BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., Vol. **11**(1)(2021), 127-134
DOI: 10.7251/BIMVI2101127R

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

ON ω-CLOSED SETS IN IDEAL NANOTOPOLOGICAL SPACES

I. Rajasekaran and O. Nethaji

Abstract. In this paper, we introduce a new class of closed set is called nI_ω-closed sets in ideal nano space. Also we are associating this with other class of sets. We have also described some properties and characteristics.

1. Introduction

An ideal I [**15**] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

(1) $A \in I$ and $B \subseteq A$ imply $B \in I$ and
(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [**3**]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [**14**] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X, τ, I). We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$. Some new notions in the concept of ideal nano topological spaces were introduced by M. Parimala *et al.* [**2, 7, 8, 9, 11**]. In this paper, we introduce a new class of closed set is called nI_ω-closed sets in ideal nano space and some new concepts are discussed. Also we are associating this with other class of sets. We have also described some properties and characteristics.

2010 *Mathematics Subject Classification.* 54A05, 54A10, 54C08, 54C10.
Key words and phrases. $n\omega$-closed, nI_ω-closed and properties of nI_ω-closed.
Communicated by Daniel A. Romano.

2. Preliminaries

Definition 2.1. ([**12**]) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not -X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. ([**4**]) Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,

(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $n\text{-}int(A)$ and $n\text{-}cl(A)$, respectively.

Definition 2.3. ([**4**]) A subset A of a space $(U, \mathcal{N})$ is called nano semi-open (briefly ns-open) if $A \subseteq n\text{-}cl(n\text{-}int(A))$. The complement of ns-open is called ns-closed .

Definition 2.4. A subset A of a space $(U, \mathcal{N}, I)$ is called

(1) nano dense (briefly n-dense) [**13**] if $n\text{-}cl(A) = U$.

(2) nano codense (briefly n-codense) [**5**] if $U - A$ is n-dense.

Definition 2.5. A subset A of a space $(U, \mathcal{N}, I)$ is called

(1) nano g-closed (briefly ng-closed) [**1**] if $n\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is n-open.

(2) nano ω-closed (briefly $n\omega$-closed) [**6**] if $n\text{-}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ns-open.

(3) nano I_g-closed (briefly nI_g-closed) [**10**] if $A_n^\star \subseteq U$ whenever $A \subseteq U$ and U is n-open.

The complement of the above mentioned sets are called their respective open sets.

Definition 2.6. ([**10**]) A subset A of a space $(U, \mathcal{N}, I)$ is called $n\star$-dense initself (resp. $n\star$-perfect and $n\star$-closed if $A \subseteq A_n^\star$ (resp. $A = A_n^\star$ and $A_n^\star \subseteq A$).

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [**8**] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [**8**] the family of nano open sets containing x.

Definition 2.7. ([**8**]) Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)_n^\star$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A_n^\star(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A_n^\star$ for $A_n^\star(I, \mathcal{N})$.

Theorem 2.1 ([**8**]). *Let $(U, \mathcal{N}, I)$ be a space and A and B be subsets of U. Then*

(1) $A \subseteq B \Rightarrow A_n^\star \subseteq B_n^\star$,
(2) $A_n^\star = n\text{-}cl(A_n^\star) \subseteq n\text{-}cl(A)$ *($A_n^\star$ is a n-closed subset of $n\text{-}cl(A)$)*,
(3) $(A_n^\star)_n^\star \subseteq A_n^\star$,
(4) $(A \cup B)_n^\star = A_n^\star \cup B_n^\star$,
(5) $V \in \mathcal{N} \Rightarrow V \cap A_n^\star = V \cap (V \cap A)_n^\star \subseteq (V \cap A)_n^\star$,
(6) $J \in I \Rightarrow (A \cup J)_n^\star = A_n^\star = (A - J)_n^\star$.

Theorem 2.2 ([**8**]). *Let $(U, \mathcal{N}, I)$ be a space with an ideal I and $A \subseteq A_n^\star$, then $A_n^\star = n\text{-}cl(A_n^\star) = n\text{-}cl(A)$.*

Definition 2.8. ([**8**]) Let $(U, \mathcal{N}, I)$ be a space. The set operator $n\text{-}cl^\star$ called a nano $\star$-closure is defined by $n\text{-}cl^\star(A) = A \cup A_n^\star$ for $A \subseteq X$.

It can be easily observed that $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A)$.

