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AN IDEAL NANO $\wedge_g$-CLOSED SETS

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Abstract. In this paper, the concept of $n\wedge_g$-closed sets is introduced in ideal topological spaces. Characterizations and properties of $nI_{\wedge_g}$-closed sets and $nI_{\wedge_g}$-open sets are given.

1. Introduction

An ideal I [**12**] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

(1) $A \in I$ and $B \subset A$ imply $B \in I$ and
(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a space (X, τ) with an ideal I on X if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [**2**]. The closure operator defined by $cl^\star(A) = A \cup A^\star(I, \tau)$ [**11**] is a Kuratowski closure operator which generates a topology $\tau^\star(I, \tau)$ called the $\star$-topology which is finer then τ. We will simply write $A^\star$ for $A^\star(I, \tau)$ and $\tau^\star$ for $\tau^\star(I, \tau)$. If I is an ideal on X, then (X, τ, I) is called an ideal topological space or an ideal space.

Some new notions in the concept of ideal nano topological spaces were introduced by Parimala et al. [**3, 4**].

In this paper, the concept of $n\wedge_g$-closed sets is introduced in ideal topological spaces. Characterizations and properties of $nI_{\wedge_g}$-closed sets and $nI_{\wedge_g}$-open sets are given.

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2. Preliminaries

Definition 2.1. ([**6**]) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. ([**9**]) Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,
(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by n-$int(A)$ and n-$cl(A)$, respectively.

Definition 2.3. ([**10**]) Let $(U, \mathcal{N})$ be a spaces and $A \subseteq U$. The nano $Ker(A) = \bigcap\{U : A \subseteq U, U \in \mathcal{N}\}$ is called the nano kernal of A and is denoted by n-$Ker(A)$.

Definition 2.4. A subset H of a space $(U, \mathcal{N})$ is called

(1) nano g-closed (briefly, ng-closed) [**1**] if n-$cl(A) \subseteq G$, whenever $A \subseteq G$ and G is n-open. The complement of a ng-open set is called ng-closed.
(2) nano $\wedge$-set (briefly, $n\wedge$-set) if [**7**] $A = n$-$Ker(A)$
(3) nano λ-closed (briefly, $n\lambda$-closed) if [**7**] $A = L \cap F$ where L is $n\wedge$-set and F is n-closed.
(4) $n\lambda$-open if [**8**] $A^c = U - A$ is $n\lambda$-closed.
(5) nano $\wedge_g$-closed set (briefly, $n\wedge_g$-closed) if [**8**] n-$cl(A) \subseteq G$, whenever $A \subseteq G$ and G is $n\lambda$-open.
The complement of $n\wedge_g$-open if $A^c = U - A$ is $n\wedge_g$-closed.

Lemma 2.1. *In a space* $(U, \mathcal{N})$,

(1) *each n-closed set is $n\lambda$-closed.*[**7**]
(2) *each n-open set is $n\wedge_g$-open.* [**8**]

THEOREM 2.1 ([**5**]). *In a space $(U,\mathcal{N},I)$, each ng-closed set is nI_g-closed.*

A nano topological space $(U,\mathcal{N})$ with an ideal I on U is called [**3**] an ideal nano topological space and is denoted by $(U,\mathcal{N},I)$. $G_n(x)=\{G_n \mid x\in G_n, G_n\in\mathcal{N}\}$, denotes [**3**] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U,\mathcal{N},I)$ is referred as a space.

DEFINITION 2.5. ([**3**]) Let $(U,\mathcal{N},I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A\subseteq U$, $A^\star_n(I,\mathcal{N})=\{x\in U : G_n\cap A\notin I$, for every $G_n\in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^\star_n$ for $A^\star_n(I,\mathcal{N})$.

THEOREM 2.2 ([**3**]). *Let $(U,\mathcal{N},I)$ be a space and A and B be subsets of U. Then*

(1) $A\subseteq B \Rightarrow A^\star_n\subseteq B^\star_n$,
(2) $A^\star_n = n\text{-}cl(A^\star_n)\subseteq n\text{-}cl(A)$ *($A^\star_n$ is a n-closed subset of $n\text{-}cl(A)$)*,
(3) $(A^\star_n)^\star_n\subseteq A^\star_n$,
(4) $(A\cup B)^\star_n = A^\star_n\cup B^\star_n$,
(5) $V\in\mathcal{N}\Rightarrow V\cap A^\star_n = V\cap(V\cap A)^\star_n\subseteq (V\cap A)^\star_n$,
(6) $J\in I\Rightarrow (A\cup J)^\star_n = A^\star_n = (A-J)^\star_n$.

