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WEAK-INTERIOR IDEALS OF Γ-SEMIGROUPS

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Abstract. In this paper, we introduce the notion of weak-interior ideal of $\Gamma-$semigroup and we characterize the simple $\Gamma-$semigroup and the regular semigroup in terms of weak-interior ideals of $\Gamma-$ semigroup.

1. Introduction

Ideals play an important role in advance studies and uses of algebraic structures. Generalization of ideals in algebraic structures is necessary for further study of algebraic structures. Many mathematicians proved important results and characterization of algebraic structures by using the concept and the properties of generalization of ideals in algebraic structures. The notion of ideals was introduced by Dedekind for the theory of algebraic numbers, was generalized by Noether for associative rings. The one and two sided ideals introduced by her, are still central concepts in ring theory and the notion of an one sided ideal of any algebraic structure is a generalization of notion of an ideal.

In 1952, the concept of bi-ideals was introduced by Good and Hughes [**1**] for semigroups. The notion of bi-ideals in rings and semigroups were introduced by Lajos and Szasz [**8, 9**]. Quasi ideals are generalization of right ideals and left ideals whereas bi-ideals are generalization of quasi ideals. Steinfeld [**30**] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [**3, 4, 5**], Izuka [**6**] introduced the concept of quasi ideal for a semigroup. Quasi ideals in $\Gamma-$semigroups were studied by Jagtap and Pawar [**7**]. Henriksen [**2**] and Shabir et al. [**29**] studied ideals in semigroups.

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M. K. Rao [**11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21**] introduced the notion of left (right) bi-quasi ideals, bi-interior ideals tri-ideals and bi-quasi interior ideals of semiring, $\Gamma-$semiring, $\Gamma-$semigroup and studied their properties. In 1981, Sen [**28**] introduced the notion of a $\Gamma-$semigroup as a generalization of semigroup. In 1995, M. K. Rao [**22, 23, 24, 25**] introduced the notion of a Γ-semiring as a generalization of Γ-ring, ring, ternary semiring and semiring. As a generalization of ring, the notion of a $\Gamma-$ring was introduced by Nobusawa [**27**] in 1964. Semigroup, as the basic algebraic structure was used in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages.

In this paper,we introduce the notion of a weak-interior ideal and we characterize the regular Γ- semigroup in terms of weak-interior ideal of $\Gamma-$ semigroup.

2. Preliminaries

In this section, we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. A semigroup is an algebraic system $(M, .)$ consisting of a non-empty set M together with an associative binary operation '$\cdot$'.

Definition 2.2. A subsemigroup T of a semigroup M is a non-empty subset T of M such that $TT \subseteq T$.

Definition 2.3. A non-empty subset T of a semigroup M is called a left (right) ideal of M if $MT \subseteq T$ $(TM \subseteq T)$.

Definition 2.4. A non-empty subset T of a semigroup M is called an ideal of M if it is both a left ideal and a right ideal of$M$.

Definition 2.5. A non-empty Q of a semigroup S is called a quasi ideal of S if $QS \cap SQ \subseteq Q$.

Definition 2.6. A subsemigroup T of a semigroup S is called a bi-ideal of S if $TST \subseteq T$.

Definition 2.7. A subsemigroup T of a semigroup M is called an interior ideal of M if $MTM \subseteq T$.

Definition 2.8. An element a of a semigroup M is called a regular element if there exists an element b of M such that $a = aba$.

Definition 2.9. A semigroup M is called a regular semigroup if every element of M is a regular element.

Definition 2.10. Let M and Γ be non-empty subsets. Then we call M a Γ-semigroup, if there exists a mapping $M \times \Gamma \times M \to M$ (images of (x, α, y) will be denoted by $x\alpha y, x, y \in M$ and $\alpha \in \Gamma$) such that it satisfies $x\alpha(y\beta z) = (x\alpha y)\beta z$. for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.11. A non-empty subset A of $\Gamma-$semigroup M is called

(i) a Γ-subsemigroup of M if $A\Gamma A \subseteq A$,
(ii) a quasi ideal of M if $A\Gamma M \cap M\Gamma A \subseteq A$,
(iii) a bi-ideal of M if $A\Gamma M\Gamma A \subseteq A$,
(iv) an interior ideal of M if $M\Gamma A\Gamma M \subseteq A$,
(v) a left (right) ideal of M if $M\Gamma A \subseteq A$ (res. $A\Gamma M \subseteq A$),
(vi) an ideal if $A\Gamma M \subseteq A$ and $M\Gamma A \subseteq A$.