Theorem 2.3 ([**9**]). *In a space $(U, \mathcal{N}, I)$, if A and B are subsets of U, then the following results are true for the set operator $n\text{-}cl^\star$.*

(1) $A \subseteq n\text{-}cl^\star(A)$,
(2) $n\text{-}cl^\star(\phi) = \phi$ *and* $n\text{-}cl^\star(U) = U$,
(3) *If* $A \subset B$, *then* $n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(B)$,
(4) $n\text{-}cl^\star(A) \cup n\text{-}cl^\star(B) = n\text{-}cl^\star(A \cup B)$,
(5) $n\text{-}cl^\star(n\text{-}cl^\star(A)) = n\text{-}cl^\star(A)$.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as an ideal nano space $(U, \mathcal{N}, I)$.

3. On nI_ω-closed sets

Definition 3.1. A subset A of an ideal nano space $(U, \mathcal{N}, I)$ is called nI_ω-closed if $A_n^\star \subseteq G$ whenever $A \subseteq G$ and G is ns-open.

The complement of nI_ω-open if U/A is nI_ω-closed.

Theorem 3.1. *In an ideal nano space* $(U, \mathcal{N}, I)$*, every* $n\star$*-closed set is* nI_ω*-closed.*

Proof. Let A be a $n\star$-closed, then $A^\star_n \subseteq A$. Let $A \subseteq G$ where G is ns-open. Hence $A^\star_n \subseteq G$ whenever $A \subseteq G$ and G is ns-open. Therefore A is nI_ω-closed. □

Theorem 3.2. *In an ideal nano space* $(U, \mathcal{N}, I)$*, every* nI_ω*-closed set,* ns*-open set is* $n\star$*-closed.*

Proof. Since A is nI_ω-closed and ns-open. Then $A^\star_n \subseteq A$ whenever $A \subseteq A$ and A is ns-open. Hence A is $n\star$-closed. □

Theorem 3.3. *In an ideal nano space* $(U, \mathcal{N}, I)$*, every* $n\omega$*-closed set is* nI_ω*-closed.*

Proof. Let A be $n\omega$-closed. Then $n - cl(A) \subseteq G$ whenever $A \subseteq G$ and G is ns-open. We have $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is ns-open. Hence A is nI_ω-closed. □

Proposition 3.1. *In an ideal nano space* $(U, \mathcal{N}, I)$*, every* nI_ω*-closed set is* nI_g*-closed.*

Remark 3.1. The following Example shows that reverse part of the Proposition 3.1 is not true.

Example 3.1. Let $U = \{1, 2, 3\}$ and $U/R = \{\{1, 2\}, \{3\}\}\}$ with $X = \{3\}$ then $\mathcal{N} = \{\phi, U, \{3\}\}$ and let $I = \{\phi\}$ be an ideal. Then nI_ω-closed sets are $\{\phi, U, \{1, 2\}\}$ and nI_g-closed sets $\{\phi, U, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$. We have the subset $\{1\}$ is nI_g-closed but not nI_ω-closed.

Remark 3.2. The following Examples are shows that the notions of ng-closed sets and the notions of nI_ω-closed sets are independent in $(U, \mathcal{N}, I)$.

Example 3.2. Let $U = \{1, 2, 3, 4\}$ and $U/R = \{\{1\}, \{2, 3\}, \{4\}\}$ with $X = \{3, 4\}$ then $\mathcal{N} = \{\phi, U, \{4\}, \{2, 3\}, \{2, 3, 4\}\}$ and let $I = \{\{\phi\}, \{1\}\}$ be an ideal. Then

(1) the subset $\{2, 3\}$ is ng-closed but not nI_ω-closed.
(2) the subset $\{2, 3, 4\}$ is nI_ω-closed but not ng-closed.

Theorem 3.4. *If* $(U, \mathcal{N}, I)$ *is any ideal nano space and* $A \subseteq U$*, then the following are equivalent.*

(1) *A is nI_ω-closed.*
(2) *$n\text{-}cl^\star(A) \subseteq G$ whenever $A \subseteq G$ and G is ns-open in U.*
(3) *For all $a \in n\text{-}cl^\star(A), n\text{-}scl(\{a\}) \cap A \neq \phi$.*
(4) *$n\text{-}cl^\star(A)/A$ contains no nonempty ns-closed.*
(5) *$A^\star_n/A$ contains no nonempty ns-closed.*

Proof. (1) ⇒ (2. If A is nI_ω-closed, then $A^\star_n \subseteq G$ whenever $A \subseteq G$ and G is ns-open in U and so $n\text{-}cl^\star(A) = A \cup A^\star_n \subseteq G$ whenever $A \subseteq G$ and G is ns-open in U. This proves (2).

