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ON GENERALIZED SYMMETRIC f-BIDERIVATIONS OF LATTICES

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Abstract. In this paper, we introduce the notion of generalized symmetric f-biderivation on lattices and investigate some related properties. We characterized the distributive and modular lattices by generalized symmetric f-biderivations.

1. Introduction

Lattices play an important role in various branches such as information theory, information access controls, information retrieval and cryptanalysis [**6, 3, 11, 15**]. After the derivation on a ring was defined by Posner in [**14**], many researchers studied the derivation theory on various algebraic structures. Recently the notion of derivation introduced in rings and near rings has been studied by various researchers in the context of lattices (see [**1, 2, 3, 8, 10, 18**]). In [**1**], Alshehri introduced the notion of generalized derivation for a lattice and investigated many properties. After the symmetric biderivation defined by Maksa [**13**], in rings various researchers [**15, 16, 17**] studied this notion. In 2009, Y. Ceven et al. introduced symmetric bi-derivations in lattices [**9**]. Many researchers studied notion of f-derivatins in different algebraic structures (see [**12, 7**]) and also Chaudhry and Khan [**10**] introduced the notion of symmetric f-biderivations on a lattice and discussed some related properties. In this paper, the notion of generalized symmetric f-biderivations, which is more general than the notion of generalized symmetric biderivations on lattices [**9**], is introduced. We apply this notion to lattices and

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investigate some related properties which are discussed in [**9**] and using it give characterization of modular and distributive lattices.

2. Preliminaries

Definition 2.1. ([**4**]) Let L be a nonempty set endowed with operations " $\wedge$ " and " $\vee$ ". If $(L, \wedge, \vee)$ satisfies the following conditions for all $x, y, z \in L$:
$(L1)$ $x \wedge x = x,\ x \vee x = x$;
$(L2)$ $x \wedge y = y \wedge x,\ x \vee y = y \vee x$;
$(L3)$ $(x \wedge y) \wedge z = x \wedge (y \wedge z),\ (x \vee y) \vee z = x \vee (y \vee z)$;
$(L4)$ $(x \wedge y) \vee x = x,\ (x \vee y) \wedge x = x$,
then L is called a lattice.

Definition 2.2. ([**4**]) A lattice $(L, \wedge, \vee)$ is called a distributive lattice if one of the following two identities hold for all $x, y, z \in L$:
$(L5)$ $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$;
$(L6)$ $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

In any lattice, the conditions $(L5)$ and $(L6)$ are equivalent.

Definition 2.3. ([**4**]) Let $(L, \wedge, \vee)$ be a lattice. A binary relation $\leqslant$ on L is defined by;

$$x \leqslant y \text{ if and only if } x \wedge y = x \text{ and } x \vee y = y.$$

Definition 2.4. ([**5**]) A lattice $(L, \wedge, \vee)$ is called a modular if for $a, b \in L$ with $a \neq b$, it satisfies the following condition:

$$x \leqslant b, \text{ implies } x \vee (a \wedge b) = (x \vee a) \wedge b \text{ for all } x \in L.$$

Definition 2.5. ([**8**]) Let $(L, \wedge, \vee)$ be a lattice. A mapping $D(.,.) : L \times L \to L$ is called symmetric if $D(x, y) = D(y, x)$ holds for all $x, y \in L$.

Definition 2.6. ([**8**]) Let $(L, \wedge, \vee)$ be a lattice. A mapping $d : L \to L$ defined by $d(x) = D(x, x)$ is called trace of $D(.,.)$.

Lemma 2.1 ([**18**]). *Let $(L, \wedge, \vee)$ be a lattice. Let the binary relation $\leqslant$ be as in Definition 2.3. Then $(L, \leqslant)$ is a partially ordered set (poset) and for any $x, y \in L$, $x \wedge y$ is the g.l.b. of $\{x, y\}$ and $x \vee y$ is the l.u.b. of $\{x, y\}$.*

Definition 2.7. ([**18**]) Let $(L, \wedge, \vee)$ be a lattice. A function $d : L \to L$ on a lattice L is called a derivation on L if for all $x, y, z \in L$ it satisfies the following condition:

$$d(x \wedge y) = (dx \wedge y) \vee (x \wedge dy).$$

Definition 2.8. ([**1**]) Let $(L, \wedge, \vee)$ be a lattice. A function $D : L \to L$ on a lattice L is called a generalized derivation on L if there exist a derivation $d : L \to L$ such that:

$$D(x \wedge y) = (Dx \wedge y) \vee (x \wedge dy)$$

for all $x, y \in L$.

