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FIRST NEIGHBOURHOOD ZAGREB INDEX OF DERIVED GRAPHS AND F-SUMS OF GRAPHS

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ABSTRACT. For a molecular graph G, the first neighbourhood Zagreb index is defined as the sum of square of neighbourhood degree sum of vertices of a graph G. In this paper, we obtain first neighbourhood Zagreb index of derived graphs and F-sums of graphs in terms of parameters of graphs considered.

1. Introduction

Let G be a finite undirected graph without loops and multiple edges on n vertices and m edges. We denote vertex set and edge set of graph G as $V(G)$ and $E(G)$, respectively. The *neighbourhood* of a vertex $u \in V(G)$ is defined as the set $N_G(u)$ consisting of all vertices v which are adjacent with u. The *degree* of a vertex $u \in V(G)$, denoted by $d_G(u)$ and is equal to $|N_G(u)|$. Let

$$S_G(v) = \sum_{u \in N_G(v)} d_G(u)$$

be the neighbourhood degree sum of vertices where and $N_G(v) = \{u : uv \in E(G)\}$. For undefined graph theoretic definitions and terminology refer to [**14**].

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Key words and phrases. F-sums of graphs; first neighbourhood Zagreb index; Zagreb indices; graph transformation.

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For a molecular graph G, *first Zagreb index* was defined by Gutman and Trinajstić [**13**] in 1972 as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2.$$

The second Zagreb index was defined in [**12**] as

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) \cdot d_G(v).$$

The first Zagreb index [**19**] can also be expressed as

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

Another vertex-degree-based graph invariant called *forgotten topological index* or F- index was put forward by Furtula and Gutman [**9**] is defined as

$$F(G) = \sum_{v \in V(G)} d_G(v)^3 = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2].$$

The *hyper Zagreb index* was introduced by Shirdel et al., in [**25**] which is defined as

$$HM(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^2.$$

The *sum-connectivity index* of a graph G was defined in [**30**] as

$$\chi(G) = \sum_{xy \in E(G)} [d_G(x) + d_G(y)]^{-\frac{1}{2}}.$$

Further, it has been extended to the *general sum-connectivity index* which is defined in [**31**] as

$$\chi_\alpha(G) = \sum_{xy \in E(G)} [d_G(x) + d_G(y)]^\alpha, \text{where } \alpha \text{ is any real number.}$$

There have been several topological indices which are direct modifications of the first and second Zagreb indices. By replacing degree of a vertex by sum of neighbourhood degrees of a vertex, new approaches to topological indices were started. The first approach in this direction was made by Graovac et al. in 2011 [**10**]. Later, Hosamani [**15**] in 2016 introduced Sanskriti index by replacing degree of a vertex by its neighbourhood degree sum in Augumented Zagreb index. Inspired by this, and to understand the effect of neighbourhood degrees on a vertex we have defined a new topological index in [**2**] called first neighbourhood Zagreb index (FNZI) of a graph defined as

$$NM_1(G) = \sum_{v \in V(G)} S_G(v)^2,$$

by replacing degree of a vertex by its neighbourhood degree sum in first form of first Zagreb index. We call this index as first neighbourhood Zagreb index because k-neighbourhood degrees can be considered, where k is distance from a given vertex. For $k = 1$, first neighbourhood Zagreb, for $k = 2$, second neighbourhood Zagreb and so on. For chemical applications of this index refer to [**2, 20**]. Bounds for this index were given in [**2**], some graph operations on this index was studied in [**20**]. Some properties of this index were given in [**23**]. FNZI of some nanostructures were given in [**2**]. Later, Kulli [**18**] extended the work on topological indices based on neighbourhood degree sum of a vertex. The fifth M-Zagreb indices [**18**] of a molecular graph G are defined as

$$M_1G_5(G) = \sum_{uv \in E(G)} (S_G(u) + S_G(v)) \quad M_2G_5(G) = \sum_{uv \in E(G)} (S_G(u)S_G(v))$$

and fifth hyper M-Zagreb indices [**18**] of a molecular graph G are defined as

$$HM_1G_5(G) = \sum_{uv \in E(G)} (S_G(u) + S_G(v))^2 \quad HM_2G_5(G) = \sum_{uv \in E(G)} (S_G(u)S_G(v))^2.$$