Definition 2.12. A$\Gamma-$semigroup M is said to be left (right) singular if for each $a \in M$ there exists $\alpha \in \Gamma$ such that $a\alpha b = a$ (res. $a\alpha b = b$), for all $b \in M$.

Definition 2.13. A $\Gamma-$semigroup M is said to be commutative if $a\alpha b = b\alpha a$, for all $a, b \in M$ and for all $\alpha \in \Gamma$.

Definition 2.14. Let M be a Γ-semigroup. An element $a \in M$ is said to be an idempotent of M if there exist $\alpha \in \Gamma$ such that $a = a\alpha a$ and a is also said to be α idempotent.

Definition 2.15. Let M be a Γ-semigroup. If every element of M is an idempotent of M, then $\Gamma-$semigroup M is said to be a band.

Definition 2.16. Let M be a Γ-semigroup. An element $a \in M$ is said to be a regular element of M if there exist $x \in M$, $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 2.17. Let M be a Γ-semigroup. Every element of M is a regular element of M then M is said to be a regular Γ-semigroup M.

Definition 2.18. Let M be a semigroup. A non-empty set L of M is said to be left bi-quasi (right bi-quasi) ideal of M if L is a subsemigroup of M and $ML \cap LML \subseteq L$ (res. $LM \cap LML \subseteq L$).

Definition 2.19. Let M be a semigroup. A nonempty subset L of M is said to be bi-quasi ideal of M if L is a subsemigroup of M ,$ML \cap LML \subseteq L$ and $LM \cap LML \subseteq L$.

Definition 2.20. ([**10**]) Let M be a semigroup. A non-empty subset L of M is said to be bi-interior ideal of M if L is a subsemigroup of M and $MLM \cap LML \subseteq L$

3. Weak-interior ideals of Γ- semigroups

In this section, we introduce the notion of weak-interior ideal as a generalization of quasi-ideal and interior ideal of $\Gamma-$ semigroupand study the properties of weak-interior ideal of Γ- semigroup. Throughout this paper, M is a $\Gamma-$ semigroup with unity element.

Definition 3.1. A non-empty subset B of a $\Gamma-$ semigroup$M$ is said to be left weak-interior ideal of M if B is a $\Gamma-$subsemigroup of M and $M\Gamma B\Gamma B \subseteq B$.

Definition 3.2. A non-empty subset B of a $\Gamma-$ semigroup$M$ is said to be right weak-interior ideal of M if B is a $\Gamma-$subsemigroup of M and $B\Gamma B\Gamma M \subseteq B$.

Definition 3.3. A non-empty subset B of a $\Gamma-$ semigroup$M$ is said to be weak-interior ideal of M if B is a $\Gamma-$subsemigroup of M and $B$is a left weak-interior ideal and a right weak-interior ideal of M.

Remark 3.1. A weak-interior ideal of a Γ- semigroup$M$ need not be quasi-ideal, interior ideal, bi-interior ideal. and bi-quasi ideal of Γ- semigroup M.

Example 3.1. Let Q be the set of all rational numbers,

$$M=\left\{\left(\begin{array}{cc} 0 & b \\ 0 & d \end{array}\right) \mid b, d \in Q\right\}$$

be the additive semigroup M of matrices and $\Gamma = M$. The ternary operation $A\alpha B$ is defined as usual matrix multiplication of A, α, B, for all $A, \alpha, B \in M$.Then M is a $\Gamma-$ semigroup If $R=\left\{\left(\begin{array}{cc} 0 & b \\ 0 & 0 \end{array}\right) \mid 0 \neq b \in Q\right\}$ then R is a left weak interior ideal of the $\Gamma-$ semigroup$M$ and R is neither a left ideal nor a right ideal, not a weak interior ideal and not a interior ideal of the $\Gamma-$ semigroup$M$.

