Y_3 - \\ &B(Y_1, Y_2)\bar{R}(\xi, X_2)Y_3 = 0. \end{aligned} \tag{3.2}$$

In view of the equation (2.9), equation (3.2) reduces to

(3.3)

$$\begin{aligned} &g(X_2, B(Y_1, Y_2)Y_3)\xi - \eta(B(Y_1, Y_2)Y_3)X_2 + (1-a)[\eta(B(Y_1, Y_2)Y_3)\eta(X_2)\xi \\ &+ g(X_2, B(Y_1, Y_2)Y_3)\xi + 2\eta(X_2)\eta(B(Y_1, Y_2)Y_3)X_2 + 2\eta(B(Y_1, Y_2)Y_3)X_2] \\ &- (1-a)^2[\eta(B(Y_1, Y_2)Y_3)X_2 + \eta(X_2)\eta(B(Y_1, Y_2)Y_3)\xi] - g(X_2, Y_1)B(\xi, Y_2)Y_3 \\ &+ \eta(U)B(X_2, Y_2)Y_3 - (1-a)\eta(Y_1)\eta(X_2)B(\xi, Y_2)Y_3 - (1-a)g(X_2, Y_1)B(\xi, Y_2)Y_3 \\ &- 2(1-a)\eta(X_2)\eta(Y_1)B(\xi, Y_2)Y_3 - 2(1-a)\eta(Y_1)B(X_2, Y_2)Y_3 \\ &+ (1-a)^2\eta(Y_1)B(X_2, Y_2)Y_3 + (1-a)^2\eta(Y_1)\eta(X_2)B(\xi, Y_2)Y_3 \end{aligned}$$

$$
\begin{aligned}
&- g(X_2,Y_2)B(Y_1,\xi)Y_3 + \eta(Y_2)B(Y_1,X_2)Y_3\\
&- (1-a)\eta(Y_2)\eta(X_2)B(Y_1,\xi)Y_3 - (1-a)g(X_2,Y_2)B(Y_1,\xi)Y_3\\
&- 2(1-a)\eta(X_2)\eta(Y_2)B(Y_1,\xi)Y_3 - 2(1-a)\eta(Y_2)B(Y_1,X_2)Y_3\\
&+ (1-a)^2\eta(Y_2)B(Y_1,X_2)Y_3 + (1-a)^2\eta(X_2)\eta(Y_2)B(Y_1,\xi)Y_3\\
&- g(X_2,Y_3)B(Y_1,Y_2)\xi + \eta(Y_3)B(Y_1,Y_2)X_2 - (1-a)\eta(Y_3)\eta(X_2)B(Y_1,Y_2)\xi\\
&- (1-a)g(X_2,Y_3)B(Y_1,Y_2)\xi - 2(1-a)\eta(X_2)\eta(Y_3)B(Y_1,Y_2)\xi\\
&- 2(1-a)\eta(Y_3)B(Y_1,Y_2)X_2 + (1-a)^2\eta(Y_3)B(Y_1,Y_2)X_2\\
&+ (1-a)^2\eta(X_2)\eta(Y_3)B(Y_1,Y_2)\xi = 0,
\end{aligned}
$$

Substituting $Y_1 = Y_3 = \xi$, the above equation reduces to

(3.4)

$$
\begin{aligned}
&g(X_2,B(\xi,Y_2)\xi)\xi + (1-a)g(X_2,B(\xi,Y_2)\xi)\xi - \eta(X_2)B(\xi,Y_2)\xi - B(X_2,Y_2)\xi\\
&+ 2(1-a)\eta(X_2)B(\xi,Y_2)\xi + 2(1-a)B(X_2,Y_2)\xi - (1-a)^2B(X_2,Y_2)\xi\\
&- (1-a)^2\eta(X_2)B(\xi,Y_2)\xi + \eta(Y_2)B(\xi,X_2)\xi - 2(1-a)\eta(Y_2)B(\xi,X_2)\xi\\
&+ (1-a)^2\eta(Y_2)B(\xi,X_2)\xi - \eta(X_2)B(\xi,Y_2)\xi - B(\xi,Y_2)X_2\\
&+ 2(1-a)\eta(X_2)B(\xi,Y_2)\xi + 2(1-a)B(\xi,Y_2)X_2\\
&- (1-a)^2B(\xi,Y_2)X_2 - (1-a)^2\eta(X_2)B(\xi,Y_2)\xi = 0.
\end{aligned}
$$

