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SEMITOTAL DOMINATION OF SOME KNOWN TREES

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Abstract. Let $G = (V, E)$ be a graph without isolated vertices with vertex set V of order $n = |V|$ and edge set E of size $m = |E|$. A vertex of a graph G is said to dominate itself and all its neighbors. A semitotal dominating set in G is a set of S such that S is a dominating set of G and each vertex in the S is within 2-distance from the another vertex of S. The semitotal domination number, denoted by $\gamma_{t2}(G)$, is the minimum cardinality of a semitotal dominating set of G. This graph parameter was introduced by Goddard, Henning and McPillan in 2014 as a generalization of the domination number and the total domination number parameters. In this paper, we examine semitotal domination number of some known trees such as comet, double comet, complete k-ary tree, binomial tree, banana tree and E_p^t graphs.

1. Introduction

Let $G = (V, E)$ be an undirected, simple and connected graphs. For notation and graph terminology we refer to [**6**]. For every vertex $v \in V$, the open neighborhood of v is the set $N(v) = \{u \in V(G)|uv \in E(G)\}$ and the closed neighborhood of v is the set $N[v] = N(v) \cup v$. If $S \subseteq V$, then $N(S) = \bigcup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$. The degree of a vertex $v \in V$ is $deg(v) = deg_G(v) = |N(v)|$. The distance $d_G(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest uv-path in G. The diameter of a graph G, denoted by $diam(G)$, is the greatest distance between two vertices of G. A tree T is a connected acyclic graph. In any tree graph T, the vertex with a degree of 1 is called the leaf vertex and the vertices with a degree of at least 2 are called the interior vertices. A support vertex of T is a vertex that has at least one leaf vertex neighbour.

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When a graph G is taken, a subset S of V is called dominating set of G if every vertex in $V-S$ is adjacent to at least one of a vertex in S. Another way of looking at domination is that a set S of vertices of G is a dominating set if $N[S]=V$. The domination number, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G, and a dominating set of such cardinality is called a γ-set [**14**].

A dominating set S of a graph G without isolated vertices is called a total dominating set of G if the induced subgraph $< S >$ has no isolated vertices. The total domination number of G, denoted by $\gamma_t(G)$, is the minimum cardinality of a total dominating set of G. Total domination was introduced by Cockayne, Dawes and Hedetniemi [**15**].

It is possible to solve many real life problems, including the design and analysis of a communication network and defence oversight, with dominance, which is one of the vulnerability concepts of graph theory. Therefore, the domination in graphs has been a topic that many researchers have been working on. There are many different domination parameters in the existing literature [**2, 3, 4, 5, 11, 14, 19, 22, 23**]. For models of interactions among the nodes in a network, these different variants of domination are very important. To illustrate this, for example, given a network and let S be a small set of main processors in the network. Let assume that the S cluster can manage all the resources of the system. Any processor in system must be in contact with at least one (neighbor) of the main nodes in order to keep it active. This is a situation that network designers desire. At the same time, for security reasons, network designers are not suitable for a processor to be directly connected to multiple main processors. Because they think that a failure in this processor could affect an important subset of S. This means that the integrity of the network is broken. In this case, S must be a network-dominant set, but must be with the additional condition that limits the maximum number of connections (neighbors) that other processors can have in S.

An area of research in domination of graphs that has received considerable attention is the neighbourhood and distances property of vertices in the graph. In this sense, one of the parameters defined recently is semitotal domination. Semitotal domination parameter is defined with the help of neighbors and distance, thus providing regional reinforcement in the graph.

As a new variant of the domination number in graphs, the semitotal domination number was defined and studied by Goddard, Henning and McPillan [**12**]. A semitotal dominating set in G is a set of S such that S is a dominating set of G and each vertex in the S is within 2-distance from the another vertex of S. The semitotal domination number, denoted by $\gamma_{t2}(G)$, is the minimum cardinality of a semitotal dominating set of G. A semitotal dominating set of G of cardinality $\gamma_{t2}(G)$ is called a $\gamma_{t2}(G)$-set. Clearly, for every graph G with no isolated vertex, $\gamma(G) \leqslant \gamma_{t2}(G) \leqslant \gamma_t(G)$. If the graph G is clear from the context, we simply write γ-set and γ_{t2}-set rather than $\gamma(G)$-set and $\gamma_{t2}(G)$-set, respectively.