In the following theorem, we mention some important properties and we omit the proofs since they are straight forward.

Theorem 3.1. *Let M be a Γ-semring. Then the following are hold:*

1. *Every left ideal is a left weak-interior ideal of M.*
2. *Every right ideal is a right weak-interior ideal of M.*
3. *Let M be a Γ- semigroupand B be a Γ-subsemigroupof M.If $M\Gamma M\Gamma B \subseteq B$ and$B\Gamma M\Gamma M \subseteq B$ then B is a weak-interior ideal of M.*
4. *Every ideal is a weak-interior ideal of M.*
5. *If B is a weak-interior ideal and T is an interior ideal of M then $B \cap T$ is a weak-interior ideal of ring M.*
6. *If L is a Γ-subsemigroupof M and R is a right ideal of a Γ- semigroup$M$ then $B = L\Gamma R$ is a weak-interior ideal of M.*
7. *Let M be a Γ- semigroupand B be a Γ-subsemigroupof M. If*

$$M\Gamma M\Gamma M\Gamma B \subseteq B$$

then B is a left weak-interior ideal of M.

Theorem 3.2. *If B be an interior ideal of a Γ- semigroup M, then B is a left weak-interior ideal of M.*

Proof. Suppose B is an interior ideal of the Γ- semigroup M. Then

$$M\Gamma B\Gamma B \subseteq M\Gamma B\Gamma M \subseteq B.$$

Hence B is a left weak-interior ideal of M. □

Corollary 3.1. *If B be an interior ideal of a Γ- semigroup M, then B is a right weak-interior ideal of M.*

Corollary 3.2. *If B be an interior ideal of a Γ- semigroup M, then B is a weak-interior ideal of M.*

Theorem 3.3. *Let M be a Γ- semigroup and B be a Γ-subsemigroupof M. B is a weak-interior ideal of M if and only if there exists left ideal L such that $L\Gamma L \subseteq B \subseteq L$.*

PROOF. Suppose B is a weak-interior ideal of the $\Gamma-$ semigroup$M$. Then $M\Gamma B\Gamma B \subseteq B$. Let $L = M\Gamma B\Gamma B$. Then L is a left ideal of M. Therefore $L\Gamma L \subseteq B \subseteq L$.

Conversely suppose that there exists left ideal L of M such that $L\Gamma L \subseteq B \subseteq L$. Then $M\Gamma B\Gamma B \subseteq M\Gamma(L)\Gamma(L) \subseteq L\Gamma(L) \subseteq B$. Hence B is a left weak-interior ideal of M. □

COROLLARY 3.3. *Let M be a $\Gamma-$ semigroupand B be a $\Gamma-$subsemigroupof M. B is a right weak-interior ideal of M if and only if there exist right ideal R such that $R\Gamma R \subseteq B \subseteq R$.*

COROLLARY 3.4. *Let M be a $\Gamma-$ semigroup and B be a $\Gamma-$subsemigroupof M. B is a weak-interior ideal of M if and only if there exist ideal R such that $R\Gamma R \subseteq B \subseteq R$.*

THEOREM 3.4. *The intersection of a left weak-interior ideal B of a Γ- semigroup M and a left ideal A of M is always a left weak-interior ideal of M.*

PROOF. Suppose $C = B \cap A$. Then $M\Gamma C\Gamma C \subseteq M\Gamma B\Gamma B \subseteq B$ and $M\Gamma C\Gamma C \subseteq M\Gamma A\Gamma A \subseteq A$ since A is a left ideal of M. Therefore $M\Gamma C\Gamma C \subseteq B \cap A = C$. Hence the intersection of a left weak-interior ideal B of a Γ- semigroup M and a left ideal A of M is always a left weak-interior ideal of M. □

COROLLARY 3.5. *The intersection of a right weak-interior ideal B of a Γ-semigroup$M$ and a right ideal A of M is always a right weak-interior ideal of M.*

COROLLARY 3.6. *The intersection of a weak-interior ideal B of a Γ- semigroup M and an ideal A of M is always a weak-interior ideal of M.*