DEFINITION 2.9. ([**10**]) Let $(L, \wedge, \vee)$ be a lattice and $D(.,.) : L \times L \to L$ be a symmetric mapping. We call D a symmetric f-biderivation on L, if for all $x, y, z \in L$ it satisfies the following condition

$$D(x \wedge y, z) = (D(x, z) \wedge f(y)) \vee (f(x) \wedge D(y, z)).$$

Obviously, a symmetric f-biderivation D on L for all $x, y, z \in L$ satisfies the relation

$$D(x, y \wedge z) = (D(x, y) \wedge f(z)) \vee (f(y) \wedge D(x, z)).$$

We remark that if D is a symmetric f-biderivation on a lattice L, then the mappings $d_1 : L \to L, d_1(x) = D(x, y)$ and $d_2 : L \to L, d_2(y) = D(x, y)$ are f-derivations on L. Further if $f = 1$, the identity on L, then symmetric 1-biderivation is a symmetric biderivation on L.

PROPOSITION 2.1 ([**10**]). *Let $(L, \wedge, \vee)$ be a lattice and $f : L \to L$ be a mapping. Let d be the trace of symmetric f-biderivation D, then the following hold for all $x, y \in L$:*
(i) $D(x, y) \leqslant f(x)$ and $D(x, y) \leqslant f(y)$,
(ii) $D(x, y) \leqslant f(x) \wedge f(y)$,
(iii) $d(x) \leqslant f(x)$.

DEFINITION 2.10. [**9**] Let $(L, \wedge, \vee)$ be a lattice. The mapping Δ satisfying

$$\Delta(x \vee y, z) = \Delta(x, z) \vee \Delta(y, z) \text{ for all } x, y \in L$$

is called a joinitive mapping.

3. Generalized Symmetric f-biderivations

The following definition introduce the notion of generalized symmetric f-biderivation related to symmetric f-biderivation for a lattice.

DEFINITION 3.1. Let $(L, \wedge, \vee)$ be a lattice, $D(.,.) : L \times L \to L$ be a symmetric f-biderivation and $\Delta(.,.) : L \times L \to L$ be a symmetric mapping. We call Δ a generalized symmetric f-biderivation related to D, if it satisfies the following condition

$$\Delta(x \wedge y, z) = (\Delta(x, z) \wedge f(y)) \vee (f(x) \wedge D(y, z))$$

for all $x, y, z \in L$. The mapping $\delta : L \to L$ defined by $\delta(x) = \Delta(x, x)$ is called the trace of generalized symmetric f-biderivation Δ.

Obviously, a generalized symmetric f-biderivation Δ on L satisfies the relation

$$\Delta(x, y \wedge z) = (\Delta(x, y) \wedge f(z)) \vee (f(y) \wedge D(x, z)) \text{ for all } x, y, z \in L.$$

We remark that if $f = 1$, the identity on L, then generalized symmetric 1-biderivation is a generalized symmetric biderivation on L and if $D = \Delta$, then Δ is symmetric f-biderivation on L.

Now we give a few examples.

EXAMPLE 3.1. Let $(L, \wedge, \vee)$ be a lattice with a least element 0 and $f : L \to L$ be a mapping satisfying $f(x \wedge y) = f(x) \wedge f(y)$. The mapping $D(x, y) = 0$ is a symmetric f-biderivation on L and $\Delta(x, y) : L \times L \to L$ be defined by $\Delta(x, y) = f(x) \wedge f(y)$ for all $x, y \in L$. Then we can easily verify that Δ is a generalized symmetric f-biderivation related to D on L.

EXAMPLE 3.2. Let $(L, \wedge, \vee)$ be a lattice with a least element 0, where $0, a \in L$ and $f : L \to L$ be a mapping satisfying $f(x \wedge y) = f(x) \wedge f(y)$. The mapping $D(x, y) = 0$ is a symmetric f-biderivation on L. Let $\Delta(x, y) : L \times L \to L$ be defined by $\Delta(x, y) = (f(x) \wedge f(y)) \wedge a$ for all $x, y \in L$. Then it is easy to verify that Δ is a generalized symmetric f-biderivation related to D on L.