(2) $\Rightarrow$ (3). Assuming that $a \in n\text{-}cl^\star(A)$. If $n\text{-}scl(\{a\}) \cap A = \phi$, then $A \subseteq U/n\text{-}scl(\{a\})$. By (2), $n\text{-}cl^\star(A) \subseteq U/n\text{-}scl(\{a\})$, a contradiction, since $a \in n\text{-}cl^\star(A)$.

(3) $\Rightarrow$ (4). Assuming that $H \subseteq n\text{-}cl^\star(A)/A$, H is ns-closed and $a \in H$. Since $H \subseteq U/A$ and H is ns-closed, then $A \subseteq U/H$ and H is ns-closed, $n\text{-}scl(\{a\}) \cap A = \phi$. Since $a \in n\text{-}cl^\star(A)$ by (3), $n\text{-}scl(\{a\}) \cap A \neq \phi$. Therefore $n\text{-}cl^\star(A)/A$ contains no nonempty ns-closed.

(4) $\Rightarrow$ (5). The following is valid
$n\text{-}cl^\star(A)/A = (A \cup A^\star_n/A = (A \cup A^\star_n) \cap A^c = (A \cap A^c) \cup (A^\star_n \cap A^c) = A^\star_n \cap A^c = A^\star_n/A$.
Therefore $A^\star_n/A$ contains no nonempty is ns-closed.

(5) $\Rightarrow$ (1). Let $A \subseteq G$ where G is ns-open. Therefore $U/G \subseteq U/A$ and so $A^\star_n \cap (U/G) \subseteq A^\star_n \cap (U/A) = A^\star_n/A$. Therefore $A^\star_n \cap (U/G) \subseteq A^\star_n/A$. Since $A^\star_n$ is always n-closed, so $A^\star_n \cap (U/G)$ is a ns-closed contained in $A^\star_n/A$. Therefore $A^\star_n \cap (U/G) = \phi$ and hence $A^\star_n \subseteq G$. Therefore A is nI_ω-closed. □

Theorem 3.5. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. For every $A \in I, A$ is nI_ω-closed.*

Proof. Let $A \subseteq G$ where G is ns-open. Since $A^\star_n = \phi$ for every $A \in I$, then $n\text{-}cl^\star(A) = A \cup A^\star_n = A \subseteq G$. Therefore, by Theorem 3.4, A is nI_ω-closed. □

Theorem 3.6. *If $(U, \mathcal{N}, I)$ is an ideal nano space, then $A^\star_n$ is always nI_ω-closed for every subset A of X.*

Proof. Let $A^\star_n \subseteq G$ where G is ns-open. Since $(A^\star_n)^\star_n \subseteq A^\star_n$, we have $(A^\star_n)^\star_n \subseteq G$ whenever $A^\star_n \subseteq G$ and G is ns-open. Hence $A^\star_n$ is nI_ω-closed. □

Corollary 3.1. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and A be a nI_ω-closed set. Then the following are equivalent.*

(1) *A is $n\star$-closed.*
(2) *$n\text{-}cl^\star(A)/A$ is ns-closed.*
(3) *$A^\star_n/A$ is ns-closed.*

Proof. (1) $\Rightarrow$ (2). If A is $n\star$-closed, then $A^\star_n \subseteq A$ and so $n\text{-}cl^\star(A)/A = (A \cup A^\star_n)/A = \phi$. Hence $n\text{-}cl^\star(A)/A$ is ns-closed.

(2) $\Rightarrow$ (3). Since $n\text{-}cl^\star(A)/A = A^\star_n/A$ and so $A^\star_n/A$ is ns-closed.