THEOREM 3.5. *Let A and C be Γ-sub semigroups of a Γ- semigroup M and $B = A\Gamma C$ and B is additively semigroup of M. If A is the left ideal of M then B is a quasi-interior ideal of M.*

PROOF. Let A and C be Γ-sub semigroups of M and $B = A\Gamma C$. Suppose A is the left ideal of M. Then $B\Gamma B = A\Gamma C\Gamma A\Gamma C \subseteq A\Gamma C = B$ and

$$M\Gamma B\Gamma B = M\Gamma A\Gamma C\Gamma A\Gamma C \subseteq A\Gamma C = B.$$

Hence B is a left weak-interior ideal of M. □

COROLLARY 3.7. *Let A and C be Γ-sub semigroups of a $\Gamma-$ semigroup$M$ and $B = A\Gamma C$and B is additively semigroup of M. If C is a right ideal then B is a right weak-interior ideal of M.*

THEOREM 3.6. *If B is a left weak-interior ideal of a Γ- semigroup$M$, $B\Gamma T$ is an additively semigroup of M and $T \subseteq B$ then $B\Gamma T$ is a left weak-interior ideal of M.*

PROOF. Suppose B is a left weak-interior ideal of the Γ- semigroup$M$, $B\Gamma T$ is an additively semigroup of M and $T \subseteq B$.Then $B\Gamma T\Gamma B\Gamma T \subseteq B\Gamma T$. Hence $B\Gamma T$ is a Γ-subsemigroupof M. We have $M\Gamma B\Gamma T\Gamma B\Gamma T \subseteq M\Gamma B\Gamma B\Gamma T \subseteq B\Gamma T$. Hence $B\Gamma T$ is a left weak-interior ideal of the Γ- semigroup$M$. □

Theorem 3.7. *Let B and I be a left weak interior ideals of a Γ- semigroup$M$. Then $B \cap I$ is a left weak-interior ideal of M.*

Proof. Suppose B and I are left weak interior ideals of M. Obviously $B \cap I$ is a Γ-subsemigroupof M. Then $M\Gamma\,(B \cap I)\,\Gamma\,(B\Gamma I) \subseteq M\Gamma B\Gamma B \subseteq B$ and $M\Gamma\,(B \cap I)\,\Gamma\,(B\Gamma I) \subseteq M\Gamma I\Gamma I \subseteq I$. Therefore $M\Gamma(B \cap I)\Gamma(B\Gamma I) \subseteq B \cap I$. Hence $B \cap I$ is a left weak-interior ideal of M. □

Theorem 3.8. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ left weak-interior ideals of a Γ- semigroup$M$ is a left weak-interior ideal of M.*

Proof. Let $B = \bigcap\limits_{\lambda \in A} B_\lambda$. Then B is a Γ-subsemigroupof M. Since B_λ is a left weak-interior ideal of M, we have $M\Gamma B_\lambda \Gamma B_\lambda \subseteq B_\lambda$, for all $\lambda \in A$ and $M\Gamma \cap B_\lambda \cap B_\lambda \subseteq \cap B_\lambda$. Thus $M\Gamma B\Gamma B \subseteq B$. Hence B is a left weak-interior ideal of M. □

Corollary 3.8. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ right weak-interior ideals of a Γ- semigroup$M$ is a right weak-interior ideal of M.*

Corollary 3.9. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ weak-interior ideals of a Γ- semigroup$M$ is a weak-interior ideal of M.*

Theorem 3.9. *Let B be a left weak-interior ideal of a Γ- semigroup$M$,$e \in B$ and e be $\beta-$idempotent. Then $e\Gamma B$ is a left weak-interior ideal of M.*

Proof. Let B be a left weak-interior ideal of the Γ- semigroup$M$. Suppose $x \in B \cap e\Gamma M$. Then $x \in B$ and $x = e\alpha y, \alpha \in \Gamma, y \in M$. Now, we have

$$x = e\alpha y = e\beta e\alpha y = e\beta(e\alpha y) = e\beta x \in e\Gamma B.$$