EXAMPLE 3.3. Let $(L, \wedge, \vee)$ be a non trivial lattice with a least element 0 and $f : L \to L$ be a mapping satisfying $f(x \vee y) = f(x) \vee f(y)$. The mapping $D(x, y) = 0$ is a symmetric f-biderivation on L. Let $\Delta(x, y) : L \times L \to L$ be defined by $\Delta(x, y) = f(x) \vee f(y)$ for all $x, y \in L$. Then it is easy to verify that Δ is not a generalized symmetric f-biderivation related to D on L.

Now, we investigate some properties for generalized symmetric f-biderivations on L.

PROPOSITION 3.1. *Let Δ is a generalized symmetric f-biderivation related to a symmetric f-biderivation D. Then the mappings $\delta_1 : L \to L, \delta_1(x) = \Delta(x, z)$ and $\delta_2 : L \to L, \delta_2(y) = \Delta(x, y)$ are generalized f-derivations on L.*

PROOF. We have

$$\begin{aligned} \delta_1(x \wedge y) &= \Delta(x \wedge y, z) \\ &= (\Delta(x, z) \wedge f(y)) \vee (f(x) \wedge D(y, z)) \\ &= (\delta_1(x) \wedge f(y)) \vee (f(x) \wedge d_1(y)). \end{aligned}$$

In the last equation, the mapping $d_1 : L \to L, d_1(y) = D(y, z)$ is a f-derivation on L, where D is the symmetric f-biderivation. Hence the mapping δ_1 is a generalized f-derivation on L. $\square$

THEOREM 3.1. *Let $(L, \wedge, \vee)$ be a lattice, $f : L \to L$ be a mapping satisfying $f(x \wedge y) = f(x) \wedge f(y)$ for all $x, y \in L$. Let Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D, δ be the trace of Δ and d be the trace of D. Then*

(i) $D(x, y) \leqslant \Delta(x, y)$ *for all* $x, y \in L$.
(ii) $\Delta(x, y) \leqslant f(x)$ *and* $\Delta(x, y) \leqslant f(y)$,
(iii) $\Delta(x, y) \leqslant f(x \wedge y)$,
(iv) $d(x) \leqslant \delta(x) \leqslant f(x)$
(v) $d(x) = f(x) \Rightarrow \delta(x) = f(x)$

for all $x, y \in L$.

PROOF. (i) Since

$$\begin{aligned}\Delta(x,y)\wedge D(x,y) &= \Delta(x\wedge x,y)\wedge D(x,y)\\ &= [(\Delta(x,y)\wedge f(x))\vee(f(x)\wedge D(x,y))]\wedge D(x,y)\\ &= [(\Delta(x,y)\wedge f(x))\vee D(x,y)]\wedge D(x,y)\\ &= D(x,y)\end{aligned}$$

by using Proposition 2.1 (i) and ($L4$), we have $D(x,y)\leqslant\Delta(x,y)$.

(ii) From (i), we get $\Delta(x,y)=(\Delta(x,y)\wedge f(x))\vee D(x,y)$ which implies

$$\begin{aligned}\Delta(x,y)\vee f(x) &= [(\Delta(x,y)\wedge f(x))\vee D(x,y)]\vee f(x)\\ &= (\Delta(x,y)\wedge f(x))\vee(D(x,y)\vee f(x))\\ &= (\Delta(x,y)\wedge f(x))\vee f(x))=f(x)\end{aligned}$$

by using ($L3$), Proposition 2.1 (i) and ($L4$), we have $\Delta(x,y)\leqslant f(x)$. Since Δ is symmetric, we have also $\Delta(x,y)\leqslant f(y)$.

(iii) It directly follows from (ii).

(iv) It follows from (i) and (ii).

(v) It directly follows from (iv). □

COROLLARY 3.1. *Let $(L,\wedge,\vee)$ be a lattice and $f:L\to L$ be a mapping. Let Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D. Let the least element be 0 and the greatest element be 1 of L and $f(0)=0$. Then $\Delta(0,x)=\Delta(x,0)=0$ and $\Delta(1,x)=\Delta(x,1)\leqslant f(x)$ for all $x\in L$.*

PROOF. Follows directly from the Theorem 3.1 (ii). □

THEOREM 3.2. *Let $(L,\wedge,\vee)$ be a modular lattice and $f:L\to L$ be a mapping. Let Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D, δ be the trace of Δ and d be the trace of D. Then*

(i) $\delta(x\wedge y)=(\delta(x)\wedge f(y))\vee(f(x)\wedge d(y))\vee D(x,y)$

(ii) $D(x,y)\leqslant\delta(x\wedge y),\delta(x)\wedge f(y)\leqslant\delta(x\wedge y),f(x)\wedge d(y)\leqslant\delta(x\wedge y)$

for all $x,y\in L$.

