BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Bull. Int. Math. Virtual Inst., Vol. **10**(3)(2020), 483-490
DOI: 10.7251/BIMVI2003483R

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

3-TOTAL QUOTIENT LABELING OF SOME GRAPHS

R. Ponraj, J. Maruthamani, and R. Kala

Abstract. In this paper we introduce a new graph labeling method called k-Total quotient cordial. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map where $k \in \mathbb{N}$ is a variable and $k > 1$. For each edge uv, assign the label $\left[\frac{f(u)}{f(v)}\right]$ or $\left[\frac{f(v)}{f(u)}\right]$ according as $f(u) \geqslant f(v)$ or $f(v) \geqslant f(u)$. f is called k-Total quotient cordial labeling of G if $\left|t_{qf}(i) - t_{qf}(j)\right| \leqslant 1$, $i, j \in \{1, 2, \ldots, k\}$ where $t_{qf}(x)$ denotes the total number of vertices and the edges labeled with x. In this paper we investigate the the 3-total quotient cordial behaviour of some graphs like corona of graphs, fan, ladder, KD_n, P_n^2 and $P_n \odot K_2$.

1. Introduction

We consider graphs in this paper are finite, simple and undirected only. Ponraj. et. al introduced the concept of quotient cordial labeling [**4**]. Ponraj.R, Maruthamani.J and Kala.R has been recently introduced the k-total quotient cordial labeling [**5**]. In [**5**], we prove that every graph is a subgraph of a k-total quotient graph and investigate the 3-total quotient cordial behaviour of path, cycle, star, bistar and some corona of graphs. In this paper we investigate the the 3-total quotient cordial behaviour of some graphs like corona of graphs, fan, ladder, KD_n, P_n^2 and $P_n \odot K_2$. Terms not defined are used from Harray [**3**].

2. k-Total quotient cordial labeling

Definition 2.1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map where $k \in \mathbb{N}$ is a variable and $k > 1$. For each edge uv, assign the label $\left[\frac{f(u)}{f(v)}\right]$ or $\left[\frac{f(v)}{f(u)}\right]$ according as $f(u) \geqslant f(v)$ or $f(v) \geqslant f(u)$. f is called k-Total quotient cordial labeling of G if $|t_{qf}(i) - t_{qf}(j)| \leqslant 1$, $i, j \in \{1, 2, \ldots, k\}$ where $t_{qf}(x)$ denotes

1991 *Mathematics Subject Classification.* 05C78.
Key words and phrases. Corona of graphs, fan, ladder, KD_n and $P_n \odot K_2$.

the total number of vertices and the edges labeled with x. A graph with k-total quotient cordial labeling is called k-total quotient cordial graph.

3. Preliminaries

Definition 3.1. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The *corona* of G_1 with G_2 is the graph $G_1 \odot G_2$ obtained by taking one copy of G_1 , p_1 copies of G_2 and joining the i^{th} vertex of G_1 by an edge to every vertex in the i^{th} copy of G_2 where $1 \leqslant i \leqslant p_1$.

Definition 3.2. The graph $F_n = P_n + K_1$ is called a *Fan graph* where P_n : $u_1u_2 \dots u_n$ is a Path and $V(K_1) = u$.

Definition 3.3. The graph $L_n = P_n \times P_2$ is called a *Ladder*.

Definition 3.4. The *cartesian product* of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ with vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$.

Definition 3.5. The graph KD_n is obtained from two copies of the path $P_n : u_1u_2 \dots u_n$ and $P_n : v_1v_2 \dots v_n$ with $V(KD_n) = \{u, u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(KD_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leqslant i \leqslant n-1\} \cup \{uu_i, vv_i : 1 \leqslant i \leqslant n\}$.

Definition 3.6. For a simple connected graph G the *square of graph G* is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

4. Main Results

Theorem 4.1. *The graph F_n is 3-total quotient cordial iff $n \not\equiv 1 \pmod 3$ where n is odd.*

Proof. The graph $F_n = P_n + K_1$ where $P_n : u_1u_2 \dots u_n$ is a Path and $V(K_1) = u$. Clearly, $|V(F_n)| + |E(F_n)| = 3n$.

Case 1. $n \equiv 0 \pmod 3$.
Let $n = 3r$, $r \in \mathbb{N}$. Assign the label 1 to the vertex u. Now we assign the labels 3, 1 and 2 respectively to the vertices u_1, u_2 and u_3. Next we assign the labels 3, 1 and 2 to the next three vertices u_4, u_5 and u_6 respectively. Proceeding like this until we reach the last vertex u_{3r}. Clearly the last vertex u_{3r} receives the label 2 in this labeling pattern. It is easy to verify that $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 3r$.