Therefore $B \cap e\Gamma M \subseteq e\Gamma B$. Thus $e\Gamma B \subseteq B$ and $e\Gamma B \subseteq e\Gamma M$. From here, it follows $e\Gamma B \subseteq B \cap e\Gamma M$ and $e\Gamma B = B \cap e\Gamma M$. Hence $e\Gamma B$ is a left weak-interior ideal of M. □

Corollary 3.10. *Let M be a Γ- semigroup$M$ and e be $\alpha-$idempotent. Then $e\Gamma M$ and $M\Gamma e$ are left weak-interior ideal and right weak-interior ideal of M respectively.*

Theorem 3.10. *Let M be a Γ-semigroup. If $M = M\Gamma a$, for all $a \in M$. Then every left weak-interior ideal of M is a quasi ideal of M.*

Proof. Let B be a left weak-interior ideal of the Γ- semigroup$M$ and $a \in B$. Then $M\Gamma a \subseteq M\Gamma B$ and $M \subseteq M\Gamma B \subseteq M$. Thus $M\Gamma B = M$ and $B\Gamma M = B\Gamma M\Gamma B \subseteq M\Gamma B\Gamma M\Gamma B \subseteq B$. Now, $M\Gamma B \cap B\Gamma M \subseteq M \cap B\Gamma M \subseteq B\Gamma M \subseteq B$. Therefore B is a quasi ideal of M. Hence the theorem. □

Theorem 3.11. *B is a left weak-interior ideal of a Γ- semigroup$M$ if and only if B is a left ideal of some ideal of a Γ- semigroup$M$.*

PROOF. Suppose B is a left ideal of ideal R of the Γ- semigroup$M$. Then $R\Gamma B \subseteq B, M\Gamma R \subseteq R$ and $M\Gamma B\Gamma M\Gamma B \subseteq M\Gamma R\Gamma M\Gamma B \subseteq R\Gamma M\Gamma B \subseteq R\Gamma B \subseteq B$. Therefore B is a left weak-interior ideal of the Γ- semigroup$M$.

Conversely suppose that B is a left weak-interior ideal of the Γ- semigroup$M$. Then $M\Gamma B\Gamma M\Gamma B \subseteq B$. Therefore B is a left ideal of ideal $M\Gamma B\Gamma M$ of the Γ-semigroup$M$. □

COROLLARY 3.11. *B is a right weak-interior ideal of a Γ- semigroup$M$ if and only if B is a right ideal of some ideal of Γ- semigroup$M$.*

COROLLARY 3.12. *B is a weak-interior ideal of a Γ- semigroup$M$ if and only if B is an ideal of some ideal of Γ- semigroup$M$.*

4. Weak-interior simple Γ-semiring

In this section,we introduce the notion of left(right) weak-interior simple Γ-semigroup and characterize the left weak-interior simple Γ- semigroupusing left weak-interior ideals of $\Gamma-$ semigroup and study the properties of minimal left weak-interior ideals of Γ- semigroup.

DEFINITION 4.1. A Γ- semigroup$M$ is a left (right) simple Γ- semigroup if M has no proper left (right) ideals of M.

DEFINITION 4.2. A Γ- semigroup$M$ is said to be simple Γ- semigroup if M has no proper ideals of M.

DEFINITION 4.3. A Γ- semigroup$M$ is said to be left(right) weak-interior simple Γ- semigroupif M has no left(right) weak-interior ideal other than M itself.

DEFINITION 4.4. A Γ- semigroup$M$ is said to be weak-interior simple Γ- semigroupif M has no weak-interior ideal other than M itself.

THEOREM 4.1. *If M is a division Γ- semigroupthen M is a right weak-interior simple $\Gamma-$ semigroup.*

PROOF. Let B be a proper right weak-interior ideal of the division Γ-semigroup M, $x \in M$ and $0 \neq a \in B$. Since M is a division $\Gamma-$ semigroup, there exist $b \in M, \alpha \in \Gamma$ such that $a\alpha b = 1$. Then there exists $\beta \in \Gamma$ such that $a\alpha b\beta x = x = x\beta a\alpha b$. Therefore $x \in B\Gamma M$ and $M \subseteq B\Gamma M$. We have $B\Gamma M \subseteq M$. Hence $M = B\Gamma M$. Thus $M = B\Gamma M = B\Gamma B\Gamma M \subseteq B$. Therefore $M = B$. Hence division Γ- semigroup M has no proper right-quasi-interior ideals. □