PROOF. (i) Using Proposition 2.1(i) and Theorem 3.1 (iv), we have

$$\begin{aligned}\delta(x\wedge y) &= \Delta(x\wedge y,x\wedge y)\\ &= (\Delta(x,x\wedge y)\wedge f(y))\vee(f(x)\wedge D(y,x\wedge y))\\ &= [\{(\Delta(x,x)\wedge f(y))\vee(f(x)\wedge D(x,y))\}\wedge f(y)]\\ &\qquad\vee[f(x)\wedge\{(D(y,x)\wedge f(y))\vee(f(x)\wedge D(y,y))\}].\end{aligned}$$

Since L is modular lattice, therefore

$$\begin{aligned}\delta(x\wedge y) &= [(\delta(x)\wedge f(y))\vee\{D(x,y)\wedge f(x)\wedge f(y)\}]\\ &\qquad\vee[\{f(x)\wedge D(x,y)\wedge f(y)\}\vee(f(x)\wedge d(y))]\end{aligned}$$

Using Proposition 2.1(i), the last equation gives

$$\delta(x\wedge y)=[(\delta(x)\wedge f(y))\vee D(x,y)\vee(f(x)\wedge d(y))]$$

for all $x,y\in L$.

(ii) Directly follows from (i). □

THEOREM 3.3. *Let $(L, \wedge, \vee)$ be a modular lattice and $f : L \to L$ be a mapping. Let Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D and δ be the trace of Δ. Then $d(x) \wedge d(y) \leqslant \delta(x) \wedge \delta(y) \leqslant \delta(x \wedge y)$ for all $x, y \in L$.*

PROOF. Since $\delta(x) \wedge f(y) \leqslant \delta(x \wedge y)$ by Theorem 3.2 (ii) and $\delta(y) \leqslant f(y)$ by Theorem 3.1 (iv), we have $\delta(x) \wedge \delta(y) \leqslant \delta(x) \wedge f(y) \leqslant \delta(x \wedge y)$. Using again Theorem 3.1 (iv), we get $d(x) \wedge d(y) \leqslant \delta(x) \wedge \delta(y)$. □

Keeping in view Example 3.3, we have the following lemma.

LEMMA 3.1. *Let $(L, \wedge, \vee)$ be a non trivial lattice with a least element 0 and $f : L \to L$ be an onto mapping satisfying $f(x \wedge y) = f(x) \wedge f(y)$. Let the mapping $\Delta(x, y) = f(x) \vee f(y)$ related to $D(x, y) = 0$ be a generalized symmetric f-biderivation. Then for $f(x) \leqslant f(z)$, the lattice L is modular.*

PROOF. Since $D(x, y) = 0$, so for all $x, y, z \in L$, we have $(f(x) \vee f(y)) \wedge f(z) = (\Delta(y, x) \wedge f(z)) \vee (f(y) \wedge D(z, x)) = \Delta(y \wedge z, x) = f(x) \vee f(y \wedge z) = f(x) \vee (f(y) \wedge f(z))$ hence L is a modular lattice. □

DEFINITION 3.2. Let $(L, \wedge, \vee)$ be a lattice, Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D, δ be the trace of Δ. If $x \leqslant y$ implies $\delta(x) \leqslant \delta(y)$, then δ is called an isotone mapping.

We remark that if $\delta(1) = 1$, since $\delta(1) \leqslant f(1)$, we have $f(1) = 1$, where 1 is the greatest element of L.