(3) $\Rightarrow$ (1). If $A^\star_n/A$ is ns-closed, since A is nI_ω-closed, by Theorem 3.4, $A^\star_n/A = \phi$ and so A is $n\star$-closed. □

Theorem 3.7. *If $(U, \mathcal{N}, I)$ is an ideal nano space and A is a $n\star$-dense in itself, nI_ω-closed subset of U, then A is $n\omega$-closed.*

Proof. Assuming that A is a $n\star$-dense in itself, nI_ω-closed subset of U. Let $A \subseteq G$ where G is ns-open. Then by Theorem 3.4 (2), $n\text{-}cl^\star(A) \subseteq G$ whenever $A \subseteq G$ and G is ns-open. Since A is $n\star$-dense in itself, $n\text{-}cl(A) = n\text{-}cl^\star(A)$. Therefore $n\text{-}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is ns-open. Hence A is $n\omega$-closed. □

Corollary 3.2. *If $(U, \mathcal{N}, I)$ is any ideal nano space where $I = \{\phi\}$, then A is nI_ω-closed $\iff$ A is $n\omega$-closed.*

PROOF. From the fact that for $I = \{\phi\}$, $A^\star_n = n\text{-}cl(A) \supseteq A$. Therefore A is $n\star$-dense in itself. Since A is nI_ω-closed, by Theorem 3.7, A is $n\omega$-closed. Conversely, by Theorem 3.3, every $n\omega$-closed set is nI_ω-closed set. □

COROLLARY 3.3. *If $(U, \mathcal{N}, I)$ is any ideal nano space where I is n-codense and A is a ns-open, nI_ω-closed subset of U, then A is $n\omega$-closed.*

PROOF. Since A is $n\star$-dense in itself. By Theorem 3.7, A is $n\omega$-closed. □

REMARK 3.3. The following Diagram for the subsets stated above discussions.Where $A \longrightarrow B$ denotes A implies B but not converse.

$$\begin{array}{ccccc} n\textbf{-closed} & \longrightarrow & n\omega\textbf{-closed} \longrightarrow & ng\textbf{-closed} \\ \downarrow & & \downarrow & \downarrow \\ n\star\textbf{-closed} & \longrightarrow & nI_\omega\textbf{-closed} \longrightarrow & nI_g\textbf{-closed} \end{array}$$

4. Further properties and characterizations

THEOREM 4.1. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $A \subseteq U$. Then A is nI_ω-closed if and only if $A = B/C$ where B is $n\star$-closed and C contains no nonempty ns-closed set.*

PROOF. If A is nI_ω-closed, then by Theorem 3.4 (5), $C = A^\star_n/A$ contains no nonempty ns-closed set. If $B = n\text{-}cl^\star(A)$, then B is $n\star$-closed such that

$$B/C = (A \cup A^\star_n)/(A^\star_n/A) = (A \cup A^\star_n) \cap (A^\star_n \cap A^c)^c = (A \cup A^\star_n) \cap ((A^\star_n)^c \cup A)$$
$$= (A \cup A^\star_n) \cap (A \cup (A^\star_n)^c) = A \cup (A^\star_n \cap (A^\star_n)^c) = A.$$

Conversely, suppose $A = B/C$ where B is $n\star$-closed and C contains no nonempty ns-closed set. Let G be a ns-open set such that $A \subseteq G$. Then $B/C \subseteq G \Rightarrow B \cap (U/G) \subseteq C$. Now $A \subseteq B$ and $B^\star_n \subseteq B$ then $A^\star_n \subseteq B^\star_n$ and so $A^\star_n \cap (U/G) \subseteq B^\star_n \cap (U/G) \subseteq B \cap (U/G) \subseteq C$. By hypothesis, since $A^\star_n \cap (U/G)$ is ns-closed, $A^\star_n \cap (U/G) = \phi$ and so $A^\star_n \subseteq G$. Hence A is nI_ω-closed. □

THEOREM 4.2. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $A \subseteq U$. If $A \subseteq B \subseteq A^\star_n$, then $A^\star_n = B^\star_n$ and B is $n\star$-dense in itself.*

PROOF. Since $A \subseteq B$, then $A^\star_n \subseteq B^\star_n$ and since $B \subseteq A^\star_n$, then $B^\star_n \subseteq (A^\star_n)^\star_n \subseteq A^\star_n$. Therefore $A^\star_n = B^\star_n$ and $B \subseteq A^\star_n \subseteq B^\star_n$. Hence proved. □

THEOREM 4.3. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. If A and B are subsets of U such that $A \subseteq B \subseteq n\text{-}cl^\star(A)$ and A is nI_ω-closed, then B is nI_ω-closed.*

PROOF. Since A is nI_ω-closed, then by Theorem 3.4 (4), $n\text{-}cl^\star(A)/A$ contains no nonempty ns-closed set. As since $n\text{-}cl^\star(B)/B \subseteq n\text{-}cl^\star(A)/A$ and so $n\text{-}cl^\star(B)/B$ contains no nonempty ns-closed set. Hence B is nI_ω-closed. □