COROLLARY 4.1. *If M is a division $\Gamma-$ semigroupthen M is a left weak-interior simple $\Gamma-$ semigroup.*

COROLLARY 4.2. *If M is a division $\Gamma-$ semigroup then M is a weak-interior left simple $\Gamma-$ semigroup.*

THEOREM 4.2. *Let M be a left simple Γ- semigroup.Then M is a left weak-interior simple Γ- semigroup.*

Proof. Let M be a simple Γ- semigroup and B be a left weak-interior ideal of M. Then $M\Gamma B\Gamma B \subseteq B$ and $M\Gamma B\Gamma B$ is an ideal of M. Since M is a simple Γ- semigroup, we have $M\Gamma B\Gamma B = M$. Therefore $M\Gamma B \subseteq M\Gamma B\Gamma B\Gamma B \subseteq B$ and $M\Gamma B \subseteq B$. □

Corollary 4.3. *Let M be a right simple Γ- semigroup. Then M is a right weak-interior simple Γ- semigroup.*

Corollary 4.4. *Let M be a simple Γ- semigroup. Then M is a weak-interior simple Γ- semigroup.*

Theorem 4.3. *Let M be a Γ- semigroup. M is a left weak-interior simple Γ- semigroup if and only if $\langle a\rangle = M$ for all $a \in M$ and where $\langle a\rangle$ is the smallest left weak-interior ideal generated by a.*

Proof. Let M be a Γ- semigroup. Suppose M is the left weak-interior simple Γ- semigroup, $a \in M$ and $B = M\Gamma a$. Then B is a left ideal of M. Therefore, by Theorem 3.5, B is a left weak-interior ideal of M. Thus $B = M$. Hence $M\Gamma a = M$, for all $a \in M$. Now, from $M\Gamma a \subseteq \langle a\rangle \subseteq M$ it follows $M \subseteq \langle a\rangle \subseteq M$. Therefore, $M = \langle a\rangle$.

Conversely suppose that $\langle a\rangle = M$ for all $a \in M$ and where $\langle a\rangle$ is the smallest left weak-interior ideal generated by a for $a \in A$. Then $A = M$. Hence M is a left weak-interior ideal simple Γ- semigroup. □

Theorem 4.4. *Let M be a Γ- semigroup. Then M is a left weak-interior simple Γ- semigroupif and only if $M\Gamma a\Gamma a = M$, for all $a \in M$.*

Proof. Suppose M is the left-weak interior simple Γ- semigroup and $a \in M$. Then $M\Gamma a\Gamma a$ is a weak-interior ideal of M. Hence $M\Gamma a\Gamma a = M$, for all $a \in M$.

Conversely suppose that $M\Gamma a\Gamma a = M$, for all $a \in M$. Let B be a left weak-interior ideal of the Γ- semigroup$M$ and $a \in B$. Then $M = M\Gamma a\Gamma a \subseteq M\Gamma B\Gamma B \subseteq B$. Therefore $M = B$. Hence M is a left weak-interior simple Γ- semigroup. □

Corollary 4.5. *Let M be a Γ- semigroup. Then M is a right weak-interior simple Γ-semriring if and only if $a\Gamma a\Gamma M = M$, for all $a \in M$.*

Corollary 4.6. *Let M be a Γ- semigroup. Then M is a weak-interior simple Γ-semriring if and only if $a\Gamma a\Gamma M = M$ and $M\Gamma aM\Gamma a = M$,for all $a \in M$.*

Theorem 4.5. *Let M be a Γ- semigroupand B be a left weak-interior ideal of M. Then B is a minimal left weak-interior ideal of M if and only if B is a left weak-interior simple Γ-subsemigroupof M.*

Proof. Let B be a minimal left weak-interior ideal of a Γ- semigroup$M$ and C be a left weak-interior ideal of B. Then $B\Gamma C\Gamma C \subseteq C$ and $B\Gamma CB\Gamma C$ is a left weak-interior ideal of M. Since C is a weak-interior ideal of B, we have $B\Gamma C\Gamma C = B$ and $B = B\Gamma C\Gamma C \subseteq C$. Then $B = C$.