PROPOSITION 3.2. *Let $(L, \wedge, \vee)$ be a lattice and Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D, δ be the trace of Δ. Then δ is an isotone mapping if and only $\delta(x) \vee \delta(y) \leqslant \delta(x \vee y)$.*

PROOF. Since $x \leqslant x \vee y$ and $y \leqslant x \vee y$ and δ is an isotone mapping, we have $\delta(x) \leqslant \delta(x \vee y)$ and $\delta(y) \leqslant \delta(x \vee y)$, so $\delta(x) \vee \delta(y) \leqslant \delta(x \vee y)$. Conversely, let $\delta(x) \vee \delta(y) \leqslant \delta(x \vee y)$ and $x \leqslant y$. Since $x \vee y = y$, we have $\delta(x) \vee \delta(y) \leqslant \delta(y)$. Also it is known that $\delta(y) \leqslant \delta(x) \vee \delta(y)$. Hence we obtain $\delta(x) \vee \delta(y) = \delta(y)$, so $\delta(x) \leqslant \delta(y)$. □

THEOREM 3.4. *Let $(L, \wedge, \vee)$ be a modular lattice with greatest element 1, $f : L \to L$ be a mapping satisfying $f(x \wedge y) = f(x) \wedge f(y)$ for all $x, y \in L$. Let Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D, δ be the trace of Δ. Then the following conditions are equivalent:*
(i) δ is an isotone mapping
(ii) $\delta(x \wedge y) = \delta(x) \wedge \delta(y)$
(iii) $\delta(x) = f(x) \wedge \delta(1)$ for all $x, y \in L$.

PROOF. $(i) \Rightarrow (ii)$. Since $x \wedge y \leqslant x$ and $x \wedge y \leqslant y$ and δ is an isotone mapping, we have

$$\delta(x \wedge y) \leqslant \delta(x) \text{ and } \delta(x \wedge y) \leqslant \delta(y).$$

So $\delta(x \wedge y) \leqslant \delta(x) \wedge \delta(y)$. By Theorem 3.3, we have $\delta(x) \wedge \delta(y) \leqslant \delta(x \wedge y)$. Hence $\delta(x) \wedge \delta(y) = \delta(x \wedge y)$.

$(ii) \Rightarrow (i)$. Let $\delta(x \wedge y) = \delta(x) \wedge \delta(y)$ and $x \leqslant y$. Since $x \wedge y = x$, we get $\delta(x) = \delta(x \wedge y) = \delta(x) \wedge \delta(y) \leqslant \delta(y)$.

$(i) \Rightarrow (iii)$. Since $y \leqslant 1$ and δ is an isotone mapping, we have $\delta(y) \leqslant \delta(1)$. Since $\delta(y) \leqslant f(y)$, by Theorem 3.1 (iv), then we have $\delta(y) \leqslant f(y) \wedge \delta(1)$. By Theorem 3.2 (ii), we have $\delta(y) \wedge f(x) \leqslant \delta(y \wedge x)$. Taking $y = 1$, we get $\delta(1) \wedge f(x) \leqslant \delta(x)$ for all $x \in L$. Hence we have $\delta(x) = f(x) \wedge \delta(1)$.

$(iii) \Rightarrow (i)$. Let $\delta(x) = f(x) \wedge \delta(1)$ and $x \leqslant y$. Since $x \wedge y = x$, we have $\delta(x) = \delta(x \wedge y) = f(x \wedge y) \wedge \delta(1) = (f(x) \wedge f(y)) \wedge \delta(1) = (f(x) \wedge \delta(1)) \wedge (f(y) \wedge \delta(1)) = \delta(x) \wedge \delta(y)$. Hence $\delta(x) \leqslant \delta(y)$. □

THEOREM 3.5. *Let $(L, \wedge, \vee)$ be a distributive lattice with greatest element 1, $f : L \to L$ be a mapping satisfying $f(x \wedge y) = f(x) \wedge f(y)$ for all $x, y \in L$. Let Δ be a generalized symmetric f-biderivation related to a symmetric f-biderivation D, δ be the trace of Δ. Then the following conditions are equivalent:*
(i) *δ is an isotone mapping*
(ii) $\delta(x \wedge y) = \delta(x) \wedge \delta(y)$
(iii) $\delta(x) = f(x) \wedge \delta(1)$ *for all $x, y \in L$.*

PROOF. Since every modular lattice is distributive therefore proof is straight forward as in Theorem 3.4. □

Note that if $f : L \to L$ is a lattice homomorphism, then $f(L)$ is a sublattice of L.