THEOREM 2.3 ([**3**]). *Let $(U,\mathcal{N},I)$ be a space with an ideal I and $A\subseteq A^\star_n$, then $A^\star_n = n\text{-}cl(A^\star_n) = n\text{-}cl(A)$.*

DEFINITION 2.6. ([**3**]) Let $(U,\mathcal{N},I)$ be a space. The set operator $n\text{-}cl^\star$ called a nano $n\star$-closure is defined by $n\text{-}cl^\star(A)=A\cup A^\star_n$ for $A\subseteq X$.

It can be easily observed that $n\text{-}cl^\star(A)\subseteq n\text{-}cl(A)$.

THEOREM 2.4 ([**4**]). *In a space $(U,\mathcal{N},I)$, if A and B are subsets of U, then the following results are true for the set operator $n\text{-}cl^\star$.*

(1) $A\subseteq n\text{-}cl^\star(A)$,
(2) $n\text{-}cl^\star(\phi)=\phi$ *and* $n\text{-}cl^\star(U)=U$,
(3) *If* $A\subset B$, *then* $n\text{-}cl^\star(A)\subseteq n\text{-}cl^\star(B)$,
(4) $n\text{-}cl^\star(A)\cup n\text{-}cl^\star(B)=n\text{-}cl^\star(A\cup B)$.
(5) $n\text{-}cl^\star(n\text{-}cl^\star(A))=n\text{-}cl^\star(A)$.

DEFINITION 2.7. ([**5**]) A subset A of a space $(U,\mathcal{N},I)$ is $n\star$-dense in itself (resp. $n\star$-perfect and $n\star$-closed) if $A\subseteq A^\star_n$ (resp. $A=A^\star_n$, $A^\star_n\subseteq A$).

The complement of a $n\star$-closed set is called $n\star$-open.

DEFINITION 2.8. ([**5**]) A subset A of a space $(U,\mathcal{N})$, is called a nano I_g-closed (briefly, nI_g-closed) if $A^\star_n\subseteq B$ whenever $A\subseteq B$ and B is n-open.

The complement of nI_g-open set is called nI_g-closed.

3. An ideal nano $\wedge_g$-closed sets

DEFINITION 3.1. A subset A of a space (X, τ, $\mathcal{I}$) is called nano $I_{\wedge_g}$-closed (briefly, $nI_{\wedge_g}$-closed) if $A_n^\star \subseteq G$ whenever $A \subseteq G$ and G is $n\lambda$-open.

The complement of $nI_{\wedge_g}$-open set is called $nI_{\wedge_g}$-closed set.

THEOREM 3.1. *In a space $(U, \mathcal{N}, I)$, for a subset A, the following relation hold.*

(1) *A is $nI_{\wedge_g}$-closed $\Rightarrow$ A is nI_g-closed.*
(2) *A is $n\star$-closed $\Rightarrow$ A is $nI_{\wedge_g}$-closed.*
(3) *A is $n\wedge_g$-closed $\Rightarrow$ A is $nI_{\wedge_g}$-closed.*

PROOF. (1) It follows from the fact that each n-open set is $n\lambda$-open.

(2) A be $n\star$-closed. To prove A is $nI_{\wedge_g}$-closed, let G be each $n\lambda$-open set such that $A \subseteq G$. Since A is $n\star$-closed, $A_n^\star \subseteq A \subseteq G$. Thus A is $nI_{\wedge_g}$-closed.

(3) A be $n\wedge_g$-closed set. Let G be each $n\lambda$-open set such that $A \subseteq G$. Since A is $n\wedge_g$-closed, n-$cl(A) \subseteq G$. So, by Theorem 2.2, $A_n^\star \subseteq n$-$cl(A) \subseteq G$ and thus A is $nI_{\wedge_g}$-closed. □

REMARK 3.1. These relations are shown in the diagram.

$$\begin{array}{ccccc} n\textbf{-closed} & \longrightarrow & n\wedge_g\textbf{-closed} & \longrightarrow & ng\textbf{-closed} \\ \downarrow & & \downarrow & & \downarrow \\ n\star\textbf{-closed} & \longrightarrow & nI_{\wedge_g}\textbf{-closed} & \longrightarrow & nI_g\textbf{-closed} \end{array}$$

The converses of each statement in Theorem 3.1 are not true as shown in the following Examples.