Case 2. $n \equiv 1 \pmod 3$.
Let $n = 3r+1$, n is even and $r \in \mathbb{N}$. Assign the label 3 to the vertices $u_1, u_2, \dots, u_{\frac{3r+1}{2}}$. Next we assign the label 2 to the vertices $u_{\frac{3r+1}{2}+1}, u_{\frac{3r+1}{2}+2}, \dots, u_{3r+1}$. Here $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 3r + 1$.

Case 3. $n \not\equiv 1 \pmod 3$, when n is odd.
Suppose f is a 3-total quotient cordial labelling of F_n then $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = n$.

Subcase 3.1. $f(u) = 1$.

To get the edge label 3, then 3 should be adjacent to the vertex which is labelled by 1. If we assign the label 3 to the first $r+1$ vertices starts from u_1 and label 2 to the remaining vertices , then $t_{qf}(3) < n$, $t_{qf}(2) > n$ and $t_{qf}(1) = n$, a contradiction. Next 3 is label to the alternating vertices and 2 is label to the remaining vertices then $t_{qf}(3) > n$, $t_{qf}(2) < n$ and $t_{qf}(1) = n$. This is the maximum possible to get the value of $t_f(3)$ and $t_{qf}(2)$ nearest to n, a contradiction.

Subcase 3.2. $f(u) = 3$.

Subcase 3.2(a).
3 is label to the consecutive vertices $u_1, u_2, \ldots, u_{r+1}$ and remaining vertices are labelled by 2, then $t_{qf}(3) < n$, $t_{qf}(2) < n$ and $t_{qf}(1) > n$, a contradiction.

Subcase 3.2(b).
1 and 3 is label to the vertices u_1 and u_2 respectively. Next label 1 to the alternating vertices starts from u_3 and the remaining vertices are labelled by 2, then $t_{qf}(3) > n$, $t_{qf}(2) > n$ and $t_{qf}(1) = n$, a contradiction.

Subcase 3.2(c).
3 is label to the alternating vertices and 2 is label to the remaining vertices, then $t_{qf}(3) < n$, $t_{qf}(2) < n$ and $t_{qf}(1) > n$, a contradiction

Subcase 3.3.
The proof is symmetry to subcase 2 of case 4.

Case 4. $n \equiv 2 \pmod 3$.
Let $n = 3r + 2$, $r \in \mathbb{N}$. As in case 1, assign the same labelling scheme for the vertices u and u_i $(1 \leqslant i \leqslant 3r)$. Finally, we assign the labels 3, 2 to the vertices u_{3r+1} and u_{3r+2} respectively. Clearly $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 3r + 2$.

Case 5. $n = 2$.
A 3-total quotient cordial labeling of F_2 is given in Figure 1.

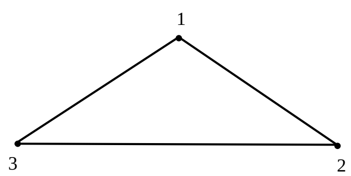

FIGURE 1

□

THEOREM 4.2. *The graph KD_n is 3-total quotient cordial for all n.*

PROOF. The graph KD_n is obtained from two copies of the path $P_n : u_1u_2 \ldots u_n$ and $P_n : v_1v_2 \ldots v_n$ with $V(KD_n) = \{u, u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(KD_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leqslant i \leqslant n-1\} \cup \{uu_i, vv_i : 1 \leqslant i \leqslant n\}$. Clearly $|V(KD_n)| + |E(KD_n)| = 6n - 1$.

Case 1. $n \equiv 0 \pmod 3$.
Let $n = 3r$, $r \in \mathbb{N}$. Assign the label 1 to the vertex u. Now we consider the vertices u_i $(1 \leqslant i \leqslant 3r)$. Then we assign the labels 3, 1 and 2 respectively to the vertices u_1, u_2 and u_3. Next we assign the labels 3, 1 and 2 to the next three vertices u_4, u_5

and u_6 respectively. Proceeding like this until we reach the last vertex u_{3r}. Clearly the last vertex u_{3r} receives the label 2 in this labeling pattern. Next we move to the pendent vertices v_i $(1 \leqslant i \leqslant 3r)$. Assign the label 1 to the vertex u. Now we assign the labels 3, 1 and 2 respectively to the vertices v_1, v_2 and v_3. Next we assign the labels 3, 1 and 2 to the next three vertices v_4, v_5 and v_6 respectively. Proceeding like this until we reach the last vertex v_{3r}. Clearly the last vertex v_{3r} receives the label 2 in this labeling scheme. Here $t_{qf}(1) = 6r - 1$ and $t_{qf}(2) = t_{qf}(3) = 6r$.