One application of domination is that of students and teachers. For good education, at least one teacher can advise each student; the concept is that of domination. However, in order to protect of students, we may also require that each teacher is seen by another teacher or headmaster; the concept is that of semitotal domination.

To date, semitotal domination parameter has been studied in many graph structures. The literature review of this parameter is as follows: In 2014, the results for path and cycle graph were defined by Goddard et al., and the upper and lower limits were given for the semitotal domination value of any G graph. Then, certain results were obtained for tree families, an important structure in graph theory [**12**]. The relationship between the concept of matching by Henning and Marcon in 2014 was studied, and claw-free cubic graphs were studied in 2016, and in the same year, vertices were determined, either in all of the minimum semitotal domination sets or not at all, for tree families [**16, 17, 18, 21**]. Henning and Pandey algorithmically studied the semitotal domination parameter [**20**]. Results for line graphs were obtained by Zhu and Liu [**24**]. The number of semitotal domination were studied by Asplund et al. for cartesian product of graphs and a lower value was obtained as a result of this study [**1**]. In 2018, Haynes and Henning worked on perfect graphs with a special graph structure [**13**].

Example 1.1. Let examine the semitotal dominating sets in given the following graph G (see Figure 1) with nine vertices.

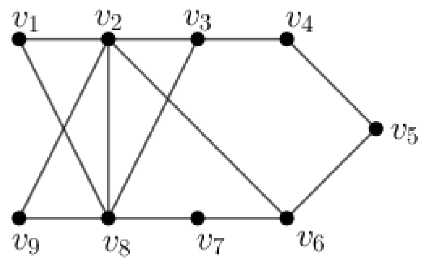

Figure 1. A graph with nine vertices

For the graph given in Figure 1, let $S_1 = \{v_5, v_8\}$ $S_2 = \{v_1, v_4, v_6, v_9\}$, $S_3 = \{v_2, v_5, v_7\}$. When considering the set S_1, it is seen that the set S_1 is a dominating set but it is not a semitotal dominating set because distance of v_5 and v_8 is 3. When considering the set S_2, it is seen that the set S_2 is both a dominating set and a semitotal dominating set. However, this set is not minimum. When considering the set S_3, it is seen that the set S_3 is a minimum semitotal dominating set. Further, this set is not single and we can get another minimum semitotal dominating sets with three vertices for example $\{v_4, v_5, v_8\}$ for the graph G. Hence, we get the semitotal domination number of G as $|S_3| = \gamma_{t2}(G) = 3$.

The paper proceeds as follows. In section 2, existing literature on semitotal domination number is reviewed. In Section 3, the semitotal domination numbers for specific tree graph types are computed and exact formulas are derived.

2. Basic Results

In this section, we recall some of the known results with respect to semitotal domination number.

OBSERVATION 2.1. ([**12**]) If G is a graph with no isolated vertices,then

$$\gamma(G) \leqslant \gamma_{t2}(G) \leqslant \begin{cases} \gamma_t(G) \\ max(\gamma_w(G), 2). \end{cases}$$

LEMMA 2.1 ([**12**]). *For $n \geqslant 3$, the semitotal domination number of a path and cycle is*

$$\gamma_{t2}(P_n) = \gamma_{t2}(C_n) = \lceil \frac{2n}{5} \rceil.$$

LEMMA 2.2 ([**12**]). *For a graph G with every component of order $n \geqslant 4$, we have*

$$\gamma_{t2}(G) \leqslant \frac{n}{2}.$$

LEMMA 2.3 ([**12**]). *If G has n vertices and maximum degree Δ, then*

$$\gamma_{t2}(G) \geqslant \frac{2n}{2\Delta + 1}.$$

LEMMA 2.4 ([**12**]). *If G is connected and nontrivial, then*

$$(a)\gamma_{t2}(G) \leqslant 2\gamma(G) - 1;$$

$$(b)\gamma_t(G) \leqslant 2\gamma_{t2}(G) - 1;$$

and these bounds are sharp.