EXAMPLE 3.1. Let

$$U = \{m_1, m_2, m_3, m_4\} \text{ with } U/R = \{\{m_1\}, \{m_3\}, \{m_2, m_4\}\}$$

and $X = \{m_1, m_2\}$. Then $\mathcal{N} = \{\phi, U, \{m_1\}, \{m_2, m_4\}, \{m_1, m_2, m_4\}\}$. Let the ideal be $I = \{\phi\}$. Then $A = \{m_3\}$ is nI_g-closed but not $nI_{\wedge_g}$-closed.

EXAMPLE 3.2. Let $U = \{m_1, m_2, m_3, m_4, m_5\}$ with $U/R = \{\{m_5\}, \{m_1, m_2\}, \{m_3, m_4\}\}$ and $X = \{m_2, m_5\}$. Then $\mathcal{N} = \{\phi, U, \{m_5\}, \{m_1, m_2\}, \{m_1, m_2, m_5\}\}$. Let the ideal be $I = \{\phi, \{m_1\}, \{m_2\}, \{m_1, m_2\}\}$.

(1) $\{m_3\}$ is $nI_{\wedge_g}$-closed set but not $n\star$-closed.
(2) $\{m_1\}$ is $nI_{\wedge_g}$-closed set but not $n\wedge_g$-closed.

THEOREM 3.2. *In a space $(U, \mathcal{N}, I)$, for a subset A, the following relations hold.*

(1) *A is $nI_{\wedge_g}$-closed,*
(2) *n-$cl^\star(A) \subseteq G$ whenever $A \subseteq G$ and G is $n\lambda$-open in U,*
(3) *n-$cl^\star(A) - A$ contains no nonempty $n\lambda$-closed set,*
(4) *$A_n^\star - A$ contains no nonempty $n\lambda$-closed set.*

PROOF. (1) $\Rightarrow$ (2) Let $A \subseteq G$ where G is $n\lambda$-open in U. Since A is $nI_{\wedge_g}$-closed, $A_n^\star \subseteq G$ and so n-$cl^\star(A) = A \cup A_n^\star \subseteq G$.

(2) $\Rightarrow$ (3) Let K be $n\lambda$-closed subset such that $K \subseteq n$-$cl^\star(A) - A$. Then $K \subseteq n$-$cl^\star(A)$. Also $K \subseteq n$-$cl^\star(A) - A \subseteq U - A$ and hence $A \subseteq U - K$ where

$U - K$ is $n\lambda$-open. By (2) $n\text{-}cl^\star(A) \subseteq U - K$ and so $K \subseteq U - n\text{-}cl^\star(A)$. Thus $K \subseteq n\text{-}cl^\star(A) \cap U - n\text{-}cl^\star(A) = \phi$.

(3) $\Rightarrow$ (4) $A^\star_n - A = A \cup A^\star_n - A = n\text{-}cl^\star(A) - A$ which has no nonempty $n\lambda$-closed subset by (3).

(4) $\Rightarrow$ (1) Let $A \subseteq G$ where G is $n\lambda$-open. Then $U - G \subseteq U - A$ and so $A^\star_n \cap (U - G) \subseteq A^\star_n \cap (U - A) = A^\star_n - A$. Since $A^\star_n$ is always n-closed subset and $U - G$ is $n\lambda$-closed, $A^\star_n \cap (U - G)$ is $n\lambda$-closed set contained in $A^\star_n - A$ and hence $A^\star_n \cap (U - G) = \phi$ by (4). Thus $A^\star_n \subseteq G$ and A is $nI_{\wedge_g}$-closed. □

Theorem 3.3. *In a space $(U, \mathcal{N}, I)$, for each $A \in I$, A is $nI_{\wedge_g}$-closed.*

Proof. Let $A \in I$ and let $A \subseteq G$ where G is $n\lambda$-open. Since $A \in I$, $A^\star_n = \phi \subseteq G$. Thus A is $nI_{\wedge_g}$-closed. □

Theorem 3.4. *In a space $(U, \mathcal{N}, I)$, then $A^\star_n$ is always $nI_{\wedge_g}$-closed for each subset A of U.*