Case 2. $n \equiv 1 \pmod 3$.
Let $n = 3r + 1$, $r \in \mathbb{N}$. As in case 1, assign the same labeling method to the vertices u, u_i $(1 \leqslant i \leqslant 3r)$ and v_i $(1 \leqslant i \leqslant 3r)$. Finally, we assign the labels 2, 3 to the vertices u_{3r+1} and v_{3r+2} respectively. Clearly $t_{qf}(1) = 6r + 1$ and $t_{qf}(2) = t_{qf}(3) = 6r + 2$.

Case 3. $n \equiv 2 \pmod 3$.
Let $n = 3r + 2$, $r \in \mathbb{N}$. Assign the same labeling pattern to the vertices u, u_i $(1 \leqslant i \leqslant 3r+1)$ and v_i $(1 \leqslant i \leqslant 3r+1)$ as in case 2. Next, we assign the labels 2, 3 respectively to the vertices u_{3r+1} and v_{3r+2} . It is easy to verify that $t_{qf}(1) = 6r+3$ and $t_{qf}(2) = t_{qf}(3) = 6r + 4$.

Case 4. $n = 2$.
A 3-total quotient cordial labeling of KD_2 is given in Figure 2.

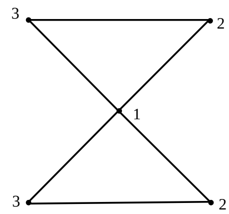

FIGURE 2

□

THEOREM 4.3. *If* $n \equiv 0, 1 \pmod 3$, *then* P_n^2 *is* 3*-total quotient cordial.*

PROOF. Let $u_1 u_2 \ldots u_n$ be the path. Let u_i is adjacent to u_{i+2}, $(1 \leqslant i \leqslant n-2)$. Clearly $\left|V(P_n^2)\right| + \left|E(P_n^2)\right| = 3n - 3$.

Case 1. $n \equiv 0 \pmod 3$.
Let $n = 3r$, $r \in \mathbb{N}$. Assign the labels 3, 1 and 2 respectively to the vertices u_1, u_2 and u_3. Next we assign the labels 3, 1 and 2 to the next three vertices u_4, u_5 and u_6 respectively. Proceeding like until we reach the last vertex u_{3r}. Clearly the last vertex u_{3r} receives the label 2 in this labeling pattern. It is easy to verify that $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 3r - 1$.

Case 2. $n \equiv 1 \pmod 3$.
Let $n = 3r + 1$, $r \in \mathbb{N}$. Assign the labels 3, 1, 1, 2 to the vertices u_1, u_2, u_3 and u_4 respectively. Next we assign the labels 3, 1 and 2 respectively to the vertices u_5, u_6 and u_7. Next we assign the labels 3, 1 and 2 to the next three vertices u_8, u_9 and u_{10} respectively. Proceeding like until we reach the last vertex u_{3r+1}.

Clearly the last vertex u_{3r+1} receives the label 2 in this labeling pattern. Clearly $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 3r$. □

THEOREM 4.4. *The ladder L_n is 3-total quotient cordial for all n.*

PROOF. Let $V(L_n) = \{u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leqslant i \leqslant n-1\} \cup \{u_iv_i : 1 \leqslant i \leqslant n\}$. Clearly $|V(L_n)| + |E(L_n)| = 5n - 2$.

Case 1. $n \equiv 0 \pmod 3$.

Let $n = 3r$, $r \in \mathbb{N}$. Assign the labels 3, 1 and 2 respectively to the vertices u_1, u_2 and u_3. Next we assign the labels 3, 1 and 2 to the next three vertices u_4, u_5 and u_6 respectively. Proceeding like until we reach the last vertex u_{3r}. Clearly the last vertex u_{3r} receives the label 2 in this labeling pattern. Next we consider the vertices v_i $(1 \leqslant i \leqslant 3r)$. Assign the labels 1, 2 and 3 respectively to the vertices v_1, v_2 and v_3. Next we assign the labels 1, 2 and 3 to the next three vertices v_4, v_5 and v_6 respectively. Proceeding like until we reach the last vertex v_{3r}. Clearly the last vertex v_{3r} receives the label 3 in this labeling scheme. It is easy to verify that $t_{qf}(1) = t_{qf}(2) = 5r$ and $t_{qf}(3) = 5r - 1$.