THEOREM 2.1 ([**21**]). *If G is a graph with $\delta \geqslant 2$ and D is a minimal dominating set of G, then $V(G) \setminus D$ is a semi-TD-set of G.*

THEOREM 2.2 ([**21**]). *If G is a graph with $\delta \geqslant 2$ and S is a minimal semi-TD-set of G, then $V(G) \setminus S$ is a dominating set of G.*

OBSERVATION 2.2. ([**18**]) If G is a connected graph that is not a star, then there is a $\gamma_{t2}(G)$-set that contains no leaf of G.

THEOREM 2.3 ([**25**]). *If T is a tree of order $n(T) \geqslant 2$ with $l(T)$ leaves, then*

$$\gamma_{t2}(T) \geqslant \frac{2[n(T) - l(T) + 2]}{5}.$$

THEOREM 2.4 ([**1**]). *For any isolate-free graphs G and H,*

$$\gamma_{t2}(G \Box H) \geqslant \frac{1}{3}\gamma_{t2}(G)\gamma_{t2}(H).$$

3. Semitotal Domination Number of Some Known Trees

In this section, we have calculated semitotal domination number of some known trees such as comet, double comet, complete k-ary trees, binomial trees, banana trees and E_p^t graphs.

DEFINITION 3.1. ([**8**]) A comet is a tree composed of a star and a pendant path. For any numbers n and $2 \leqslant k \leqslant n-1$, we denote by $T_{n,k}$ the comet of order n with k pendant vertices, i.e., a tree formed by a path P_{n-k-1} of which one end vertex coincides with k pendant vertex of a star $K_{1,k}$ of order $k+1$. $T_{n,k}$ is shown in Figure 2.

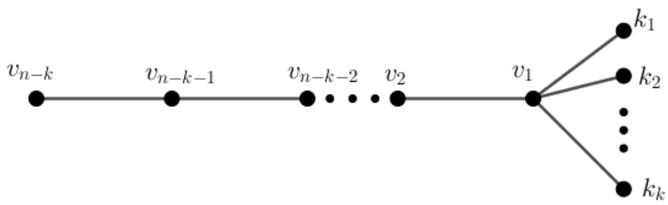

FIGURE 2. A comet $T(n,k)$

THEOREM 3.1. *If* $T_{n,k}$ *is a comet graph of order* n*, then*

$$\gamma_{t2}(T_{n,k}) = \begin{cases} \lceil \frac{2(n-k)}{5} \rceil + 1, & \textit{if } (n-k) \equiv 0, 2 (mod 5) \\ \lceil \frac{2(n-k)}{5} \rceil, & \textit{otherwise.} \end{cases}$$

PROOF. Let S is a semitotal dominating set of $T_{n,k}$. Let $V(T_{n,k}) = V(P_{n-k-1})$ $\cup V(K_{1,k})$ and v_1 is central vertex of $K_{1,k}$. Let label vertices of P_{n-k-1} graph from right to left in $v_2, v_3, ..., v_{n-k}$. Hence we obtain a graph P_{n-k} with P_{n-k-1} and the vertex v_1. In $K_{1,k}$, we can dominate to all leaves with the vertex v_1 so we must start the vertex v_1 to form S. For this reason, let $v_1 \in S$. By definiton of semitotal dominating set there must be one vertex in S which is within distance 2 to semitotally dominate the vertex v_1. Therefore we must choose for the vertex v_1 either one vertex in P_{n-k} (v_2 or v_3) or one leaf in $K_{1,k}$. Since the set S has minimum cardinality, vertex v_3 must be added to S. In order to semitotally dominate remaining vertices of P_{n-k} the set S contains v_i, where i equivalent to 1 and 3 modulo 5 and $i \in \{4, 5, ..., n-k\}$. We know that $\gamma_{t2}(P_{n-k}) = \lceil \frac{(2(n-k))}{5} \rceil$ by Lemma 2.1 but we have five cases depending on $n-k$.