Proof. Let $A^\star_n \subseteq G$ where G is $n\lambda$-open. Since $(A^\star_n)^\star_n \subseteq A^\star_n$ (3) of By Theorem 2.2, we have $(A^\star_n)^\star_n \subseteq G$. Hence $A^\star_n$ is $nI_{\wedge_g}$-closed. □

Theorem 3.5. *In a space $(U, \mathcal{N}, I)$, each $nI_{\wedge_g}$-closed, $n\lambda$-open set is $n\star$-closed.*

Proof. Let A be $nI_{\wedge_g}$-closed and $n\lambda$-open. We have $A \subseteq A$ where A is $n\lambda$-open. Since A is $nI_{\wedge_g}$-closed, $A^\star_n \subseteq A$. Thus A is $n\star$-closed. □

Corollary 3.1. *In a space $(U, \mathcal{N}, I)$ and A be $nI_{\wedge_g}$-closed set. Then the following are equivalent.*

(1) *A is $n\star$-closed set.*
(2) *$n\text{-}cl^\star(A) - A$ is $n\lambda$-closed set.*
(3) *$A^\star_n - A$ is $n\lambda$-closed set.*

Proof. (1) $\Rightarrow$ (2) By (1) A is $n\star$-closed. Hence $A^\star_n \subseteq A$ and $n\text{-}cl^\star(A) - A = (A \cup A^\star_n) - A = \phi$ which is $n\lambda$-closed set.

(2) $\Rightarrow$ (3) $A^\star_n - A = A \cup A^\star_n - A = n\text{-}cl^\star(A) - A$ which is $n\lambda$-closed set by (2).

(3) $\Rightarrow$ (1) Since A is $nI_{\wedge_g}$-closed, by Theorem 3.2 $A^\star_n - A$ contains no nonempty $n\lambda$-closed set. By assumption (3) $A^\star_n - A$ is $n\lambda$-closed and hence $A^\star_n - A = \phi$. Thus $A^\star_n \subseteq A$ and A is $n\star$-closed. □

Theorem 3.6. *In a space $(U, \mathcal{N}, I)$ and A is $n\star$-dense in itself, $nI_{\wedge_g}$-closed subset of U, then A is $n\wedge_g$-closed.*

Proof. Let $A \subseteq G$ where G is $n\lambda$-open. Since A is $nI_{\wedge_g}$-closed, $A^\star_n \subseteq G$. Since A is $n\star$-dense in itself, by Theorem 2.3, $n\text{-}cl(A) = A^\star_n$. Hence $n\text{-}cl(A) \subseteq G$ and thus A is $n\wedge_g$-closed. □

Corollary 3.2. *If a space $(U, \mathcal{N}, I)$ where $I = \{\phi\}$, then A is $nI_{\wedge_g}$-closed if and only if A is $n\wedge_g$-closed.*

Proof. In a space $(U, \mathcal{N}, I)$, if $I = \{\phi\}$ then $A^\star_n = n\text{-}cl(A)$ for the subset A. A is $nI_{\wedge_g}$-closed $\Leftrightarrow$ $A^\star_n \subseteq G$ whenever $A \subseteq G$ and G is $n\lambda$-open $\Leftrightarrow$ $n\text{-}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $n\lambda$-open $\Leftrightarrow$ A is $n\wedge_g$-closed. □

REMARK 3.2. In a space $(U, \mathcal{N}, I)$, the family of ng-closed sets and the family of $nI_{\wedge_g}$-closed sets are independent of each other.

EXAMPLE 3.3. In Example 3.1, then $A = \{m_1, m_3\}$ is ng-closed set but not $nI_{\wedge_g}$-closed.

EXAMPLE 3.4. In Example 3.2, $A = \{m_1\}$ is $nI_{\wedge_g}$-closed set but not ng-closed.

THEOREM 3.7. *Let be a space $(U, \mathcal{N}, I)$ and $A \subseteq U$. Then A is $nI_{\wedge_g}$-closed if and only if $A = K - L$ where K is $n\star$-closed and L contains no nonempty $n\lambda$-closed set.*

PROOF. If A is $nI_{\wedge_g}$-closed, then by Theorem 3.2 (4), $L = A^\star_n - A$ contains no nonempty $n\lambda$-closed set. If $K = n\text{-}cl^\star(A)$, then K is $n\star$-closed such that $K - L = (A \cup A^\star_n) - (A^\star_n - A) = (A \cup A^\star_n) \cap (A^\star_n \cap A^c)^c = (A \cup A^\star_n) \cap ((A^\star_n)^c \cup A) = (A \cup A^\star_n) \cap (A \cup (A^\star_n)^c) = A \cup (A^\star_n \cap (A^\star_n)^c) = A$.