For chemical applications of topological indices refer [**11, 26**]. For more on topological indices of graph operations refer to [**1, 3, 5, 8, 16, 17, 21, 22, 27, 28, 29**]. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, there are four related transformation graphs as follows:

• *Subdivision graph* $S = S(G)$ [**14**]; $V(S) = V(G) \cup E(G)$ and the vertex of S corresponding to the edge uv of G is inserted in the edge uv of G;
• *Semitotal-point graph* $T_2 = T_2(G)$ [**24**]; $V(T_2) = V(G) \cup E(G)$ and $E(T_2) = E(S) \cup E(G)$;
• *Semitotal-line graph* $T_1 = T_1(G)$ [**24**]; $V(T_1) = V(G) \cup E(G)$ and $E(T_1) = E(S) \cup E(L)$;
• *Total graph* $T = T(G)$ [**4**]; $V(T) = V(G) \cup E(G)$ and $E(T) = E(S) \cup E(G) \cup E(L)$.

Here $L = L(G)$ is the line graph of G. In the recent paper [**7**], Eliasi and Taeri

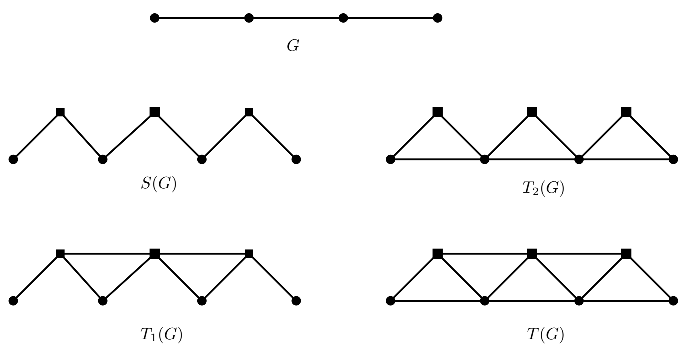

FIGURE 1. Graph G and its transformations $S(G), T_2(G), T_1(G)$ and $T(G)$.

introduced four new operations on graphs as follows:

Definition 1.1. Let $F \in \{S, T_2, T_1, T\}$. The F-sums of G_1 and G_2, denoted by $G_1 +_F G_2$, is a graph with the set of vertices $V(G_1 +_F G_2) = (V(G_1) \cup E(G_1)) \times V(G_2)$ and two vertices (u_1, u_2) and (v_1, v_2) of $G_1 +_F G_2$ are adjacent if and only if $[u_1 = v_1 \in V(G_1)$ and $u_2v_2 \in E(G_2)]$ or $[u_2 = v_2 \in V(G_2)$ and $u_1v_1 \in E(F(G_1))]$.

