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YANG-BAXTER EQUATION IN KU-ALGEBRAS

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Abstract. In this paper, we give some important notions for the Yang-Baxter equation after presenting the definitions of KU-algebra and its some properties. Then we build some set-theoretical solutions to the Yang-Baxter equation by using these properties.

1. Introduction

The concept of KU-algebra was introduced by C. Prabpayak and U. Leerawat, and they studied ideals and congruences of this algebraic structure [**14**]. Indeed, they presented a homomorphism of KU-algebra, and the relationships between quotient KU-algebras and an isomorphism.[**15**] Recently, Rezaei et al. showed that a KU-algebra is equivalent to a commutative self-distributive BE-algebra and a self-distributive KU-algebra is equivalent to a Hilbert algebra [**18**]. Mostafa et al. investigated the implicative and commutative ideals of KU-algebras and their properties [**5, 17**]. They also introduced the associated graph of commutative KU-algebra [**6**].

Besides, the Yang-Baxter equation which was originally used in theoretical physics [**16**] and in statistical mechanics ([**1**], [**2**], [**19**]) can be applied to many different parts of science, technology and industry. In addition to these areas, there exist some applications of this equation on scientific areas such as quantum groups, quantum mechanics, quantum computing, knot theory, integrable systems, non-commutative geometry, C*-algebras (see, for instance, [**3**]-[**4**] and ([**7**], [**10**], [**19**]). This also shows to us the significance of Yang-Baxter equation. Therefore, many researchers benefit from various algebraic structures to find (set-theoretical) solutions to this equation. Recently, Oner et al. have built new set-theoretical solutions to the Yang–Baxter equation using some algebraic such as BL-algebras

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[12], Wajsberg Algebras [11] and MTL-algebras [13]. We wish to examine new solutions to the Yang-Baxter equation in relation with KU-algebras rather than aforementioned algebraic, quantum or geometric structures.

After presenting two definitions of a KU-algebra and its properties, it is given a definition of the Yang-Baxter equation which plays important role in many popular areas such as quantum mechanics and integrable systems. Then we investigate some solutions to the set-theoretical Yang-Baxter equation by taking advantage of the axioms of KU-algebra. Besides, we sampled those that usually are not usually solutions to the set-theoretical Yang-Baxter equation.

2. Preliminaries

The following fundamental notions are taken from [5] and [18].

Definition 2.1. A KU-algebra is a triple $(X;*,1)$ where X is a non-empty set, $*$ is a binary operation on X and 1 is a fixed element of X such that the following axioms are satisfied:
(KU1) $(x*y)*((y*z)*(x*z))=1$,
(KU2) $1*x=x$,
(KU3) $x*1=1$,
(KU4) $x*y=1$ and $y*x=1$, then $x=y$, for all $x,y,z\in X$.

In what follows, let X denote a KU-algebra unless otherwise specified.

Proposition 2.1. *Let $(X;*,1)$ be a KU-algebra. Then*
(i) $x*x=1$,
(ii) $x*(y*x)=1$,
(iii) $x*(y*z)=y*(x*z)$
(iv) $y*((y*x)*x)=1$, *for all* $x,y,z\in X$.

Definition 2.2. A KU-algebra X is said to be self-distributive if

$$x*(y*z) \quad = \quad (x*y)*(x*z)$$

for all $x,y,z\in X$.

Definition 2.3. A KU-algebra is a triple $(X;*,0)$ where X is a non-empty set, $*$ is a binary operation on X and 1 is a fixed element of X such that the following axioms are satisfied:
(KU1') $(x*y)*((y*z)*(x*z))=0$,
(KU2') $0*x=x$,
(KU3') $x*0=0$,
(KU4') $x*y=0$ and $y*x=0$, then $x=y$, for all $x,y,z\in X$.

3. Solutions to the Yang-Baxter Equation in KU-Algebras

In this paper, we present some results in connection with (the set-theoretical) Yang-Baxter equation in KU-algebras.