Taking the inner product with ξ, equation (3.4) reduces to

$$
\begin{aligned}
&- g(X_2,B(\xi,Y_2)\xi) - (1-a)g(X_2,B(\xi,Y_2)\xi) - \eta(B(X_2,Y_2)\xi) + 2(1-a)\\
&\eta(B(X_2,Y_2)\xi) - (1-a)^2\eta(B(X_2,Y_2)\xi) - \eta(B(\xi,Y_2)X_2) + 2(1-a)\\
&\eta(B(\xi,Y_2)X_2) - (1-a)^2\eta(B(\xi,Y_2)X_2) = 0.
\end{aligned}
\tag{3.5}
$$

In view of (2.13), (2.14) and (2.16), equation (3.5) reduces to

$$
A_1(a^2 + a - 2)g(\phi Y_2, \phi X_2) = 0, \tag{3.6}
$$

since $g(\phi Y_2, \phi X_2) \neq 0$, therefore the equation (3.6) leads to $A_1(a^2 + a - 2) = 0$. Hence, we can state the following theorem:

Theorem 3.1. *If a M^{2n+1} dimensional D-homothetic deformation of LP-Sasakian manifold satisfying $B(\xi, Y_1).B = 0$, then $A_1(a^2 + a - 2) = 0$.*

4. D-Homothetic Deformation of LP-Sasakian manifold satisfying $B(\xi, Y_1).B = 0$

Let M^{2n+1} be a D-homothetic deformation of LP-Sasakian manifold which satisfies $(B(\xi, Y_1).B)(X_1, X_2)X_3 = 0$. It imples that

$$
\begin{aligned}
&B(\xi,Y_1)B(X_1,X_2)X_3 - B(B(\xi,Y_1)X_1,X_2)X_3\\
&- B(X_1,B(\xi,Y_1)X_2)X_3 - B(X_1,X_2)B(\xi,Y_1)X_3 = 0,
\end{aligned}
\tag{4.1}
$$

In view of (2.14), (4.1) reduces to

(4.2)
$$A_1[g(Y_1, B(X_1, X_2)X_3)\xi - \eta(B(X_1, X_2)X_3)Y_1 - g(Y_1, X_1)B(\xi, X_2)X_3 + \eta(X_1) B(Y_1, X_2)X_3 - g(Y_1, X_2)B(X_1, \xi)X_3 + \eta(X_2)B(X_1, Y_1)X_3 - g(Y_1, X_3)B(X_1, X_2)\xi + \eta(X_3)B(X_1, X_2)Y_1] = 0,$$

where $A_1 \neq 0$ and substituting $X_3 = \xi$, (4.2) reduces to

$$g(Y_1, B(X_1, X_2)\xi)\xi - \eta(B(X_1, X_2)\xi)Y_1 - g(Y_1, X_1)B(\xi, X_2)\xi + \eta(X_1)B(Y_1, X_2)\xi - g(Y_1, X_2)B(X_1, \xi)\xi + \eta(X_2)B(X_1, Y_1)\xi - g(Y_1, \xi)B(X_1, X_2)\xi + \eta(\xi)B(X_1, X_2)Y_1 = 0. \tag{4.3}$$

Taking inner product with ξ and using (2.13), (2.16), the equation (4.3) reduces to

$$B(X_1, X_2)Y_1 = A_1[g(Y_1, X_2)X_1 - g(Y_1, X_1)X_2]. \tag{4.4}$$

Hence we can state the following theorem:

Theorem 4.1. *If a M^{2n+1} dimensional D-homothetic deformation of LP-Sasakian manifold satisfying $B(\xi, Y_1).B = 0$ then M^{2n+1} is isometric to a hyperbolic space.*

5. D-Homothetic Deformation of LP-Sasakian manifold satisfying $B(\xi, Y_1).\bar{R} = 0$

Let M^{2n+1} be a D-homothetic deformation of LP-Sasakian manifold which satisfies $(B(\xi, Y_1).\bar{R})(X_1, X_2)X_3 = 0$. It imples that

$$B(\xi, Y_1)\bar{R}(X_1, X_2)X_3 - \bar{R}(B(\xi, Y_1)X_1, X_2)X_3 - \bar{R}(X_1, B(\xi, Y_1)X_2)X_3 - \bar{R}(X_1, X_2)B(\xi, Y_1)X_3 = 0. \tag{5.1}$$

In view of (2.14), (5.1) reduces to

$$A_1[g(Y_1, \bar{R}(X_1, X_2)X_3)\xi - \eta(\bar{R}(X_1, X_2)X_3)Y_1 - g(Y_1, X_1)\bar{R}(\xi, X_2)X_3 + \eta(X_1)\bar{R}(Y_1, X_2)X_3 - g(Y_1, X_2)\bar{R}(X_1, \xi)X_3 + \eta(X_2)\bar{R}(X_1, Y_1)X_3 - g(Y_1, X_3)\bar{R}(X_1, X_2)\xi + \eta(X_3)\bar{R}(X_1, X_2)Y_1] = 0, \tag{5.2}$$

where $A_1 \neq 0$, substituting $X_3 = \xi$ and using (2.9), equation (5.2) reduces to