Case 2. $n \equiv 1 \pmod 3$.

Let $n = 3r + 1$, $r \in \mathbb{N}$. As in case 1, assign the same labeling technique to the vertices u_i $(1 \leqslant i \leqslant 3r)$ and v_i $(1 \leqslant i \leqslant 3r)$. Finally we assign the labels 2, 1 to the vertices u_{3r+1} and v_{3r+1} respectively. Clearly $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 5r + 1$.

Case 3. $n \equiv 2 \pmod 3$.

Let $n = 3r+2$, $r \in \mathbb{N}$. As in case 2, assign the same labeling pattern to the vertices u_i $(1 \leqslant i \leqslant 3r + 1)$ and v_i $(1 \leqslant i \leqslant 3r + 1)$. Finally we assign the labels 3, 2 to the vertices u_{3r+2} and v_{3r+2} respectively. Here $t_{qf}(1) = t_{qf}(3) = 5r + 3$ and $t_{qf}(2) = 5r + 2$.

Case 4. $n = 2$.

A 3-total quotient cordial labeling of L_2 is given in Figure 3.

FIGURE 3

□

THEOREM 4.5. *The graph $P_n \odot K_2$ is 3-total quotient cordial for all n.*

PROOF. Let $V(P_n \odot K_2) = \{u_i, v_i, w_i : 1 \leqslant i \leqslant n\}$ and $E(P_n \odot K_2) = \{u_iv_i, u_iw_i, v_iw_i : 1 \leqslant i \leqslant n\} \cup \{u_iu_{i+1} : 1 \leqslant i \leqslant n-1\}$. Obviously $|V(P_n \odot K_2)| + |E(P_n \odot K_2)| = 7n - 1$.

Case 1. $n \equiv 0 \pmod 3$.

Let $n = 3r$, $r \in \mathbb{N}$. Assign the labels 3, 1 and 2 respectively to the vertices u_1, u_2 and u_3. Next we assign the labels 3, 1 and 2 to the next three vertices u_4, u_5 and u_6 respectively. Proceeding like until we reach the last vertex u_{3r}. Clearly the last vertex u_{3r} receives the label 2 in this labeling pattern. Next we consider the vertices v_i $(1 \leqslant i \leqslant 3r)$. Assign the labels 1, 3 and 1 respectively to the vertices v_1, v_2 and v_3. Next we assign the labels 1, 3 and 1 to the next three vertices v_4, v_5 and v_6 respectively. Proceeding like until we reach the last vertex v_{3r}. Clearly the last vertex v_{3r} receives the label 1 in this labeling scheme. Now we move to the vertices w_i $(1 \leqslant i \leqslant 3r)$. Assign the labels 3, 2 and 2 respectively to the vertices w_1, w_2 and w_3. Next we assign the labels 3, 2 and 2 to the next three vertices w_4, w_5 and w_6 respectively. Proceeding like until we reach the last vertex w_{3r}. Clearly the last vertex w_{3r} receives the label 2 in this labeling technique. It is easy to verify that $t_{qf}(1) = 7r - 1$ and $t_{qf}(2) = t_{qf}(3) = 7r$.

Case 2. $n \equiv 1 \pmod 3$.
Let $n = 3r + 1$, $r \in \mathbb{N}$. As in case 1, assign the same labeling technique to the vertices u_i $(1 \leqslant i \leqslant 3r)$, v_i $(1 \leqslant i \leqslant 3r)$ and w_i $(1 \leqslant i \leqslant 3r)$. Finally we assign the labels 1, 3, 2 to the vertices u_{3r+1}, v_{3r+1} and w_{3r+1} respectively. Clearly $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 7r + 2$.

Case 3. $n \equiv 2 \pmod 3$.
Let $n = 3r + 2$, $r \in \mathbb{N}$. Assign the same labeling pattern to the vertices u_i $(1 \leqslant i \leqslant 3r + 1)$, v_i $(1 \leqslant i \leqslant 3r + 1)$ and w_i $(1 \leqslant i \leqslant 3r + 1)$ by case 2. Finally we assign the labels 1, 3, 2 to the vertices u_{3r+2}, v_{3r+2} and w_{3r+2} respectively. Clearly $t_{qf}(1) = 7r + 5$ and $t_{qf}(2) = t_{qf}(3) = 7r + 4$.