Let V be a vector space over a filed F, which is algebraically closed and of characteristic zero.

Definition 3.1. ([**8**]) A linear automorphism φ of $V \otimes V$ is a solution to the Yang-Baxter equation, if the following equality holds in the automorphism group of $V \otimes V \otimes V$:

$$(\varphi \otimes id_V) \circ (id_V \otimes \varphi) \circ (\varphi \otimes id_V) = (id_V \otimes \varphi) \circ (\varphi \otimes id_V) \circ (id_V \otimes \varphi). \tag{3.1}$$

In the following definitions φ^{nm} means φ acting on the n-th and m-th component.

Definition 3.2. ([**9**]) φ is a solution to the Yang-Baxter equation if

$$\varphi^{12} \circ \varphi^{23} \circ \varphi^{12} = \varphi^{23} \circ \varphi^{12} \circ \varphi^{23}, \tag{3.2}$$

Definition 3.3. ([**8**]) φ is a solution to the quantum Yang-Baxter equation if

$$\varphi^{12} \circ \varphi^{13} \circ \varphi^{23} = \varphi^{23} \circ \varphi^{13} \circ \varphi^{12}. \tag{3.3}$$

Let T be the twist map $T : V \otimes V \to V \otimes V$ defined by $T(u \otimes v) = v \otimes u$. Then, φ satisfies (3.2) if and only if $\varphi \circ T$ satisfies (3.3) if and only if $T \circ \varphi$ satisfies (3.3).

A connection between the set-theoretical Yang-Baxter equation and KU-algebras is constituted by the following definition.

Definition 3.4. ([**9**]) Let X be a set and $\varphi : X^2 \to X^2, \varphi(p,q) = (p', q')$ be a map. The map φ is a solution for the set-theoretical Yang-Baxter equation if it satisfies (3.2), which is also equivalent to (3.3), where

$$\begin{aligned}
\varphi^{12} : X^3 \to X^3, &\quad \varphi^{12}(s_1, s_2, s_3) = (s_1', s_2', s_3), \\
\varphi^{23} : X^3 \to X^3, &\quad \varphi^{23}(s_1, s_2, s_3) = (s_1, s_2', s_3'), \\
\varphi^{13} : X^3 \to X^3, &\quad \varphi^{13}(s_1, s_2, s_3) = (s_1', s_2, s_3').
\end{aligned}$$

Now we construct solutions to the set theoretical Yang-Baxter equation by using KU-algebras.

Lemma 3.1. *Let $\mathcal{X} = (X; *, 1)$ be a KU-algebra. Then the following are solutions to the set-theoretical Yang-Baxter equation:*
(a) $\varphi(k_1, k_2) = (1 * k_1, 1 * k_2)$,
(b) $\varphi(k_1, k_2) = (0 * k_1, 0 * k_2)$.

Proof. The proof is obtained from (KU2) and (KU2'). □

Theorem 3.1. *Let $\mathcal{X} = (X; *, 1)$ be a KU-algebra. If the condition*

$$k_1 * (k_2 * k_3) \quad = \quad (k_1 * k_2) * k_3$$

holds for $k_1, k_2, k_3 \in X$, then the following are solutions to the set-theoretical Yang-Baxter equation:
(a) $\varphi(k_1, k_2) = (k_1 * k_2, 1)$,
(b) $\varphi(k_1, k_2) = (k_1 * k_2, 0)$.