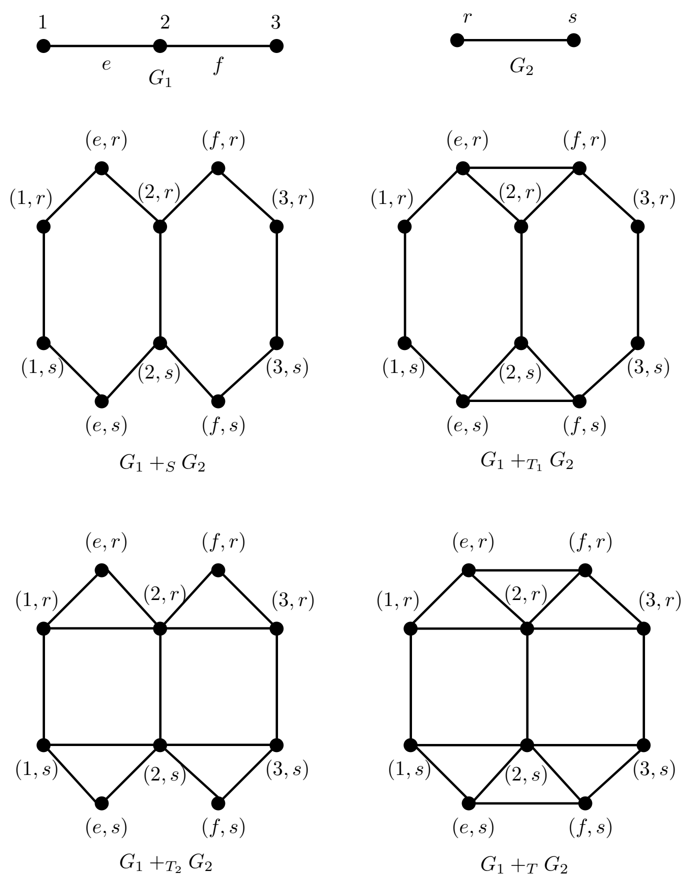

Figure 2. F-sums of graphs

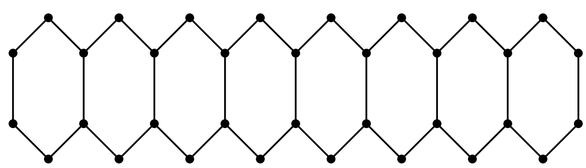

Figure 3. The hexagonal chain H_n with $n = 8$.

Thus, authors obtained four new operations as $G_1 +_S G_2$, $G_1 +_{T_2} G_2$, $G_1 +_{T_1} G_2$ and $G_1 +_T G_2$ and studied the Wiener indices of these graphs. These graphs are depicted in Figure 2. The advantage of these new graph operations based on transformation graphs yield some chemically important structures. For example, the hexagonal chain H_n depicted in Fig. 3 is obtained by $P_{n+1} +_S P_2$. In [**6**], Deng et al., gave the expressions for first and second Zagreb indices of these new graphs. Motivated by this, in this paper, we consider the problem of finding first neighbourhood Zagreb index of these new operations. In section 2, we obtain first neighbourhood Zagreb index of derived graphs and in section 3, we obtain explicit formula for finding first neighbourhood Zagreb index of F-sums of graphs.

2. First Neighbourhood Zagreb index of Derived Graphs

In this section, we proceed to obtain FNZI of derived graphs.

THEOREM 2.1. *If $S(G)$ is the subdivision graph of graph G, then*

$$NM_1(S(G)) = 4M_1(G) + HM(G).$$

PROOF. By definitions of FNZI and subdivision graph, we have

$$\begin{aligned} NM_1(S(G)) &= \sum_{v \in V(S(G))} S_G(v)^2 \\ &= \sum_{v \in V(S(G)) \cap V(G)} S_G(v)^2 + \sum_{v \in V(S(G)) \cap E(G)} S_G(v)^2. \end{aligned}$$

For $v \in V(S(G)) \cap V(G)$, $S_G(v) = 2d_G(v)$ and for $e = uw \in V(S(G)) \cap E(G)$, $S_G(e) = d_G(u) + d_G(w)$. Therefore,

$$\begin{aligned} NM_1(S(G)) &= 4 \sum_{v \in V(G)} d_G(v)^2 + \sum_{uw \in E(G)} (d_G(u) + d_G(w))^2 \\ &= 4M_1(G) + HM(G). \end{aligned}$$

□

THEOREM 2.2. *If $T_2(G)$ is the semitotal-point graph of a graph G, then*

$$NM_1(T_2(G)) = 4\Big(NM_1(G) + M_1(G) + 4M_2(G) + HM(G)\Big).$$

PROOF. By definitions of FNZI and semitotal-point graph, we have

$$\begin{aligned} NM_1(T_2(G)) &= \sum_{v \in V(T_2(G))} S_G(v)^2 \\ &= \sum_{v \in V(T_2(G)) \cap V(G)} S_G(v)^2 + \sum_{v \in V(T_2(G)) \cap E(G)} S_G(v)^2. \end{aligned}$$

For $v \in V(T_2(G)) \cap V(G)$, $S_G(v) = 2(S_G(v) + d_G(v))$ and for $e = uw \in V(T_2(G)) \cap E(G)$, $S_G(e) = 2(d_G(u) + d_G(w))$. Therefore,

$$\begin{aligned} NM_1(T_2(G)) &= \sum_{v \in V(G)} (2(S_G(v) + d_G(v)))^2 + \sum_{uw \in E(G)} (2(d_G(u) + d_G(w)))^2 \\ &= 4\Big(NM_1(G) + M_1(G) + 4M_2(G) + HM(G)\Big). \end{aligned}$$

