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ANALYTIC FUNCTION CLASSES INVOLVING POISSON DISTRIBUTION SERIES IN CONIC DOMAINS

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ABSTRACT. In this paper the authors estimate the bounds for probability distribution series by means of q-difference operator using subordination as well as quasi-subordination.

1. Introduction

Let A indicate an analytic function family, which is normalized under the condition of $f(0) = f'(0) - 1 = 0$ in the open unit disk $U = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$ and given by the following Taylor-Maclaurin series:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Further, by S we shall denote the class of all functions in A which are univalent in U. Let $h(z)$ be an analytic function in U and $|h(z)| \leqslant 1$, such that

$$h(z) = h_0 + h_1 z + h_2 z^2 + \cdots ,$$

where all coefficients are real. Also, let φ be an analytic and univalent function with positive real part in U with $\varphi(0) = 1$, $\varphi'(0) > 0$ and φ maps the unit disk U onto a region starlike with respect to 1 and symmetric with respect to the real axis. Taylor's series expansion of such function is of the form

$$\varphi(z) = 1 + B_1 z + B_2 z^2 + \cdots , \tag{1.2}$$

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where all coefficients are real and $B_1 > 0$. Let P be the class of functions consisting of the form (1.2). If the functions f and g are analytic in U, then f is said to be subordinate to g, written as

$$f(z) \prec g(z) \quad (z \in U)$$

if there exists a Schwarz function $w \in \Omega$, where

$$\Omega = \{w : w(0) = 0, \ |w(z)| < 1, \ z \in U\},$$

such that

$$f(z) = g(w(z)) \qquad (z \in U).$$

In the year 1970, Robertson [**26**] introduced the concept of quasi-subordination. For two analytic functions f and g, the function f is said to be quasi-subordinate to g in U and written as

$$f(z) \prec_q g(z) \qquad (z \in U)$$

if there exists an analytic function $|h(z)| \leqslant 1$ such that $\frac{f(z)}{h(z)}$ analytic in U and

$$\frac{f(z)}{h(z)} \prec g(z) \qquad (z \in U)$$

that is, there exists a Schwarz function $w(z)$ such that $f(z) = h(z)g(w(z))$. Observe that if $h(z) = 1$, then $f(z) = g(w(z))$ so that $f(z) \prec g(z)$ in U. Also notice that if $w(z) = z$, then $f(z) = h(z)g(z)$ and it is said that is majorized by g and written $f(z) \ll g(z)$ in U. Hence it is obvious that quasi-subordination is a generalization of subordination as well as majorization. (see, e.g. [**4**], [**8**], [**11**], [**24**], [**26**], [**25**], [**18**] for works related to quasi-subordination).

Let also $P(p_k)$ $(0 \leqslant k < \infty)$ denote the family of functions P, such that $p \in P$, and $p \prec p_k$ in U, where the function P_k maps the unit disk conformally onto the region Ω_k such that $1 \in \Omega_k$ and

$$\partial\Omega_k = \left\{u + iv : u^2 = k^2(u-1)^2 + k^2v^2\right\}.$$

REMARK 1.1. (See [**11**]) The domain Ω_k is elliptic for $k > 1$, hyperbolic when $0 < k < 1$, parabolic for $k = 1$ and covers the right half plane wnen $k = 0$. Sim et al. [**29**] extended original definition to the $p-$valent functions generalizing the domains Ω_k to $\Omega_{k,\alpha}$ $0 \leqslant k < \infty, 0 \leqslant \alpha < 1$ as follows

$$\Omega_{k,\alpha} = \{w = u + iv : (u-\alpha)^2 > k^2(u-1)^2 + k^2v^2\}, \ \Omega_{k,0} = \Omega_k.$$

In the sequel we shall make use of $q-$operators to the functions related to the conic sections, that were introduced and studied by Kanas et al. [**13**], [**14**], [**15**], [**16**], [**17**] and investigated by several other authors.

2. Preliminaries

The Poisson distribution is one of the most well-utilized discrete distributions in multivariate data research fields. However, nowadays, the elementary distributions such as the Poisson, the Pascal, the Logarithmic, the Binomial have been partially studied in the Geometric Function Theory from a theoretical point of view (see [**1**], [**2**], [**3**], [**19**], [**20**], [**28**], [**21**]). Very recently, Porwal [**22**] introduced and studied Poisson distribution series for analytic functions.

A variable x is said to have Poisson distribution if it takes the values $0, 1, 2, 3, \ldots$ with probabilities

$$e^{-m},\ \frac{me^{-m}}{1!},\ \frac{m^2e^{-m}}{2!},\ \frac{m^3e^{-m}}{3!}, \ldots,$$

respectively, where m is called the parameter. Thus

$$P(x=k) = \frac{m^k e^{-m}}{k!},\ k = 0, 1, 2, 3, \ldots.$$

Now we introduce a power series whose coefficients are probabilities of the Poisson distribution:

$$K(m, z) = z + \sum_{n=2}^{\infty} \frac{e^{-m} m^{n-1}}{(n-1)!} z^n. \tag{2.1}$$

It was pointed out that by ratio test, the radius of convergence of the series in (2.1) is infinity.