Proof. (a) We define

$$\begin{aligned}
\varphi^{12}(k_1, k_2, k_3) &= (k_1 * k_2, 1, k_3) \\
\varphi^{23}(k_1, k_2, k_3) &= (k_1, k_2 * k_3, 1).
\end{aligned}$$

By using (KU2) and (KU3), for all $(k_1, k_2, k_3) \in X^3$, we obtain

$$\begin{aligned}(\varphi^{12} \circ \varphi^{23} \circ \varphi^{12})(k_1, k_2, k_3) &= \varphi^{12}(\varphi^{23}(\varphi^{12}(k_1, k_2, k_3))) \\ &= \varphi^{12}(\varphi^{23}(k_1 * k_2, 1, k_3)) \\ &= \varphi^{12}(k_1 * k_2, 1 * k_3, 1) \\ &= \varphi^{12}(k_1 * k_2, k_3, 1) \\ &= ((k_1 * k_2) * k_3, 1, 1)\end{aligned}$$

and

$$\begin{aligned}(\varphi^{23} \circ \varphi^{12} \circ \varphi^{23})(k_1, k_2, k_3) &= \varphi^{23}(\varphi^{12}(\varphi^{23}(k_1, k_2, k_3))) \\ &= \varphi^{23}(\varphi^{12}(k_1, k_2 * k_3, 1)) \\ &= \varphi^{23}(k_1 * (k_2 * k_3), 1, 1) \\ &= (k_1 * (k_2 * k_3), 1 * 1, 1) \\ &= (k_1 * (k_2 * k_3), 1, 1) \\ &= ((k_1 * k_2) * k_3, 1, 1).\end{aligned}$$

(b) The proof is similiar to (a). □

Corollary 3.1. *Let $(X, *, 1)$ be a KU-algebra. If the condition*

$$k_1 * (k_2 * k_3) = (k_1 * k_2) * k_3$$

holds for $k_1, k_2, k_3 \in X$, then the following are solutions to the set-theoretical Yang-Baxter equation.

(a) $\varphi(k_1, k_2) = (k_1 * (1 * k_2), 1)$,

(b) $\varphi(k_1, k_2) = ((1 * k_1) * k_2, 1)$.

Lemma 3.2. *Let $(X; *, 1)$ be a KU-algebra. Then the following are solutions of the set-theoretical Yang-Baxter equation in self-distributive BE-algebras and Hilbert algebras while they are not solutions in KU-algebras.*

(a) $\varphi(k_1, k_2) = (k_1 * k_2, k_1)$,

(b) $\varphi(k_1, k_2) = (k_1 * (k_1 * k_2), k_1)$.

Proof. (a) We define φ^{12} and φ^{23} as follows:

$$\begin{aligned}\varphi^{12}(k_1, k_2, k_3) &= (k_1 * k_2, k_1, k_3), \\ \varphi^{23}(k_1, k_2, k_3) &= (k_1, k_2 * k_3, k_2).\end{aligned}$$

For all $(k_1, k_2, k_3) \in X^3$ we have

$$\begin{aligned}(\varphi^{12} \circ \varphi^{23} \circ \varphi^{12})(k_1, k_2, k_3) &= \varphi^{12}(\varphi^{23}(\varphi^{12}(k_1, k_2, k_3))) \\ &= \varphi^{12}(\varphi^{23}(k_1 * k_2, k_1, k_3)) \\ &= \varphi^{12}(k_1 * k_2, k_1 * k_3, k_1) \\ &= ((k_1 * k_2) * (k_1 * k_3), k_1 * k_2, k_1) \\ &= (k_1 * (k_2 * k_3), k_1 * k_2, k_1)\end{aligned}$$

and

$$\begin{aligned}(\varphi^{23}\circ\varphi^{12}\circ\varphi^{23})(k_1,k_2,k_3) &= \varphi^{23}(\varphi^{12}(\varphi^{23}(k_1,k_2,k_3)))\\ &= \varphi^{23}(\varphi^{12}(k_1,k_2*k_3,k_2))\\ &= \varphi^{23}(k_1*(k_2*k_3),k_1,k_2)\\ &= (k_1*(k_2*k_3),k_1*k_2,k_1).\end{aligned}$$