□

THEOREM 2.3. *If $T_1(G)$ is the semitotal-line graph of a graph G, then*

$$\begin{aligned} NM_1(T_1(G)) &= NM_1(G) + HM(G) + HM_1G_5(G) + \sum_{v \in V(G)} d_G^4(v) \\ &\quad +2 \sum_{v \in V(G)} d_G^2(v) S_G(v) + \sum_{uw \in E(G)} \big(d_G^2(u) + d_G^2(w)\big)^2 \\ &\quad +2 \sum_{uw \in E(G)} \Big((S_G(u) + S_G(w))((d_G^2(u) + d_G^2(w) \\ &\quad -(d_G(u) + d_G(w))\Big) \\ &\quad -2 \sum_{vw \in E(G)} \big((d_G^2(u) + d_G^2(w))(d_G(u) + d_G(w))\big). \end{aligned}$$

PROOF. By definitions of FNZI and semitotal-line graph, we have

$$\begin{aligned} NM_1(T_1(G)) &= \sum_{v \in V(T_1(G))} S_G(v)^2 \\ &= \sum_{v \in V(T_1(G)) \cap V(G)} S_G(v)^2 + \sum_{v \in V(T_1(G)) \cap E(G)} S_G(v)^2. \end{aligned}$$

For $v \in V(T_1(G)) \cap V(G)$, $S_G(v) = d_G^2(v) + S_G(v)$ and for

$$e = uw \in V(T_1(G)) \cap E(G),$$
$$S_G(e) = d_G^2(u) + d_G^2(w) + S_G(u) + S_G(w) - (d_G(u) + d_G(w)).$$

Therefore,

$$\begin{aligned} NM_1(T_1(G)) &= \sum_{v \in V(G)} \Big(d_G^2(v) + S_G(v)\Big)^2 \\ &\quad + \sum_{uw \in E(G)} \Big(d_G^2(u) + d_G^2(w) + S_G(u) + S_G(w) \\ &\quad -(d_G(u) + d_G(w))\Big)^2 \end{aligned}$$

and

$$\begin{aligned}NM_1(T_1(G)) &= NM_1(G)+HM(G)+HM_1G_5(G)+\sum_{v\in V(G)} d_G^4(v)\\&+2\sum_{v\in V(G)} d_G^2(v)S_G(v)+\sum_{uw\in E(G)} \big(d_G^2(u)+d_G^2(w)\big)^2\\&+2\sum_{uw\in E(G)} \Big((S_G(u)+S_G(w))((d_G^2(u)+d_G^2(w))\\&-(d_G(u)+d_G(w))\Big)\\&-2\sum_{vw\in E(G)} \big((d_G^2(u)+d_G^2(w))(d_G(u)+d_G(w))\big).\end{aligned}$$

□

THEOREM 2.4. *If $T(G)$ is the total graph of a graph G, then*

$$\begin{aligned}NM_1(T(G)) &= 9NM_1(G)+\sum_{v\in V(G)} d_G^4(v)+6\sum_{v\in V(G)} d_G^2(v)S_G(v)\\&+\sum_{uw\in E(G)} (d_G^4(u)+d_G^4(w))+2\sum_{uw\in E(G)} (d_G^2(u)d_G^2(w))\\&+\sum_{uw\in E(G)} (S_G^2(u)+S_G^2(w))\\&+2\sum_{uw\in E(G)} \Big(S_G(u)S_G(w)+(d_G^2(u)+d_G^2(w))(S_G(u)+S_G(w)\Big).\end{aligned}$$

PROOF. By definitions of FNZI and total graph, we have

$$\begin{aligned}NM_1(T(G)) &= \sum_{v\in V(T(G))} S_G(v)^2\\&= \sum_{v\in V(T(G))\cap V(G)} S_G(v)^2+\sum_{v\in V(T(G))\cap E(G)} S_G(v)^2.\end{aligned}$$