Next, the study of operators plays an important role in Geometric Function Theory in Complex Analysis and its related fields. The interest in this area has been increasing because it permits detailed investigations of problems with physical applications. Some integral transforms in the classical analysis have their q-analogues in the theory of q-calculus. This has led various researchers in the field of q-theory for extending all the important results involving the classical analysis to their q-analogs.

For the convenience, we provide some basic definitions and concept details of q-calculus which are used in this paper. Throughout this paper, we will assume that q satisfies the condition $0 < q < 1$. We shall follow the notation and terminology of [**6**]. We first recall the definitions of fractional q-calculus operators of complex valued function f.

Definition 2.1. Let $q \in (0, 1)$ and let $\lambda \in \mathbb{C}$. The q-number, denoted $[\lambda]_q$, we define as

$$[\lambda]_q = \frac{1 - q^\lambda}{1 - q}.$$

In the case when $\lambda = n \in \mathbb{N}$ we obtain $[\lambda]_q = 1 + q + q^2 + \cdots + q^{n-1}$, and when $q \to 1^-$ then $[n]_q = n$.

Applying above q-number we define q-derivative below.

Definition 2.2. ([**10**]) The q-derivative of a function f, defined on a subset of $\mathbb{C}$, is given by

$$(D_q f)(z) = \begin{cases} \dfrac{f(z) - f(qz)}{(1-q)z} & for \quad z \neq 0, \\ f'(0) & for \quad z = 0. \end{cases} \tag{2.2}$$

We note that $\lim_{q \to 1^-} (D_q f)(z) = f'(z)$ if f is differentiable at z. Additionally, in view of (2.2), we deduce that

$$(D_q f)(z) = 1 + \sum_{n=2}^{\infty} [n]_q \, a_n z^{n-1}. \tag{2.3}$$

For a function $h(z) = z^m$, we obtain

$$D_q h(z) = D_q z^m = \frac{1-q^m}{1-q} z^m = [m]_q z^{m-1}$$

and

$$lim_{q \to 1^-} D_q h(z) = lim_{q \to 1^-} \left([m]_q z^{m-1} \right) = m z^{m-1} = h'(z),$$

where h' is the ordinary derivative. Jackson $q-$derivative satisfies known rules of differentiation, for example a $q-$analoque of Leibniz's rule. As a right inverse, Jackson [**9**] introduced the $q-$ integral of a function f as follows

$$\int_0^z f(t) d_q t = z(1-q) \sum_{n=0}^{\infty} q^n f(q^n z) = z(1-q) \sum_{n=0}^{\infty} a_n q^n z^n,$$

provided the series converges. For a function $h(z) = z^m$ we obtain

$$\int_0^z h(t) d_q t = \int_0^z t^m d_q t = \frac{z^{m+1}}{[m+1]_q} \quad (m \neq -1),$$

and

$$lim_{q \to 1^-} \int_0^z h(t) d_q t = lim_{q \to 1^-} \frac{z^{m+1}}{[m+1]_q} = \frac{z^{m+1}}{[m+1]} = \int_0^z h(t) dt,$$

where $\int_0^z h(t)dt$ is the ordinary integral, (see also [**7**], [**12**], [**23**], [**24**]).
In view of (2.3) and (2.1), $K(m,z) \in P_{ds}$ we obtain

$$D_q K(m,z) = 1 + \sum_{n=2}^{\infty} \frac{[n]_q m^{n-1}}{(n-1)!} e^{-m} z^{n-1}.$$

The defined above fractional q-calculus is an important tool used in a study of various families of analytic functions, and in the context of univalent functions was first used in a book chapter by Srivastava [**27**]. In contrast to the Leibniz notation, being a ratio of two infinitisemals, the notions of q-derivatives are plain ratios. Therefore, it appeared soon a generalization of q-calculus in many subjects, such as hypergeometric series, complex analysis, and particle physics. It is also widely applied in an approximation theory, especially on various operators, which includes convergence of operators to functions in real and complex domain. In the last twenty years q-calculus served as a bridge between mathematics and physics.

The field has expanded explosively, due to the fact that applications of basic hypergeometric series to the diverse subjects of combinatorics, quantum theory, number theory, statistical mechanics, are constantly being uncovered. Specially, the theory of univalent functions can be described by using the theory of the q-calculus. In recent years, such q-calculus operators as the fractional q-integral and fractional q-derivative operators were used to construct several subclasses of analytic functions. In the present paper we study the q-operator, and related problems involving univalent functions.