Case 1. Let $(n-k) \equiv 0 (mod 5)$. Note that the vertex v_{n-k-2} added to S finally. Since the vertex v_{n-k} is not dominated, we must add this vertex to S. Thus $S \cup \{v_{n-k}\}$ is γ_{t2}-set for the comet graph $T_{n,k}$ and $\gamma_{t2}(T_{n,k}) = \lceil \frac{2(n-k)}{5} \rceil + 1$.

Case 2. Let $(n-k) \equiv 1 (mod 5)$. In this case the vertex v_{n-k} added to S finally. All vertices are dominated by S but no vertex in S is within distance 2 for the vertex v_{n-k}. If we remove the vertex v_{n-k} from the set S and add the vertex

v_{n-k-1} to S then the new set $S-\{v_{n-k}\}\cup\{v_{n-k-1}\}$ as γ_{t2}-set for the comet graph $T_{k,n}$ and $\gamma_{t2}(T_{k,n}) = \lceil\frac{2(n-k)}{5}\rceil$.

Case 3. Let $(n-k) \equiv 2(mod5)$. Since the vertex v_{n-k-1} added to S finally and no vertex in S is within distance 2 for the vertex v_{n-k-1}, one of the the vertices v_{n-k},v_{n-k-2},v_{n-k-3} must be added to S. Thus the new set is γ_{t2}-set for $T_{n,k}$ and $\gamma_{t2}(T_{n,k}) = \lceil\frac{2(n-k)}{5}\rceil + 1$.

Case 4. Let $(n-k) \equiv 3,4(mod5)$. The set S is a γ_{t2}-set for the comet graph $T_{n,k}$ and $\gamma_{t2}(T_{n,k}) = \lceil\frac{2(n-k)}{5}\rceil$.

By Case 1, 2, 3 and 4, the proof is completed. □

DEFINITION 3.2. ([**10**]) For $a, b \geqslant 1$, $n \geqslant a+b+2$ by $DC(n, a, b)$ we denote a double comet, which is a tree composed of a path containing $n-a-b$ vertices with a pendant vertices attached to one of the ends of the path and b pendant vertices attached to the other end of the path. Thus, $DC(n, a, b)$ has n vertices and $a+b$ leaves. $DC(n, a, b)$ is shown in Figure 3.

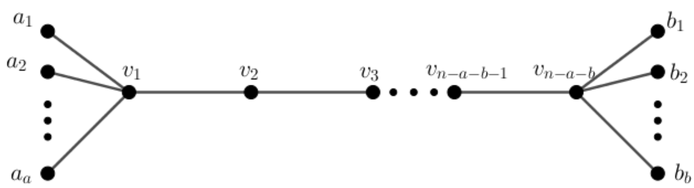

FIGURE 3. A double comet $DC(n, a, b)$

THEOREM 3.2. *Let $DC(n, a, b)$ be a double comet graph of order n. Then semitotal domination number of $DC(n, a, b)$ is*

$$\gamma_{t2}(DC(n, a, b)) = \lceil\frac{2(n-a-b)+4}{5}\rceil.$$

PROOF. Let S be a semitotal dominating set of $DC(n, a, b)$. Let $V(DC(n, a, b)) = \{a_1, ..., a_a, v_1, v_2..., v_{n-a-b-1}, v_{n-a-b}, b_1, ..., b_b, \}$, where v_1 and v_{n-a-b} vertices are central vertices. Firstly, similarly the comet graph $T_{n,k}$ the vertices v_1 and v_{n-a-b} must be added to S to dominate all pendant vertices. We consider the path P with v_1 and $v_{n-a-b-1}$ end vertices. Since the vertex v_1 is in S we must choose the vertices of this path to S in accordance with the definition of semitotal dominating set starting form v_1 to dominate remaining vertices in $DC(n, a, b)$. Thus

$$S = \{v_1, v_{n-a-b}\} \cup \bigcup_{i=0}^{\lceil\frac{n-a-b-3}{5}\rceil-1} v_{5i+3} \bigcup_{i=0}^{\lceil\frac{n-a-b-6}{5}\rceil-1} v_{5i+6}.$$