COROLLARY 4.1. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. If A and B are subsets of U such that $A \subseteq B \subseteq A^\star_n$ and A is nI_ω-closed, then A and B are $n\omega$-closed sets.*

PROOF. Let A and B be subsets of U such that $A \subseteq B \subseteq A^\star_n \Rightarrow A \subseteq B \subseteq A^\star_n \subseteq n\text{-}cl^\star(A)$ and A is nI_ω-closed, B is nI_ω-closed. Since $A \subseteq B \subseteq A^\star_n$, then $A^\star_n = B^\star_n$ and so A and B are $n\star$-dense in itself. By Theorem 3.7, A and B are $n\omega$-closed. □

THEOREM 4.4. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $A \subseteq U$. Then A is nI_ω-open if and only if $G \subseteq n\text{-}int^\star(A)$ whenever G is ns-closed and $G \subseteq A$.*

PROOF. Suppose A is nI_ω-open. If G is ns-closed and $G \subseteq A$, then $U/A \subseteq U/G$ and so $n\text{-}cl^\star(U/A) \subseteq U/G$ by Theorem 3.4 (2). Therefore $G \subseteq U/n\text{-}cl^\star(U/A) = n\text{-}int^\star(A)$. Hence $G \subseteq n\text{-}int^\star(A)$.

Conversely, suppose the condition holds. Let G be a ns-open set such that $U/A \subseteq G$. Then $U/G \subseteq A$ and so $U/G \subseteq n\text{-}int^\star(A)$. Therefore $n\text{-}cl^\star(U/A) \subseteq G$. By Theorem 3.4 (2), U/A is nI_ω-closed. Hence A is nI_ω-open. □

COROLLARY 4.2. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $A \subseteq U$. If A is nI_ω-open, then $G \subseteq n\text{-}int^\star(A)$ whenever G is n-closed and $G \subseteq A$.*

THEOREM 4.5. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $A \subseteq U$. If A is nI_ω-open and $n\text{-}int^\star(A) \subseteq B \subseteq A$, then B is nI_ω-open.*

PROOF. Since A is nI_ω-open, then U/A is nI_ω-closed. Follows from the Theorem 3.4 (4), $n\text{-}cl^\star(U/A)/(U/A)$ contains no nonempty ns-closed set. As since $int^\star(A) \subseteq int^\star(B)$ which implies that

$$n\text{-}cl^\star(U/B) \subseteq n\text{-}cl^\star(U/A) \text{ and so } n\text{-}cl^\star(U/B)/(U/B) \subseteq n\text{-}cl^\star(U/A)/(U/A).$$

Hence B is nI_ω-open. □

THEOREM 4.6. *Let $(U, \mathcal{N}, I)$ be an ideal nano space and $A \subseteq U$. Then the following are equivalent.*

(1) *A is nI_ω-closed.*
(2) *$A \cup (U/A^\star_n)$ is nI_ω-closed.*
(3) *$A^\star_n/A$ is nI_ω-open.*

PROOF. (1) ⇒ (2). Suppose A is nI_ω-closed. If G is any ns-open set such that $A \cup (U/A^\star_n) \subseteq G$, then $U/G \subseteq U/(A \cup (U/A^\star_n)) = U \cap (A \cup (A^\star_n)^c)^c = A^\star_n \cap A^c = A^\star_n/A$. Since A is nI_ω-closed, by Theorem 3.4 (5), it follows that $U/G = \phi$ and so $U = G$. Therefore $A \cup (U/A^\star_n) \subseteq G \Rightarrow A \cup (U/A^\star_n) \subseteq U$ and so $(A \cup (U/A^\star_n))^\star_n \subseteq U^\star_n \subseteq U = G$. Hence $A \cup (U/A^\star_n)$ is nI_ω-closed.

(2) ⇒ (1). Suppose $A \cup (U/A^\star_n)$ is nI_ω-closed. If G is any ns-closed set such that $G \subseteq A^\star_n/A$, then $G \subseteq A^\star_n$ and $G \nsubseteq A \Rightarrow U/A^\star_n \subseteq U/G$ and $A \subseteq U/G$. Therefore $A \cup (U/A^\star_n) \subseteq A \cup (U/G) = U/G$ and U/G is ns-open. Since $(A \cup (U/A^\star_n))^\star_n \subseteq U/G \Rightarrow A^\star_n \cup (U/A^\star_n)^\star_n \subseteq U/G$ and so $A^\star_n \subseteq U/G \Rightarrow G \subseteq U/A^\star_n$. Since $G \subseteq A^\star_n$, it follows that $G = \phi$. Hence A is nI_ω-closed.