(5.3)
$$g(Y_1, a^2(\eta(X_2)X_1 - \eta(X_1)X_2))\xi - \eta(a^2(\eta(X_2)X_1 - \eta(X_1)X_2))Y_1 - g(Y_1, X_1)(a^2 (\eta(X_2)\xi + X_2)) + \eta(X_1)a^2(\eta(X_2)Y_1 - \eta(Y_1)X_2) + g(Y_1, X_2)a^2(X_1 + \eta(X_1)\xi) + \eta(X_2)a^2(\eta(Y_1)X_1 - \eta(X_1)Y_1) - \eta(Y_1)a^2(\eta(X_2)X_1 - \eta(X_1)X_2) - \bar{R}(X_1, X_2)Y_1 = 0.$$

On simplification, the equation (5.3) reduces to

$$\bar{R}(X_1, X_2)Y_1 = a^2[g(Y_1, X_2)X_1 - g(Y_1, X_1)X_2]. \tag{5.4}$$

In the view of (2.9) and (5.4), we have:

(5.5)

$$R(X_1, X_2)Y_1 + (1-a)[g(X_1, Y_1)\eta(X_2)\xi - g(X_2, Y_1)\eta(X_1)\xi + 2\eta(X_2)\eta(Y_1)X_1 -$$
$$2\eta(X_1)\eta(Y_1)X_2 + g(\phi X_1, Y_1)\phi X_2 - g(\phi X_2, Y_1)\phi X_1] + \frac{a-1}{a}[g(\phi Y_1, X_2)\phi X_1 -$$
$$g(\phi Y_1, X_1)\phi X_2] + (1-a)^2[\eta(X_1)\eta(Y_1)X_2 - \eta(X_2)\eta(Y_1)X_1] - \frac{(1-a)^2}{a}[g(\phi Y_1, X_1)$$
$$\phi X_2 - g(\phi Y_1, X_2)\phi X_1] = a^2(g(Y_1, X_2)X_1 - g(Y_1, X_1)X_2),$$

where a is a positive integer and if $a = 1$, then the above equation reduces to

$$R(X_1, X_2)Y_1 = g(Y_1, X_2)X_1 - g(Y_1, X_1)X_2$$

. Hence, we can state the following theorem:

THEOREM 5.1. *If a M^{2n+1} dimensional D-Homothetic Deformation of LP-Sasakian manifold satisfying $B(\xi, Y_1).\bar{R} = 0$ then M^{2n+1} is isometric to a sphere.*

6. D-Homothetic Deformation of LP-Sasakian manifold satisfying $B(\xi, Y_1).\bar{S} = 0$

Let M^{2n+1} be a D-homothetic deformation of LP-Sasakian manifold which satisfies $(B(\xi, X_1).\bar{S} = 0$. It imples that

$$\bar{S}(B(\xi, X_1)X_2, \xi) + \bar{S}(X_2, B(\xi, X_1)\xi) = 0, \tag{6.1}$$

In view of (2.14), (6.1) reduces to

$$A_1[g(X_1, X_2)\bar{S}(\xi, \xi) - \eta(X_2)\bar{S}(X_1, \xi) + \eta(X_1)\bar{S}(X_2, \xi) + \bar{S}(X_1, X_2)] = 0, \tag{6.2}$$

where $A_1 \neq 0$ and using (2.10), (6.2) reduces to

(6.3)

$$[-2n + (2(1-a) + (1-a)^2) - (2(1-a)(n-1) + (1-a)^2)]g(X_1, X_2) + S(X_1, X_2)$$
$$-[2(1-a) + (1-a)^2]g(X_1, X_2) - [2(1-a)(n-1) + (1-a)^2]\eta(X_1)\eta(X_2) = 0,$$

where a is a positive integer and if $a = 1$, then the above equation reduces to

$$S(X_1, X_2) = 2ng(X_1, X_2). \tag{6.4}$$

Hence we can state the following theorem:

THEOREM 6.1. *If a M^{2n+1} dimensional D-homothetic deformation of LP-Sasakian manifold satisfying $B(\xi, Y_1).\bar{S} = 0$ then M^{2n+1} is an Einstein manifold.*

Acknowledgements

The authors would like to thank the referees for their invaluable comments and suggestions which led to the improvement of the manuscript.

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Received by editors 18.12.2019; Revised version 30.06.2020; Available online 06.07.2020.

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