We have five cases depending on $n-a-b$.

Case 1. Let $(n-a-b) \equiv 0(mod5)$. In this case S is a minimum semitotal dominating set and thus $\gamma_{t2}(DC(n,a,b)) = |S| = 2 + \lceil \frac{n-a-b-3}{5} \rceil + \lceil \frac{n-a-b-6}{5} \rceil = 2 + \frac{n-a-b}{5} + \frac{n-a-b-5}{5} = \frac{2(n-a-b)+5}{5}$.

Case 2. Let $(n-a-b) \equiv 1(mod5)$ be holds. Since the vertex $v_{n-a-b-3}$ added to S from the path P finally and $d(v_{n-a-b}, v_{n-a-b-3}) = 3$, definition of semitotal dominating set is not provided for this vertex. Therefore we must add either the vertex $v_{n-a-b-1}$ or the vertex $v_{n-a-b-2}$ to S and then $S \cup \{v_{n-a-b-1}\}$ or $S \cup \{v_{n-a-b-2}\}$ is a semitotal dominating set. Thus $\gamma_{t2}(DC(n,a,b)) = 2 + \frac{n-a-b-1}{5} + \frac{n-a-b-6}{5} + 1 = \frac{2(n-a-b)+8}{5}$.

Case 3. Let $(n-a-b) \equiv 2,3,4(mod5)$. In these cases, S is a minimum semitotal dominating set. Further if $n-a-b \equiv 2(mod5)$, then $\gamma_{t2}(DC(n,a,b)) = \frac{2(n-a-b)+6}{5}$. If $n-a-b \equiv 3(mod5)$, then $\gamma_{t2}(DC(n,a,b)) = \frac{2(n-a-b)+4}{5}$. If $n-a-b \equiv 4(mod5)$, then $\gamma_{t2}(DC(n,a,b)) = \frac{2(n-a-b)+7}{5}$.

By Case 1, 2 and 3, the proof is completed. □

DEFINITION 3.3. ([**9**]) A complete k-ary tree with depth n is all leaves have the same depth and all internal vertices have degree k. A complete k-ary tree has $\frac{k^{n+1}-1}{k-1}$ vertices and $\frac{k^{n+1}-1}{k-1} - 1 = \frac{k^{n+1}-k}{k-1}$ edges. A complete k-art tree H_k^n is shown in Figure

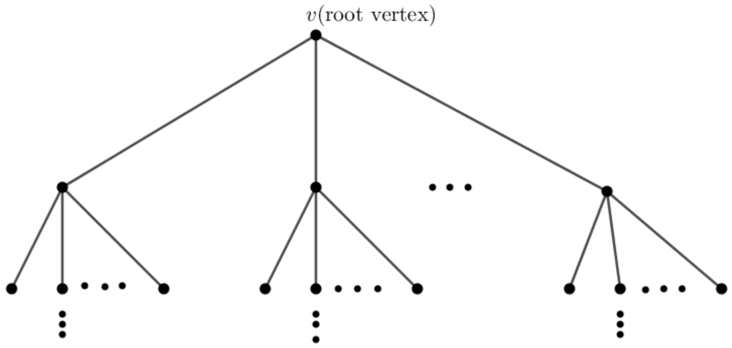

FIGURE 4. A complete k-ary tree H_k^n

THEOREM 3.3. *If H_k^n is a complete k-ary tree of order $\frac{k^{n+1}-1}{k-1}$, then semitotal domination number of H_k^n is*

$$\gamma_{t2}(H_k^n) = \begin{cases} \frac{k^2(1-k^n)}{1-k^3} & \text{if } n \equiv 0(mod3), \\ \frac{1-k^{n+2}}{1-k^3} & \text{if } n \equiv 1(mod3), \\ \frac{k(1-k^{n+1})}{1-k^3} & \text{if } n \equiv 2(mod3). \end{cases}$$