Let us recall now the following lemma required in sequel.

LEMMA 2.1 ([**24**]). *Let the Schwarz function* $w(z)$ *be given by*

$$w(z) = w_1 z + w_2 z^2 + \cdots, \qquad (z \in \Delta) \tag{2.4}$$

then

$$|w_1| \leqslant 1, \quad \left|w_2 - \vartheta w_1^2\right| \leqslant 1 + (|\vartheta| - 1)\,|w_1|^2 \leqslant \max\{1, |\vartheta|\}$$

where $\vartheta \in \mathbb{C}$.

Motivated by each of the above definitions, we now define the following classes of analytic univalent functions involving the q-derivative operator.

DEFINITION 2.3. A function $f \in A$ given by (1.1) is said to be in the class

$$P_{ds}S^q_{p_{k,\alpha}}(\theta, b) \qquad (0 \leqslant k < \infty, 0 \leqslant \alpha < 1, \frac{-\pi}{2} < \theta < \frac{\pi}{2},\ b \in \mathbb{C}\backslash\{0\}, z \in U)$$

the following condition is satisfied:

$$1 + \frac{1}{b}\left((1 + itan\theta)\left(\frac{zD_qK(m,z)}{K(m,z)}\right) - itan\theta - 1\right) \prec p_{k,\alpha}(z)$$

and a function $f \in A$ given by (1.1) is said to be in the class

$$P_{ds}C^q_{p_{k,\alpha}}(\theta, b) \qquad (0 \leqslant k < \infty, 0 \leqslant \alpha < 1, \frac{-\pi}{2} < \theta < \frac{\pi}{2},\ b \in \mathbb{C}\backslash\{0\}, z \in U)$$

the following condition is satisfied:

$$1 + \frac{1}{b}\left((1 + itan\theta)\left(\frac{(zD_qK(m,z))'}{D_qK(m,z)}\right) - itan\theta - 1\right) \prec p_{k,\alpha}(z).$$

DEFINITION 2.4. A function $f \in A$ given by (1.1) is said to be in the class

$$\overline{P_{ds}S^q_{p_{k,\alpha}}(\theta, b)} \qquad (0 \leqslant k < \infty, 0 \leqslant \alpha < 1, \frac{-\pi}{2} < \theta < \frac{\pi}{2},\ b \in \mathbb{C}\backslash\{0\}, z \in U)$$

the following condition is satisfied:

$$\frac{1}{b}\left((1 + itan\theta)\left(\frac{zD_qK(m,z)}{K(m,z)}\right) - itan\theta - 1\right) \prec_q \phi(z) - 1$$

and a function $f \in A$ given by (1.1) is said to be in the class

$$\overline{P_{ds}C^q_{p_{k,\alpha}}(\theta, b)} \qquad (0 \leqslant k < \infty, 0 \leqslant \alpha < 1, \frac{-\pi}{2} < \theta < \frac{\pi}{2},\ b \in \mathbb{C}\backslash\{0\}, z \in U)$$

the following condition is satisfied:

$$\frac{1}{b}\left((1+itan\theta)\left(\frac{(zD_qK(m,z))'}{D_qK(m,z)}\right)-itan\theta-1\right)\prec_q \phi(z)-1.$$

3. The Fekete-Szegö functional associated with conical domains

The Fekete-Szegö functional $\left|a_3-\mu a_2^2\right|$ for normalized univalent functions of the form given by (1.1) is well known for its rich history in Geometric Function Theory. Its origin was in the disproof by Fekete and Szegö [**5**] of the 1933 conjecture of Littlewood and Paley that the coefficients of odd univalent functions are bounded by unity (see, for details, [**5**]). The Fekete-Szegö functional has $\left|a_3-\mu a_2^2\right|$ since received great attention, particularly in connection with many subclasses of the class of normalized analytic and univalent functions.

Here we shall consider Fekete-Szegö functional for the classes $P_{ds}S^q_{p_{k,\alpha}}(\theta,b)$ and $P_{ds}C^q_{p_{k,\alpha}}(\theta,b)$.

THEOREM 3.1. *For any complex μ, the function $K(m,z)$, given by (2.1), is in the class $P_{ds}S^q_{p_{k,\alpha}}(\theta,b)$ if the following holds*

$$m^2e^{-m}(\frac{1}{2}-\mu e^{-m})\leqslant\frac{|b|\,p_1}{|\sigma|([3]_q-1)}max\left\{1,\left|\frac{p_2}{p_1}+\frac{([2]_q-1)-\mu([3]_q-1)}{\sigma([2]_q-1)^2}bp_1\right|\right\},$$

where $\sigma=1+itan\theta$.