(b) Since self-distributive BE-algebra and Hilbert algebra are distributive, $\varphi(k_1,k_2)$ $= (k_1*(k_1*k_2),k_1)$ is equivalent to $\varphi(k_1,k_2)=(k_1*k_2,k_1)$, then the proof is obtained from (a). □

EXAMPLE 3.1. Let $(X,*,0)$ be a KU-algebra, where $X=\{0,k_1,k_2\}$. We define $*$ by the following Cayley table:

$*$	0	k_1	k_2
0	0	k_1	k_2
k_1	0	0	k_2
k_2	0	k_1	0

Then it is easy to show that $\varphi(k_1,k_2)=(k_1*k_2,0)$ is a solution to the set-theoretical Yang-Baxter equation in this KU-algebra. However, it is not generally a solution in KU-algebras.

φ^{12} and φ^{23} are defined as follows:

$$\begin{aligned}\varphi^{12}(k_1,k_2,k_3) &= (k_1*k_2,0,k_3),\\ \varphi^{23}(k_1,k_2,k_3) &= (k_1,k_2*k_3,0).\end{aligned}$$

By using (KU2'), we get

$$\begin{aligned}(\varphi^{12}\circ\varphi^{23}\circ\varphi^{12})(k_1,k_2,k_3) &= \varphi^{12}(\varphi^{23}(\varphi^{12}(k_1,k_2,k_3)))\\ &= \varphi^{12}(\varphi^{23}(k_1*k_2,0,k_3))\\ &= \varphi^{12}(k_1*k_2,0*k_3,0)\\ &= \varphi^{12}(k_1*k_2,k_3,0)\\ &= ((k_1*k_2)*k_3,0,0) \qquad (5)\end{aligned}$$

and

$$\begin{aligned}(\varphi^{23}\circ\varphi^{12}\circ\varphi^{23})(k_1,k_2,k_3) &= \varphi^{23}(\varphi^{12}(\varphi^{23}(k_1,k_2,k_3)))\\ &= \varphi^{23}(\varphi^{12}(k_1,k_2*k_3,0))\\ &= \varphi^{23}(k_1*(k_2*k_3),0,0)\\ &= (k_1*(k_2*k_3),0*0,0)\\ &= (k_1*(k_2*k_3),0,0). \qquad (6)\end{aligned}$$

Since the equation (5) is equal to equation (6) for all distinct $k_1,k_2,k_3\in X$, it is not generally a solution in KU-algebras.

EXAMPLE 3.2. Let $(X, *, 0)$ be a KU-algebra, where $X = \{0, 1, 2\}$. We define $k_1 * k_2$ as follows [**18**]:

$$k_1 * k_2 = \begin{cases} 0, & \text{if } k_1 \geqslant k_2 \\ k_2 - k_1, & \text{if } k_2 > k_1 \end{cases}$$

Then $\varphi(k_1, k_2) = (0, k_1 * (k_1 * k_2))$ is a solution to the set-theoretical Yang-Baxter equation in this KU-algebra while it is not a solution in KU-algebras.

φ^{12} and φ^{23} are defined as follows:

$$\begin{aligned} \varphi^{12}(k_1, k_2, k_3) &= (0, k_1 * (k_1 * k_2), k_3), \\ \varphi^{23}(k_1, k_2, k_3) &= (k_1, 0, k_2 * (k_2 * k_3)). \end{aligned}$$

By using (KU2')and (KU3'), we get

$$\begin{aligned} (\varphi^{12} \circ \varphi^{23} \circ \varphi^{12})(k_1, k_2, k_3) &= \varphi^{12}(\varphi^{23}(\varphi^{12}(k_1, k_2, k_3))) \\ &= \varphi^{12}(\varphi^{23}(0, k_1 * (k_1 * k_2), k_3)) \\ &= \varphi^{12}(0, 0, (k_1 * (k_1 * k_2)) * \\ &\quad ((k_1 * (k_1 * k_2)) * k_3)) \\ &= (0, 0 * (0 * 0), (k_1 * (k_1 * k_2)) \\ &\quad *((k_1 * (k_1 * k_2)) * k_3)) \\ &= (0, 0, (k_1 * (k_1 * k_2)) * \\ &\quad ((k_1 * (k_1 * k_2)) * k_3)) \end{aligned} \tag{7}$$