PROOF. Let S be a semitotal dominating set of H_k^n. Let label v to root vertex at zeroth level. Firstly, to dominate to k^n vertices at nth level, all k^{n-1} vertices at $(n-1)$st level must be added to S. Thus all vertices at nth level and $(n-2)$th level are dominated by the set S. Further, since each k vertices of the choosen vertices have same parent, distance of these vertices is 2 and so definition of semitotal dominating set becomes available. Then with the taking of all vertices at $(n-4)$th level to S all vertices at $(n-3)$rd level and at $(n-5)$th level are dominated. Note that the distance between any two vertices at different levels is 3 in S. When this logic is proceeded, we have three cases depending on n.

Case 1. Let $n \equiv 0(mod3)$. In this case, all vertices at $n-1, n-4, ..., n-1-3\lceil\frac{n-3}{3}\rceil$ levels are added to S. Thus $|S| = \sum_{i=0}^{\lceil\frac{n-3}{3}\rceil} k^{n-1-3i}$. Since vertices at $n-1-3\lceil\frac{n-3}{3}\rceil = n-1-n+3 = 2$. level added to S finally, the vertex v is not dominated and we must add this vertex to S. Thus $S \cup \{v\}$ is γ_{t2}-set for H_k^n and $\gamma_{t2}(H_k^n) = \sum_{i=0}^{\lceil\frac{n-3}{3}\rceil} k^{n-1-3i} + 1 = \frac{k^2(1-k^n)}{1-k^3}$.

Case 2. $n \equiv 1(mod3)$. When a similar examination is made, seen that the vertex v added to S finally. In this case, since no vertex have same parent with the vertex v, any vertex must be added in S which is within distance 2 to semitotally dominate the vertex v. Thus the new set is γ_{t2}-set for H_k^n and $\gamma_{t2}(H_k^n) = \sum_{i=0}^{\lceil\frac{n-3}{3}\rceil} k^{n-1-3i} + 1 = \frac{1-k^{n+2}}{1-k^3}$.

Case 3. $n \equiv 2(mod3)$. When a similar examination is made, seen that all vertices at first level added to S finally. Therefore all vertices of H_k^n are dominated by S and the set S is a semitotal dominating set. Thus $|S| = \gamma_{t2}(H_k^n) = \sum_{i=0}^{\lceil\frac{n-3}{3}\rceil} k^{n-1-3i} = \frac{k(1-k^{n+1})}{1-k^3}$.

By Case 1, 2 and 3, the proof is completed. □

DEFINITION 3.4. ([**9**]) The binomial tree of order $n > 0$ with root R is the tree B_n defined as follows.

If $n = 0$, $B_n = B_0 = R$, i.e., the binomial tree of order zero consists of a single node R.

If $n > 0$, $B_n = R, B_0, B_1, ..., B_{n-1}$, i.e., the binomial tree of order $n > 0$ comprises the root R, and n binomial subtrees, $B_0, B_1, ..., B_{n-1}$. $B_0, B-1, B-2, B-3$ and B_n graphs are shown in Figure 5.