PROPOSITION 3.3. *Let $(L, \wedge, \vee)$ be a lattice and $f : L \to L$ be a lattice homomorphism. Let $\Delta : L \times L \to L$ be a generalized symmetric f-biderivation related to D on L defined by $\Delta(x, y) = f(x) \wedge f(y)$. Then $f(L)$ is a distributive lattice if and only if Δ is joinitive.*

PROOF. Let Δ be a joinitive. By definition of Δ, we have $\Delta(x \vee y, z) = f(x \vee y) \wedge f(z) = (f(x) \vee f(y)) \wedge f(z)$. Since Δ is joinitive, therefore $\Delta(x \vee y, z) = \Delta(x, z) \vee \Delta(y, z) = (f(x) \wedge f(z)) \vee (f(y) \wedge f(z))$. Hence $(f(x) \vee f(y)) \wedge f(z) = (f(x) \wedge f(z)) \vee (f(y) \wedge f(z))$. Thus $f(L)$ is distributive lattice. Conversely let $f(L)$ be a distributive lattice. Then $f(x) \wedge f(y \vee z) = f(x) \wedge (f(y) \vee f(z)) = (f(x) \wedge f(y)) \vee (f(x) \wedge f(z))$, which along with definition of Δ implies $\Delta(x, y \vee z) = \Delta(x, y) \vee \Delta(x, z)$. Hence Δ is joinitive. □

Taking $f = 1$, the identity on L, we get the following corollary, which is an improvement of the result of Y. Ceven []Ceven19 Proposition 3.

COROLLARY 3.2. *Let $(L, \wedge, \vee)$ be a lattice and $\Delta : L \times L \to L$ be a symmetric biderivation on L defined by $\Delta(x, y) = x \wedge y$. Then L is a distributive lattice if and only if Δ is joinitive.*

THEOREM 3.6. *Let $(L, \wedge, \vee)$ be a distributive lattice, $f : L \to L$ be a mapping. Let Δ_1 and Δ_2 be generalized symmetric f-biderivations related to a same symmetric f-biderivation D. The mapping $\Delta_1 \wedge \Delta_2$ defined by $(\Delta_1 \wedge \Delta_2)(x, y) =$*

$\Delta_1(x,y) \wedge \Delta_2(x,y)$, *is a generalized symmetric f-biderivation related to the symmetric f-biderivation D.*

PROOF.

$$\begin{aligned}
(\Delta_1 \wedge \Delta_2)(x \wedge y, z) &= \Delta_1(x \wedge y, z) \wedge \Delta_2(x \wedge y, z) \\
&= [(\Delta_1(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z))] \\
&\quad \wedge [(\Delta_2(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z))] \\
&= [(\Delta_1(x,z) \wedge f(y)) \wedge (\Delta_2(x,z) \wedge f(y))] \vee (f(x) \wedge D(y,z)) \\
&= [\Delta_1(x,z) \wedge \Delta_2(x,z) \wedge f(y)] \vee (f(x) \wedge D(y,z)) \\
&= ((\Delta_1 \wedge \Delta_2)(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z))
\end{aligned}$$

This completes the proof. □

THEOREM 3.7. *Let $(L, \wedge, \vee)$ be a distributive lattice, $f : L \to L$ be a mapping. Let Δ_1 and Δ_2 be generalized symmetric f-biderivations related to a same symmetric f-biderivation D. The mapping $\Delta_1 \vee \Delta_2$ defined by $(\Delta_1 \vee \Delta_2)(x,y) = \Delta_1(x,y) \vee \Delta_2(x,y)$, is a generalized symmetric f-biderivation related to the symmetric f-biderivation D.*

PROOF.

$$\begin{aligned}
(\Delta_1 \vee \Delta_2)(x \wedge y, z) &= [\Delta_1(x \wedge y, z) \vee \Delta_2(x \wedge y, z)] \\
&= (\Delta_1(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z)) \\
&\quad \vee [(\Delta_2(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z))] \\
&= [(\Delta_1(x,z) \wedge f(y)) \vee (\Delta_2(x,z) \wedge f(y))] \vee (f(x) \wedge D(y,z)) \\
&= [(\Delta_1(x,z) \vee \Delta_2(x,z)) \wedge f(y)] \vee (f(x) \wedge D(y,z)) \\
&= ((\Delta_1 \vee \Delta_2)(x,z) \wedge f(y)) \vee (f(x) \wedge D(y,z)),
\end{aligned}$$

This completes the proof. □

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