(2) ⇔ (3). $U/(A^\star_n/A) = U \cap (A^\star_n \cap A^c)^c = U \cap ((A^\star_n)^c \cup A)$
$= (U \cap (A^\star_n)^c) \cup (U \cap A) = A \cup (U/A^\star_n)$. □

THEOREM 4.7. *Let $(U, \mathcal{N}, I)$ be an ideal nano space. Then every subset of U is nI_ω-closed if and only if every ns-open set is $n\star$-closed.*

PROOF. Suppose every subset of U is nI_ω-closed. If $G \subseteq U$ is ns-open, then G is nI_ω-closed and so $G^\star_n \subseteq G$. Hence G is $n\star$-closed.

Conversely, suppose that every ns-open set is $n\star$-closed. If G is ns-open set such that $A \subseteq G \subseteq U$, then $A^\star_n \subseteq G^\star_n \subseteq G$ and so A is nI_ω-closed. □

References

[1] K. Bhuvaneshwari and K. Mythili Gnanapriya. Nano generalized closed sets in nano topological Space. *International Journal of Scientific and Research Publications*, **4**(5)(2014), Available at http://www.ijsrp.org/research-paper-0514/ijsrp-p2984.pdf

[2] R. Jeevitha, M. Parimala and R. Udhaya Kumar. Nano $A\psi$-connectedness and compactness in nano topological spaces. *International Journal of Recent Technology and Engineering (IJRTE)*, **8**(2)(2019), 788–791.

[3] K. Kuratowskii. *Topology, Vol I.* Academic Press, New York, 1966.

[4] M. L. Thivagar and C. Richard. On nano forms of weakly open sets. *International Journal of Mathematics and Statistics Invention*, **1**(1)(2013), 31–37.

[5] O. Nethaji, R. Asokan and I. Rajasekaran. New generalized classes of ideal nanotopological spaces. *Bull. Int. Math. Virtual Inst.*, **9**(3)(2019), 543–552.

[6] O. Nethaji and I. Rajasekaran. On ω-closed sets in nano topological spaces. (To appear).

[7] M. Parimala, D. Arivuoli and R. Udhayakumar. $nI\alpha g$-closed sets and normality via $nI\alpha g$-closed sets in nano ideal topological spaces. *J. Math., Punjab Univ.*, **52**(4)(2020), 41–51.

[8] M. Parimala, T. Noiri and S. Jafari. New types of nano topological spaces via nano ideals. https://www.researchgate.net/publication/315892279.

[9] M. Parimala and S. Jafari. On some new notions in nano ideal topological spaces. *Eurasian Bulletin of Mathematics*, **1**(3)(2018), 85–93.

[10] M. Parimala, S. Jafari and S. Murali. Nano ideal generalized closed sets in nano ideal topological spaces. *Annales Univ. Sci. Budapest. Sec. Math.*, **60**(2017), 3–11.

[11] M. Parimala, R. Jeevitha and R. Udhayakumar. Nano contra $\alpha\psi$ continuous and nano contra $\alpha\psi$ irresolute in nano topology. *Global journal of engineering science and researches*, **5**(9)(2018), 64–71.

[12] Z. Pawlak. Rough sets. *International journal of computer and Information Sciences*, **11**(5)(1982), 341–356.

[13] P. Sathishmohan, V. Rajendran, C. Vignesh Kumar and P. K. Dhanasekaran. On nano semi pre neighbourhoods in nano topological spaces. *Malaya J. Mat.*, **6**(1)(2018), 294–298.

[14] R. Vaidyanathaswamy. The localization theory in set topology. *Proc. Indian Acad. Sci.*, **20**(1945), 51–61.

[15] R. Vaidyanathaswamy. *Set topology.* Chelsea Publishing Company, New York, 1946.

Received by editors 03.08.2020; Revised version 12.08.2020; Available online 24.08.2020.

I. Rajasekaran: Department of Mathematics, Tirunelveli Dakshina Mara Nadar Sangam College, T. Kallikulam - 627 113, Tirunelveli District, Tamil Nadu, India.
E-mail address: sekarmelakkal@gmail.com

O. Nethaji: Research Scholar, Department of Mathematics, School of Mathematics, Madurai Kamaraj University, Madurai-21, Tamil Nadu, India.
E-mail address: jionetha@yahoo.com