For $v\in V(T(G))\cap V(G)$, $S_G(v)=d_G^2(v)+3S_G(v)$ and for

$$e=uw\in V(T(G))\cap E(G),$$
$$S_G(e)=d_G^2(u)+d_G^2(w)+S_G(u)+S_G(w).$$

Therefore,

$$\begin{aligned}NM_1(T(G)) &= \sum_{v\in V(G)} \Big(d_G^2(v)+3S_G(v)\Big)^2\\&+\sum_{uw\in E(G)} \Big(d_G^2(u)+d_G^2(w)+S_G(u)+S_G(w)\Big)^2\end{aligned}$$

$$\begin{aligned}NM_1(T(G)) &= \sum_{v\in V(G)}\Big(d_G^4(v)+9S_G(v)^2+6d_G^2(v)S_G(v)\Big)\\ &+\sum_{uw\in E(G)}\Big((d_G^4(u)+d_G^4(w))+2d_G^2(u)d_G^2(w)+(S_G^2(u)+S_G^2(w))\\ &+2\Big(S_G(u)S_G(w)+(d_G^2(u)+d_G^2(w))(S_G(u)+S_G(w))\Big)\end{aligned}$$

$$\begin{aligned}NM_1(T(G)) &= 9NM_1(G)+\sum_{v\in V(G)}d_G^4(v)+6\sum_{v\in V(G)}d_G^2(v)S_G(v)\\ &+\sum_{uw\in E(G)}(d_G^4(u)+d_G^4(w))+2\sum_{uw\in E(G)}(d_G^2(u)d_G^2(w))\\ &+\sum_{uw\in E(G)}(S_G^2(u)+S_G^2(w))\\ &+2\sum_{uw\in E(G)}\Big(S_G(u)S_G(w)+(d_G^2(u)+d_G^2(w))(S_G(u)+S_G(w)\Big)\end{aligned}$$

□

3. First Neighbourhood Zagreb index of F-sums of graphs

We usually consider two finite and simple graphs, one graph is G with vertex set $V(G)$ and edge set $E(G)$ having n_1, m_1 as the order and size. Let H be another graph with vertex set $V(H)$ and edge set $E(H)$ having n_2, m_2 as the order and size. In this section, first neighbourhood Zagreb index of F-sums of graphs are obtained.

Theorem 3.1. *If G and H are (n_1, m_1) and (n_2, m_2) graphs respectively, then*

$$\begin{aligned}NM_1(G+_S H) &= n_2NM_1(G)+n_1NM_1(H)+M_1(G)M_1(H)+4M_1(G)M_2(G)\\ &+(4m_2-n_2)F(G)+(n_2-4m_2)M_1(G)-8m_2M_2(G)\\ &-8m_1M_2(H)+2n_2M_2(G)+4m_1M_1(H)+n_2\sum_{v\in V(G)}d_G^4(v)\\ &-2n_2\sum_{v\in V(G)}d_G^2(v)S_G(v).\end{aligned}$$

Proof. By definition of FNZI we have,

$$NM_1(G+_S H) = \sum_{(u,v)\in V(G+_S H)} S_{G+_S H}(u,v)^2$$

$$NM_1(G+_S H) = \sum_{u\in V(G)}\sum_{v\in V(H)} S_{G+_S H}(u,v)^2+\sum_{e=uv\in E(G)}\sum_{w\in V(H)} S_{G+_S H}(e,w)^2$$

$$\begin{aligned} NM_1(G +_S H) &= \sum_{u\in V(G)}\sum_{v\in V(H)}\Big(d_G(u)[d_G(u)+d_H(v)] - d_G(u) + S_G(u) \\ &\quad +S_H(v)\Big)^2 + \sum_{e=uv\in E(G)}\sum_{w\in V(H)}\Big(d_G(u)+d_G(v)+2d_H(w)\Big)^2. \end{aligned}$$

Expanding and by applying summation, we get

$$\begin{aligned} NM_1(G +_S H) &= n_2NM_1(G) + n_1NM_1(H) + M_1(G)M_1(H) + 4M_1(G)M_2(G) \\ &\quad +(4m_2-n_2)F(G) + (n_2-4m_2)M_1(G) - 8m_2M_2(G) \\ &\quad -8m_1M_2(H) + 2n_2M_2(G) + 4m_1M_1(H) + n_2\sum_{v\in V(G)} d_G^4(v) \\ &\quad -2n_2\sum_{v\in V(G)} d_G^2(v)S_G(v). \end{aligned}$$