Case 4. $n = 2$.
A 3-total quotient cordial labeling of $P_n \odot K_2$ is given in Figure 4.

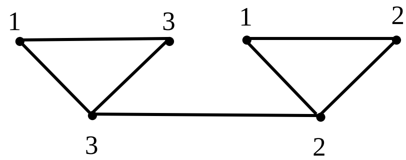

FIGURE 4

□

COROLLARY 4.1. *The graph $C_n \odot K_2$ is 3-total quotient cordial for all n.*

PROOF. Let $V(P_n \odot K_2) = \{u_i, v_i, w_i : 1 \leqslant i \leqslant n\}$ and $E(P_n \odot K_2) = \{u_1u_n, v_1v_n\} \cup \{u_iv_i, u_iw_i, v_iw_i : 1 \leqslant i \leqslant n\} \cup \{u_iu_{i+1} : 1 \leqslant i \leqslant n - 1\}$. Clearly $|V(C_n \odot K_2)| + |E(C_n \odot K_2)| = 7n$.

Case 1. $n \equiv 0 \pmod 3$.
Let $n = 3r$, $r \in \mathbb{N}$. The vertex labelled in case 1 of Theorem 4.5 is also a 3-total quotient cordial of $C_n \odot K_2$. Here $t_{qf}(1) = t_{qf}(2) = t_{qf}(3) = 7r$.

Case 2. $n \equiv 1 \pmod 3$.

Let $n = 3r + 1$, $r \in \mathbb{N}$. Assign the same labeling technique in case 2 of Theorem 4.5 is also a 3-total quotient cordial of $C_n \odot K_2$. It is easy to verify that $t_{qf}(1) = t_{qf}(2) = 7r + 2$ and $t_{qf}(3) = 7r + 3$.

Case 3. $n \equiv 3 \pmod 3$.

Let $n = 3r + 2$, $r \in \mathbb{N}$. Assign the same labeling pattern in case 3 of Theorem 4.5 is also a 3-total quotient cordial of $C_n \odot K_2$. Clearly $t_{qf}(1) = 7r + 4$ and $t_{qf}(2) = t_{qf}(3) = 7r + 5$. □

EXAMPLE 4.1. A 3-total quotient cordial labeling of $C_5 \odot K_2$ is given in Figure 5.

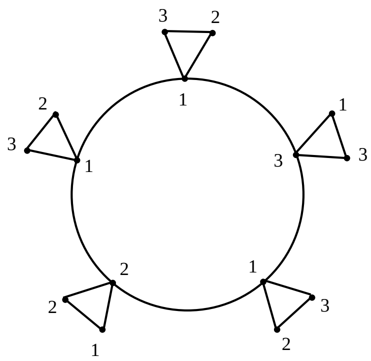

FIGURE 5

References

[1] I. Cahit. Cordial graphs: A weaker version of graceful and harmonious graphs. *Ars Combinatoria* **23** (1987), 201–207.

[2] J. A. Gallian. A dynamic survey of graph labeling. *The Electronic Journal of Combinatorics* **19** (2019), 000–000.

[3] F. Harary. *Graph theory.* Addision wesley, New Delhi, 1969.

[4] R. Ponraj, M. Sivakumar and R. Singh. k-quotient cordial labeling of graphs. *Palestine Journal of Mathematics* **8**(1) (2019), 361–366.

[5] R. Ponraj, J. Maruthamani and R. Kala. k-total quotient cordial labeling of graphs. *Journal of Applied and Pure Mathematics*, **1**(3-4)(2019), 157–165.

Received by editors 09.07.2019; Revised version 19.03.2020; Available online 30.03.2020.

R. Ponraj. Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India

E-mail address: ponrajmaths@gmail.com

J. Maruthamani. Research Scholar, Register number: 18124012091054, Department of Mathematics, Manonmaniam sundarnar university, Abishekapatti, Tirunelveli-627 012, Tamilnadu, India

E-mail address: mmani2011@gmail.com

R. Kala. Department of Mathematics, Manonmaniam sundarnar university, Abishekapatti, Tirunelveli-627 012, Tamilnadu, India

E-mail address: karthipyi91@yahoo.co.in