and

$$\begin{aligned} (\varphi^{23} \circ \varphi^{12} \circ \varphi^{23})(k_1, k_2, k_3) &= \varphi^{23}(\varphi^{12}(\varphi^{23}(k_1, k_2, k_3))) \\ &= \varphi^{23}(\varphi^{12}(k_1, 0, k_2 * (k_2 * k_3))) \\ &= \varphi^{23}(0, k_1 * (k_1 * 0), k_2 * (k_2 * k_3)) \\ &= \varphi^{23}(0, 0, k_2 * (k_2 * k_3)) \\ &= (0, 0, 0 * (k_2 * (k_2 * k_3))) \\ &= (0, 0, k_2 * (k_2 * k_3)). \end{aligned} \tag{8}$$

Since the equation (7) is equal to equation (8) for all distinct $k_1, k_2, k_3 \in X$, it is not generally a solution in KU-algebras.

LEMMA 3.3. *Let $(X, *, 1)$ be a self distributive KU-algebra. If the condition*

$$k_1 * k_2 \quad = \quad k_2 * k_1$$

holds for $k_1, k_2 \in X$, then $\varphi(k_1, k_2) = (k_1 * k_2, k_2)$ is a solution to the set-theoretical Yang-Baxter equation.

PROOF. Let φ^{12} and φ^{23} be defined as follows:

$$\begin{aligned} \varphi^{12}(k_1, k_2, k_3) &= (k_1 * k_2, k_2, k_3), \\ \varphi^{23}(k_1, k_2, k_3) &= (k_1, k_2 * k_3, k_3). \end{aligned}$$

By using (KU3), Proposition (i) and (iii), for all $(k_1, k_2, k_3) \in X^3$ we get

$$\begin{aligned}(\varphi^{12} \circ \varphi^{23} \circ \varphi^{12})(k_1, k_2, k_3) &= \varphi^{12}(\varphi^{23}(\varphi^{12}(k_1, k_2, k_3))) \\ &= \varphi^{12}(\varphi^{23}(k_1 * k_2, k_2, k_3)) \\ &= \varphi^{12}(k_1 * k_2, k_2 * k_3, k_3) \\ &= ((k_1 * k_2) * (k_2 * k_3), k_2 * k_3, k_3)\end{aligned}$$

and

$$\begin{aligned}(\varphi^{23} \circ \varphi^{12} \circ \varphi^{23})(k_1, k_2, k_3) &= \varphi^{23}(\varphi^{12}(\varphi^{23}(k_1, k_2, k_3))) \\ &= \varphi^{23}(\varphi^{12}(k_1, k_2 * k_3, k_3)) \\ &= \varphi^{23}(k_1 * (k_2 * k_3), k_2 * k_3, k_3) \\ &= (k_1 * (k_2 * k_3), (k_2 * k_3) * k_3, k_3) \\ &= (k_1 * (k_2 * k_3), k_3 * (k_2 * k_3), k_3) \\ &= (k_2 * (k_1 * k_3), k_3 * (k_2 * k_3), k_3) \\ &= ((k_2 * k_1) * (k_2 * k_3), (k_3 * k_2) * (k_3 * k_3), k_3) \\ &= ((k_2 * k_1) * (k_2 * k_3), (k_3 * k_2) * 1, k_3) \\ &= ((k_2 * k_1) * (k_2 * k_3), k_3 * k_2, k_3) \\ &= ((k_1 * k_2) * (k_2 * k_3), k_2 * k_3, k_3).\end{aligned}$$

Then $\varphi(x, y) = (k_1 * k_2, k_2)$ is a solution to the set-theoretical Yang-Baxter equation in the self-distributive KU-algebra X under the given condition. □

Lemma 3.4. *Let $\mathcal{X} = (X; *, 1)$ be a KU-algebra. If the condition*

$$(k_1 * k_2) * k_3 = (k_1 * k_3) * (k_2 * k_3)$$

holds for $k_1, k_2, k_3 \in X$, then $\varphi(k_1, k_2) = (k_2 * k_1, k_1)$ is a solution of the set-theoretical Yang-Baxter equation.