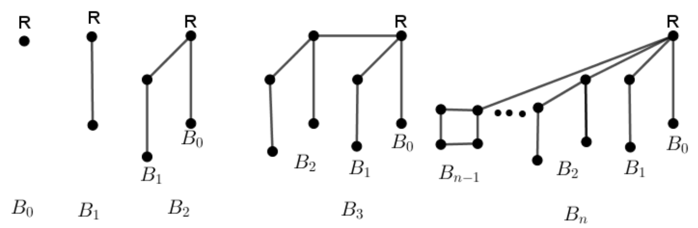

FIGURE 5. Binomial trees B_0, B_1, B_2, B_3, B_n

THEOREM 3.4. *If B_n is a binomial tree, then semitotal domination number of B_n is $\gamma_{t2}(B_n) = 2^{n-1}$.*

PROOF. Let S be a semitotal dominating set of B_n. We know that the structure of B_n includes two B_{n-1} and the structure of B_{n-1} includes two B_{n-2}. With this logic, the recursive structure of the B_n binomial tree obtained as the following.

$$B_n = 2(B_{n-1})$$
$$B_n = 2(2(B_{n-2})) = 2^2(B_{n-2})$$
$$...$$
$$B_n = 2^{n-2}(B_{n-(n-2)}).$$

Therefore $B_n = 2^i(B_{n-i})$ is obtained for $1 \leqslant i \leqslant n-2$. For $n < 3$, it is easily seen that $\gamma_{t2}(B_1) = 2$ and also $\gamma_{t2}(B_2) = 2$ since $B_2 \cong P_4$. Further, since B_3 includes two B_2, when we add all vertices of semitotal dominating set of any B_2 to S, the set S is a semitotal dominating set for B_3 and this set which dominate all leaves is minimum set. Thus $\gamma_{t2}(B_3) = 2\gamma_{t2}(B_2) = 4$. For $n > 3$, when we continue to do same logic, we obtain that the semitotal dominating set of B_n is

$$\gamma_{t2}(B_n) = 2\gamma_{t2}(B_{n-1}). \tag{3.1}$$

Let put this result in recursive formula of B_n. Thus we have

$$\gamma_{t2}(B_n) = 2^i(\gamma_{t2}(B_{n-i})),\ \ 1 \leqslant i \leqslant n-2.$$

We must prove this formula by induction on i for $n > 3$.

- Let $i = 1$. In this case $\gamma_{t2}(B_n) = 2(\gamma_{t2}(B_{n-1}))$ and it is true by (3.1).
- Let $i = k$ and the result is true. That is, we assume that $\gamma_{t2}(B_n) = 2^k(\gamma_{t2}(B_{n-k}))$.

Now, we prove it for $i = k+1$. By induction hypotesis we know that $\gamma_{t2}(B_n) = 2^k(\gamma_{t2}(B_{n-k}))$. Then we get

$$\gamma_{t2}(B_n) = 2^k(2(\gamma_{t2}(B_{n-k-1}))) = 2^{k+1}(\gamma_{t2}(B_{n-(k+1)})).$$

Hence the formula is true for $i = k+1$. Hence, we have

$$\gamma_{t2}(B_n) = 2^i(\gamma_{t2}(B_{n-i})),\ \text{for } 1 \leqslant i \leqslant n-2. \tag{3.2}$$

Initial condition $n = 2$ is achieved for $i = n-2$. We obtain the following formula by putting $i = n-2$ in (3.2).

$$\gamma_{t2}(B_n) = 2^i(\gamma_{t2}(B_{n-i}))$$
$$\gamma_{t2}(B_n) = 2^{n-2}(\gamma_{t2}(B_{n-(n-2)}))$$
$$\gamma_{t2}(B_n) = 2^{n-2}(\gamma_{t2}(B_2))$$
$$\gamma_{t2}(B_n) = 2^{n-2}.2$$
$$\gamma_{t2}(B_n) = 2^{n-1}.$$

The proof is completed. □

Definition 3.5. ([**7**]) The Banana tree graph $B_{n,k}$ is the graph obtained by connecting one leaf of each of n copies of a $k-$star graph with a single root vertex that is distinct for all the stars. The $B_{n,k}$ has order $nk+1$ and size nk. $B_{3,4}$ is shown in Figure 6.