Conversely, suppose $A = K - L$ where K is $n\star$-closed and L contains no nonempty $n\lambda$-closed set. Let G be $n\lambda$-open set such that $A \subseteq G$. Then $K - L \subseteq G$ which implies that $K \cap (U - G) \subseteq L$. Now $A \subseteq K$ and $K^\star_n \subseteq L$ then $A^\star_n \subseteq K^\star_n$ and so $A^\star_n \cap (U - G) \subseteq K^\star_n \cap (U - G) \subseteq K \cap (X - G) \subseteq L$. Since $A^\star_n \cap (U - G)$ is $n\lambda$-closed, by hypothesis $A^\star_n \cap (U - G) = \phi$ and so $A^\star_n \subseteq G$. Hence A is $nI_{\wedge_g}$-closed. □

THEOREM 3.8. *Let be a space $(U, \mathcal{N}, I)$ and $A \subseteq U$. If $A \subseteq B \subseteq A^\star_n$, then $A^\star_n = B^\star_n$ and B is $n\star$-dense in itself.*

PROOF. Since $A \subseteq B$, then $A^\star_n \subseteq B^\star_n$ and since $B \subseteq A^\star_n$, then $B^\star_n \subseteq (A^\star_n)^\star_n \subseteq A^\star_n$. Therefore $A^\star_n = B^\star_n$ and $B \subseteq A^\star_n \subseteq B^\star_n$. □

THEOREM 3.9. *Let be a space $(U, \mathcal{N}, I)$. If A and B are subsets of U such that $A \subseteq B \subseteq n\text{-}cl^\star(A)$ and A is $nI_{\wedge_g}$-closed, then B is $nI_{\wedge_g}$-closed.*

PROOF. Since A is $nI_{\wedge_g}$-closed, then by Theorem 3.2 (3), $n\text{-}cl^\star(A) - A$ contains no nonempty $n\lambda$-closed set. But $n\text{-}cl^\star(B) - B \subseteq n\text{-}cl^\star(A) - A$ and so $n\text{-}cl^\star(B) - B$ contains no nonempty $n\lambda$-closed set. Hence B is $nI_{\wedge_g}$-closed. □

COROLLARY 3.3. *Let be a space $(U, \mathcal{N}, I)$. If A and B are subsets of U such that $A \subseteq B \subseteq A^\star_n$ and A is $nI_{\wedge_g}$-closed, then A and B are $n\wedge_g$-closed sets.*

PROOF. Let A and B be subsets of U such that $A \subseteq B \subseteq A^\star_n$. Then $A \subseteq B \subseteq A^\star_n \subseteq n\text{-}cl^\star(A)$. Since A is $nI_{\wedge_g}$-closed, by Theorem 3.9, B is $nI_{\wedge_g}$-closed. Since $A \subseteq B \subseteq A^\star_n$, we have $A^\star_n = B^\star_n$. Hence $A \subseteq A^\star_n$ and $B \subseteq B^\star_n$. Thus A is $n\star$-dense in itself and B is $n\star$-dense in itself and by Theorem 3.6, A and B are $n\wedge_g$-closed. □

THEOREM 3.10. *Let be a space $(U, \mathcal{N}, I)$ and $A \subseteq U$. Then A is $nI_{\wedge_g}$-open if and only if $K \subseteq n\text{-}int^\star(A)$ whenever K is $n\lambda$-closed and $K \subseteq A$.*

PROOF. Suppose A is $nI_{\wedge_g}$-open. If K is $n\lambda$-closed and $K \subseteq A$, then $U - A \subseteq U - K$ and so $n\text{-}cl^\star(U - A) \subseteq U - K$ by Theorem 3.2(2). Therefore $K \subseteq U - n\text{-}cl^\star(U - A) = n\text{-}int^\star(A)$. Hence $K \subseteq n\text{-}int^\star(A)$.