Conversely suppose that B is the left weak-interior simple Γ-subsemigroup of M. Let C be a left weak-interior ideal of M and $C \subseteq B$. Then $B\Gamma C\Gamma C \subseteq M\Gamma C\Gamma C \subseteq M\Gamma B\Gamma B \subseteq B$. Therefore C is a left weak-interior of B. Thus $B = C$

since B is a left weak-interior simple Γ-subsemigroup of M. Hence B is a minimal left weak-interior ideal of M. □

Corollary 4.7. *Let M be a* Γ- *semigroupand B be a right weak-interior ideal of M. Then B is a minimal right weak-interior ideal of M if and only if B is a right weak-interior simple* Γ-*subsemigroupof M.*

Corollary 4.8. *Let M be a* Γ- *semigroupand B be a weak-interior ideal of M. Then B is a minimal weak-interior ideal of M if and only if B is a weak-interior simple* Γ-*subsemigroupof M.*

Theorem 4.6. *Let M be a* Γ- *semigroupand $B = R\Gamma L$, where,R is an ideal and L is a minimal left ideal of M . Then B is a minimal left weak-interior ideal of M.*

Proof. Obviously $B = R\Gamma L$ is a left weak-interior ideal of M. Let A be a left weak-interior ideal of M such that $A \subseteq B$. We have $M\Gamma A$ is a left ideal of M. Then $M\Gamma A \subseteq M\Gamma B$ and $M\Gamma A = M\Gamma R\Gamma L \subseteq L$ since L is a left ideal of M. Therefore $M\Gamma A = L$ since L is a minimal left ideal of M. Hence

$$B = M\Gamma A\Gamma A \subseteq M\Gamma B\Gamma B \subseteq M\Gamma R\Gamma L\Gamma R\Gamma L \subseteq L = M\Gamma A\Gamma A \subseteq A.$$

Therefore $A = B$. Hence B is a minimal left weak-interior ideal of M. □

Corollary 4.9. *Let M be a* Γ- *semigroup and $B = R\Gamma L$, where R is a minimal right ideal and L is an right ideal of M. Then B is a minimal right weak-interior ideal of M.*

Theorem 4.7. *M is regular* $\Gamma-$ *semigroup if and only if $A\Gamma B = A \cap B$ for any right ideal A and left ideal B of* Γ-*semigroup M.*

Theorem 4.8. *Let M be a commutative* Γ-*semigroup Then B is a weak-interior ideal of regular* Γ-*semigroup$M$ if and only if $B\Gamma B\Gamma M = B$ for all weak-interior ideals B of M.*

Proof. Suppose M is the commutative regular semigroup, B is the weak-interior ideal of M and $x \in B$. Then $M\Gamma B\Gamma B \subseteq B$, $y \in M$, $\alpha, \beta \in \Gamma$ such that $x = x\alpha y\beta x = y\alpha x\beta x \in M\Gamma B\Gamma B$. Thus $x \in M\Gamma B\Gamma B$. Hence $M\Gamma B\Gamma B = B$.

Conversely suppose that $M\Gamma B\Gamma B = B$ for all weak-interior ideals B of M. Let $B = R \cap L$ and $C = R\Gamma L$, where R, L are ideals of M. Then B and C are weak-interior ideals of M. Thus $M\Gamma(R \cap L)\Gamma(R \cap L) = R \cap L$. From here, it follows $R \cap L = M\Gamma(R \cap L)\Gamma(R \cap L) \subseteq R\Gamma L\Gamma M$ and $R \cap L = M\Gamma(R \cap L)\Gamma(R \cap L) \subseteq M\Gamma R\Gamma L\Gamma M\Gamma L \subseteq R\Gamma L \subseteq R \cap L$ since $R\Gamma L \subseteq L$ and $R\Gamma L \subseteq R$. Therefore $R \cap L = R\Gamma L$. Hence M is a regular $\Gamma-$ semigroup. □

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