Proof. We define

$$\begin{aligned}\varphi^{12}(k_1, k_2, k_3) &= (k_2 * k_1, k_1, k_3) \\ \varphi^{23}(k_1, k_2, k_3) &= (k_1, k_3 * k_2, k_2).\end{aligned}$$

For all $(k_1, k_2, k_3) \in X^3$, we obtain

$$\begin{aligned}(\varphi^{12} \circ \varphi^{23} \circ \varphi^{12})(k_1, k_2, k_3) &= \varphi^{12}(\varphi^{23}(\varphi^{12}(k_1, k_2, k_3))) \\ &= \varphi^{12}(\varphi^{23}(k_2 * k_1, k_1, k_3)) \\ &= \varphi^{12}(k_2 * k_1, k_3 * k_1, k_1) \\ &= ((k_3 * k_1) * (k_2 * k_1), k_2 * k_1, k_1)\end{aligned}$$

and

$$\begin{aligned}(\varphi^{23}\circ\varphi^{12}\circ\varphi^{23})(k_1,k_2,k_3) &= \varphi^{23}(\varphi^{12}(\varphi^{23}(k_1,k_2,k_3)))\\ &= \varphi^{23}(\varphi^{12}(k_1,k_3*k_2,k_2))\\ &= \varphi^{23}((k_3*k_2)*k_1,k_1,k_2)\\ &= ((k_3*k_2)*k_1,k_2*k_1,k_1)\\ &= ((k_3*k_1)*(k_2*k_1),k_2*k_1,k_1).\end{aligned}$$

Then, it is a solution under the above condition. □

THEOREM 3.2. *Let $\mathcal{X}=(X;*,1)$ be a self-distributive KU-algebra. Then $\varphi(k_1,k_2)=(k_2,k_2*k_1)$ is a solution to the set-theoretical Yang-Baxter equation.*

PROOF. We define

$$\begin{aligned}\varphi^{12}(k_1,k_2,k_3) &= (k_2,k_2*k_1,k_3)\\ \varphi^{23}(k_1,k_2,k_3) &= (k_1,k_3,k_3*k_2).\end{aligned}$$

For all $(k_1,k_2,k_3)\in X^3$, we obtain

$$\begin{aligned}(\varphi^{12}\circ\varphi^{23}\circ\varphi^{12})(k_1,k_2,k_3) &= \varphi^{12}(\varphi^{23}(\varphi^{12}(k_1,k_2,k_3)))\\ &= \varphi^{12}(\varphi^{23}(k_2,k_2*k_1,k_3))\\ &= \varphi^{12}(k_2,k_3,k_3*(k_2*k_1))\\ &= (k_3,k_3*k_2,k_3*(k_2*k_1))\\ &= (k_3,k_3*k_2,(k_3*k_2)*(k_3*k_1))\end{aligned}$$

and

$$\begin{aligned}(\varphi^{23}\circ\varphi^{12}\circ\varphi^{23})(k_1,k_2,k_3) &= \varphi^{23}(\varphi^{12}(\varphi^{23}(k_1,k_2,k_3)))\\ &= \varphi^{23}(\varphi^{12}(k_1,k_3,k_3*k_2))\\ &= \varphi^{23}(k_3,k_3*k_1,k_3*k_2)\\ &= (k_3,k_3*k_2,(k_3*k_2)*(k_3*k_1)).\end{aligned}$$

□

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