□

Theorem 3.2. *If G and H are (n_1, m_1) and (n_2, m_2) graphs respectively, then*

$$\begin{aligned} NM_1(G +_{T_2} H) &= 4n_2NM_1(G) + n_1NM_1(H) + 5M_1(G)M_1(H) + 16n_2M_1(G) \\ &\quad +16n_2M_2(G) + 8m_1M_2(H) + 17m_1M_1(H) + 16m_2M_1(G) \\ &\quad +16m_2M_2(G) + 4n_2F(G) + 2n_2M_2(G) + 4m_2M_1(G). \end{aligned}$$

Proof. By definition of FNZI we have,

$$\begin{aligned} NM_1(G +_{T_2} H) &= \sum_{(u,v)\in V(G+_{T_2}H)} S_{G+_{T_2}H}(u,v)^2 \\ &= \sum_{u\in V(G)}\sum_{v\in V(H)} S_{G+_{T_2}H}(u,v)^2 \\ &\quad + \sum_{e=uv\in E(G)}\sum_{w\in V(H)} S_{G+_{T_2}H}(e,w)^2 \\ &= \sum_{u\in V(G)}\sum_{v\in V(H)}\Big(4d_G(u) + 2S_G(u) + S_H(v) + d_G(u)d_H(v)\Big)^2 \\ &\quad + \sum_{e=uv\in E(G)}\sum_{w\in V(H)}\Big(2(d_G(u)+d_G(v)+d_H(w))\Big)^2. \end{aligned}$$

Expanding and by applying summation, we get

$$\begin{aligned} NM_1(G +_{T_2} H) &= 4n_2NM_1(G) + n_1NM_1(H) + 5M_1(G)M_1(H) + 16n_2M_1(G) \\ &\quad +16n_2M_2(G) + 8m_1M_2(H) + 17m_1M_1(H) + 16m_2M_1(G) \\ &\quad +16m_2M_2(G) + 4n_2F(G) + 2n_2M_2(G) + 4m_2M_1(G). \end{aligned}$$

□

THEOREM 3.3. *If G and H are (n_1, m_1) and (n_2, m_2) graphs respectively, then*

$$\begin{aligned}
NM_1(G +_{T_1} H) &= n_2 NM_1(G) + n_1 NM_1(H) + 2M_1(G)M_1(H) + 8m_1M_2(H) \\
&+4m_2M_1(G) + 8m_2M_2(G) + 4m_2F(G) + 8m_1M_1(H) \\
&+32m_2M_1(G) - 4n_2F(G) + 16m_2M_1(G) - 8M_2(G) \\
&-4m_1M_1(G) - 2n_2M_2(G) + n_2 \sum_{uv\in E(G)} (S_G^2(u) + S_G^2(v)) \\
&+2n_2M_2G_5(G) + \sum_{uv\in E(G)} (d_G^4(u) + d_G^4(v)) \\
&+2n_2 \sum_{uv\in E(G)} (d_G^2(u)d_G^2(v)) + n_2 \sum_{uv\in E(G)} (d_G^2(u) + d_G^2(v)) \\
&+2n_2 \sum_{uv\in E(G)} (S_G(u) + S_G(v))(d_G^2(u) + d_G^2(v)) \\
&-4n_2 \sum_{uv\in E(G)} (d_G^2(u)d_G(v)) - 2n_2 \sum_{uv\in E(G)} (d_G(u)S_G(v)).
\end{aligned}$$

PROOF. By definition of FNZI we have,

$$\begin{aligned}
NM_1(G +_{T_1} H) &= \sum_{(u,v)\in V(G+_{T_1}H)} S_{G+_{T_1}H}(u,v)^2 \\
&= \sum_{u\in V(G)} \sum_{v\in V(H)} S_{G+_{T_1}H}(u,v)^2 \\
&\quad + \sum_{e=uv\in E(G)} \sum_{w\in V(H)} S_{G+_{T_1}H}(e,w)^2 \\
&= \sum_{u\in V(G)} \sum_{v\in V(H)} \Big(d_G^2(u) + S_G(u) + S_H(v) + d_G(u)d_H(v)\Big)^2 \\
&\quad + \sum_{e=uv\in E(G)} \sum_{w\in V(H)} \Big(S_G(u) + S_G(v) + d_G^2(u) + d_G^2(v) \\
&\quad +2d_H(w) - d_G(u) - d_G(v)\Big)^2.
\end{aligned}$$