Theorem 3.5. *If $B_{n,k}$ is a banana tree of order nk+1, then semitotal domination number of $B_{n,k}$ is $\gamma_{t2}(B_{n,k}) = n+1$.*

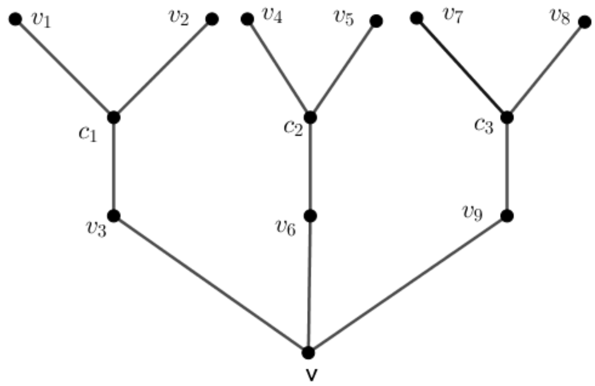

FIGURE 6. A banana tree $B_{n,k}$ for $n = 3$, $k = 4$

PROOF. Let S be a semitotal dominating set of the banana tree $B_{n,k}$. Since the banana tree $B_{n,k}$ includes n star graph, central vertex of each star graph must be added to S to dominate all leaves. However, in this case distance between these central vertices is 4. Further, root vertex is not dominated by S. Thus, if we add the root vertex to S, then the set S which includes all central vertices and root vertex semitotally dominate all vertices of $B_{n,k}$. Hence we get $\gamma_{t2}(B_{n,k}) = n + 1$. □

DEFINITION 3.6. ([**9**]) The graph E_p^t is a tree which has t legs and each leg has p vertices. Thus, E_p^t has $pt + 2$ vertices. The graph E_7^5 is shown in Figure 7.

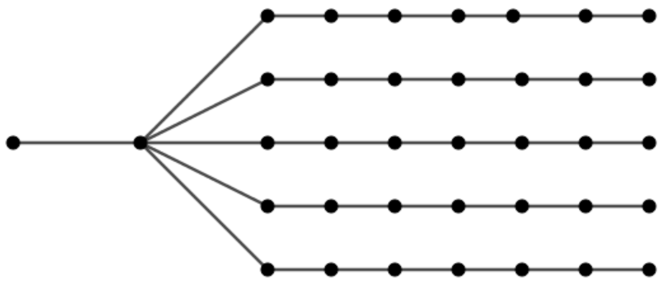

FIGURE 7. E_p^t graph for $p = 7$, $t = 5$

THEOREM 3.6. *If E_p^t is a graph of order $pt + 2$, then semitotal domination number of E_p^t is*

$$\gamma_{t2}(E_p^t) = \lceil \frac{2(p+2)}{5} \rceil + (t-1)\lceil \frac{2(p-1)}{5} \rceil.$$

PROOF. Let S be a semitotal dominating set of E_p^t graph. Let label the vertices of E_p^t by x,y,v_{ij}, where $i \in \{1,2,...,t\}$, $j \in \{1,2,...,p\}$, x is the vertex having maximum degree and y is the adjacent to x of degree 1. As the vertex x has maximum degree of the graph E_p^t, we must take this vertex to S to have minimum cardinality. Further, another vertex that is distance 2 from the vertex x must be added to S. Without loss of generality, we may assume taht the vertex is v_{12} and then $v_{12} \in S$. We know that the vertices v_{i1} where $i \in \{1,2,...,t\}$ are dominated by the vertex x. Then to dominate v_{i2}, we take the vertices v_{i3} to S, where $i \in \{2,...,t\}$. Similarly, the vertices v_{i5} that are distance 2 from v_{i3} must be added to S. When we continue to do same logic according to semitotal dominating set of path, we obtain t paths. One of them has $p+2$ vertices while others have $p-1$ vertices. We know that semitotal domination number of path by lemma 2.1. And so we obtain that semitotal domination number of E_p^t graph is $\lceil \frac{2(p+2)}{5} \rceil + (t-1)\lceil \frac{2(p-1)}{5} \rceil$. □

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