PROOF. Suppose that $K(m,z)\in P_{ds}S^q_{p_{k,\alpha}}(\theta,b)$. Then there exists a Schwarz function $w\in\Omega$ of the form (2.4) such that

$$1+\frac{1}{b}\left((1+itan\theta)\left(\frac{zD_qK(m,z)}{K(m,z)}\right)-itan\theta-1\right)=p_{k,\alpha}(w(z)).$$

Observe that

$$\sigma\left[me^{-m}([2]_q-1)z+\frac{m^2}{2!}e^{-m}([3]_q-1)z^2+\dots\right] \tag{3.1}$$
$$=b\left[p_1w_1z+(p_1w_2+p_2w_1^2+me^{-m}p_1w_1)z^2+...\right].$$

Comparing the coefficients of z and z^2 in (3.1), we obtain

$$me^{-m}=\frac{bp_1w_1}{\sigma([2]_q-1)}$$

and

$$m^2e^{-m}=\frac{2bp_1}{\sigma([3]_q-1)}\left(w_2-\left(\frac{-p_2}{p_1}-\frac{bp_1}{\sigma([2]_q-1)}\right)w_1^2\right).$$

Hence, by the last two equations becomes

$$m^2e^{-m}(\frac{1}{2}-\mu e^{-m})=\frac{bp_1}{\sigma([3]_q-1)}\left(w_2-tw_1^2\right),$$

where $t=-\frac{p_2}{p_1}-\frac{([2]_q-1)-\mu([3]_q-1)}{\sigma([2]_q-1)^2}bp_1$. The desired inequality is obtained by applying Lemma 2.1. □

COROLLARY 3.1. *For any complex μ, the function $K(m,z)$ given by (2.1) is in the class $P_{ds}C^{q}_{p_{k,\alpha}}(\theta,b)$ if the following holds*

$$m^2e^m(1-\mu e^m) \leqslant \frac{|b|\,p_1}{[3]_q|\sigma|} max\left\{1, \left|\frac{p_2}{p_1}+\frac{[2]_q^2-\mu[3]_q}{\sigma[2]_q^2}bp_1\right|\right\},$$

where $\sigma = 1 + itan\theta$.

4. The Fekete-Szegö functional associated with quasi-subordination

In this section, we will focus on Fekete-Szegö functional for the classes

$$\overline{P_{ds}S^{q}_{p_{k,\alpha}}(\theta,b)} \text{ and } \overline{P_{ds}C^{q}_{p_{k,\alpha}}(\theta,b)}$$

associated with quasi-subordination.

COROLLARY 4.1. *The function $K(m,z)$, given by (2.1), generated by the function f given in (1.1), is in the class $\overline{P_{ds}S^{q}_{p_{k,\alpha}}(\theta,b)}$ if the following condition holds*

$$me^{-m} \leqslant \frac{|b|\,c_1}{|\sigma|\,([2]_q-1)},$$

$$m^2e^{-m} \leqslant \frac{2\,|b|}{|\sigma|\,([3]_q-1)}\left\{c_1 + max\left\{c_1, |c_2| + \left|\frac{bc_1^2}{\sigma([2]_q-1)}\right|\right\}\right\}$$

and for any complex μ

$$m^2e^{-m}(1-\mu e^{-m}) \leqslant \frac{2\,|b|}{|\sigma|\,([3]_q-1)}\left\{c_1 + max\left(c_1, \left|\frac{2([2]_q-1)-\mu([3]_q-1)}{2\sigma([2]_q-1)^2}\right| |b|c_1^2+|c_2|\right)\right\}$$

where $\sigma = 1 + itan\theta$.

COROLLARY 4.2. *The function $K(m,z)$ given by (2.1), generated by the function f given in (1.1), is in the class $\overline{P_{ds}C^{q}_{p_{k,\alpha}}(\theta,b)}$ if the following condition holds*

$$me^{-m} \leqslant \frac{|b|\,c_1}{|\sigma|\,[2]_q},$$

$$m^2e^{-m} \leqslant \frac{|b|}{|\sigma|\,[3]_q}\left\{c_1 + max\left\{c_1, |c_2| + \left|\frac{bc_1^2}{\sigma}\right|\right\}\right\}$$

and for any complex μ

$$m^2e^{-m}(1-\mu e^{-m}) \leqslant \frac{|b|}{|\sigma|\,[3]_q}\left\{c_1 + max\left(c_1, \left|\frac{[2]_q^2-\mu[3]_q}{\sigma[2]_q^2}\right| |b|c_1^2+|c_2|\right)\right\}.$$

where $\sigma = 1 + itan\theta$.

5. Concluding Remark

Various choices of the function $h(z)$ as mentioned above and by specializing on the parameters, we state some interesting results analogous to Theorem 3.1. The details involved may be left as an exercise for the interested reader.

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