$$\begin{aligned}
NM_1(G+_{T_1}H) &= n_2NM_1(G)+n_1NM_1(H)+2M_1(G)M_1(H)+8m_1M_2(H)\\
&+4m_2M_1(G)+8m_2M_2(G)+4m_2F(G)+8m_1M_1(H)\\
&+32m_2M_1(G)-4n_2F(G)+16m_2M_1(G)-8M_2(G)\\
&-4m_1M_1(G)-2n_2M_2(G)+n_2\sum_{uv\in E(G)}(S_G^2(u)+S_G^2(v))\\
&+2n_2M_2G_5(G)+\sum_{uv\in E(G)}(d_G^4(u)+d_G^4(v))\\
&+2n_2\sum_{uv\in E(G)}(d_G^2(u)d_G^2(v))+n_2\sum_{uv\in E(G)}(d_G^2(u)+d_G^2(v))\\
&+2n_2\sum_{uv\in E(G)}(S_G(u)+S_G(v))(d_G^2(u)+d_G^2(v))\\
&-4n_2\sum_{uv\in E(G)}(d_G^2(u)d_G(v))-2n_2\sum_{uv\in E(G)}(d_G(u)S_G(v)).
\end{aligned}$$

□

THEOREM 3.4. *If G and H are (n_1,m_1) and (n_2,m_2) graphs respectively, then*

$$\begin{aligned}
NM_1(G+_TH) &= 9n_2NM_1(G)+n_1NM_1(H)+15M_1(G)M_1(H)\\
&+24m_1M_2(H)+72m_2M_2(G)+20m_2F(G)+2(M_1(G))^2\\
&+4m_1M_1(H)+8m_2M_1G_5(G)+n_2HM_1G_5(G)\\
&+n_2\sum_{u\in V(G)}d_G^4(u)+6n_2\sum_{u\in V(G)}d_G^2(u)S_G(u)\\
&+n_2\sum_{uv\in E(G)}\Big(d_G^4(u)+d_G^4(v)\Big)+2n_2\sum_{uv\in E(G)}\Big(d_G^2(u)d_G^2(v)\Big)\\
&+2n_2\sum_{uv\in E(G)}\Big((d_G^2(u)+d_G^2(v))(S_G(u)+S_G(v))\Big).
\end{aligned}$$

PROOF. By definition of FNZI we have,

$$\begin{aligned}
NM_1(G+_TH) &= \sum_{(u,v)\in V(G+_TH)} S_{G+_TH}(u,v)^2\\
&= \sum_{u\in V(G)}\sum_{v\in V(H)} S_{G+_TH}(u,v)^2+\sum_{e=uv\in E(G)}\sum_{w\in V(H)} S_{G+_TH}(e,w)^2\\
&= \sum_{u\in V(G)}\sum_{v\in V(H)}\Big(d_G^2(u)+3S_G(u)+S_H(v)+3d_G(u)d_H(v)\Big)^2\\
&+\sum_{e=uv\in E(G)}\sum_{w\in V(H)}\Big(d_G^2(u)+d_G^2(v)+S_G(u)+S_G(v)\\
&+2d_H(w)\Big)^2.
\end{aligned}$$

Expanding and by applying summation, we get

$$\begin{aligned} NM_1(G +_T H) &= 9n_2NM_1(G) + n_1NM_1(H) + 15M_1(G)M_1(H) + 24m_1M_2(H) \\ &+72m_2M_2(G) + 20m_2F(G) + 2(M_1(G))^2 + 4m_1M_1(H) \\ &+8m_2M_1G_5(G) + n_2HM_1G_5(G) + n_2 \sum_{u\in V(G)} d_G^4(u) \\ &+6n_2 \sum_{u\in V(G)} d_G^2(u)S_G(u) + n_2 \sum_{uv\in E(G)} \left(d_G^4(u) + d_G^4(v)\right) \\ &+2n_2 \sum_{uv\in E(G)} \left(d_G^2(u)d_G^2(v)\right) \\ &+2n_2 \sum_{uv\in E(G)} \left((d_G^2(u) + d_G^2(v))(S_G(u) + S_G(v))\right). \end{aligned}$$

□

4. Conclusion

In this paper, we have obtained first neighbourhood Zagreb index of derived graphs and F-sums of graphs (graph operations based on derived graphs) in terms of parameters of graphs considered.

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