Conversely, suppose the condition holds. Let G be a $n\lambda$-open set such that $U-A \subseteq G$. Then $U-G \subseteq A$ and so $U-G \subseteq n\text{-}int^\star(A)$. Therefore $n\text{-}cl^\star(U-A) \subseteq G$. By Theorem 3.2(2), $U-A$ is $nI_{\wedge_g}$-closed. Hence A is $nI_{\wedge_g}$-open. □

Corollary 3.4. *Let be a space $(U, \mathcal{N}, I)$ and $A \subseteq U$. If A is $nI_{\wedge_g}$-open, then $K \subseteq n\text{-}int^\star(A)$ whenever K is closed and $K \subseteq A$.*

Theorem 3.11. *Let be a space $(U, \mathcal{N}, I)$ and $A \subseteq U$. If A is $nI_{\wedge_g}$-open and $n\text{-}int^\star(A) \subseteq B \subseteq A$, then B is $nI_{\wedge_g}$-open.*

Proof. Since $n\text{-}int^\star(A) \subseteq B \subseteq A$, we have $U-A \subseteq X-B \subseteq X-n\text{-}int^\star(A) = n\text{-}cl^\star(U-A)$. By assumption A is $nI_{\wedge_g}$-open and so $U-A$ is $nI_{\wedge_g}$-closed. Hence by Theorem 3.9, $X-B$ is $nI_{\wedge_g}$-closed and B is $nI_{\wedge_g}$-open. □

Theorem 3.12. *Let be a space $(U, \mathcal{N}, I)$ and $A \subseteq U$. Then the following are equivalent.*

(1) *A is $nI_{\wedge_g}$-closed,*
(2) *$A \cup (U - A_n^\star)$ is $nI_{\wedge_g}$-closed,*
(3) *$A_n^\star - A$ is $nI_{\wedge_g}$-open.*

Proof. (1)⇒(2) Let G be any $n\lambda$-open set such that $A \cup (U - A_n^\star) \subseteq G$. Then $G^c \subseteq (A \cup (U - A_n^\star))^c = (A \cup (A_n^\star)^c)^c = A_n^\star \cap A^c = A_n^\star - A$ where G^c is $n\lambda$-closed. Since A is $nI_{\wedge_g}$-closed, by Theorem 3.2(4), $G^c = \phi$ and $U = G$. Thus U is the only $n\lambda$-open set containing $A \cup (U - A_n^\star)$ and hence $A \cup (U - A_n^\star)$ is $nI_{\wedge_g}$-closed.

(2)⇒(3) $(A_n^\star - A)^c = (A_n^\star \cap A^c)^c = A \cup A_n^{\star c} = A \cup (U - A_n^\star)$ which is $nI_{\wedge_g}$-closed by (2). Hence $A_n^\star - A$ is $nI_{\wedge_g}$-open.

(3) ⇒ (1) Since $A_n^\star - A$ is $nI_{\wedge_g}$-open, $(A_n^\star - A)^c = A \cup A_n^{\star c}$ is $nI_{\wedge_g}$-closed. Hence by Theorem 3.2(4) $(A \cup (A_n^\star)^c)_n^\star - (A \cup A_n^{\star c})$ contains no nonempty $n\lambda$-closed subset. But $(A \cup (A_n^\star)^c)_n^\star - (A \cup (A_n^\star)^c) = (A \cup (A_n^\star)^c)_n^\star \cap (A \cup (A_n^\star)^c)^c = (A \cup (A_n^\star)^c)_n^\star \cap (A_n^\star \cup A^c) = (A_n^\star \cup ((A_n^\star)^c)_n^\star) \cap (A_n^\star \cap A^c) = A_n^\star \cap A^c = A_n^\star - A$. Thus $A_n^\star - A$ has no nonempty $n\lambda$-closed subset. Hence by Theorem 3.2(4), A is $nI_{\wedge_g}$-closed. □

Theorem 3.13. *Let be a space $(U, \mathcal{N}, I)$, then each subset of U is $nI_{\wedge_g}$-closed if and only if each $n\lambda$-open set is $n\star$-closed.*

Proof. Suppose each subset of U is $nI_{\wedge_g}$-closed. Let G be $n\lambda$-open in U. Then $G \subseteq G$ and G is $nI_{\wedge_g}$-closed by assumption implies $G_n^\star \subseteq G$. Hence G is $n\star$-closed.

Conversely, let $A \subseteq U$ and G be $n\lambda$-open such that $A \subseteq G$. Since G is $n\star$-closed by assumption, we have $A_n^\star \subseteq G_n^\star \subseteq G$. Thus A is $nI_{\wedge_g}$-closed. □

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