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SOME DEGREE BASED TOPOLOGICAL INDICES OF TRANSFORMATION GRAPHS

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ABSTRACT. In this paper, an explicit formula for sum-connectivity index, atom-bond connectivity index and augmented Zagreb index of eight distinct transformation graphs are obtained and also extended the same for transformation graphs of r-regular graph.

1. Introduction

Let G be finite, undirected simple graph having n vertices and m edges. Let $V(G)$ and $E(G)$ be the vertex set and edge set of G respectively. The degree of a vertex v is denoted by $d_G(v)$. Let uv represent an edge between the two vertices u and v. For undefined terminologies we refer to [**5**].

A topological index is a numerical parameter mathematically derived from the graph representing a molecule. The topological indices of molecular graphs are widely used for establishing correlations between the structure of a molecular compound and its physicochemical properties or biological activity in Quantitative Structure Property Relationships (QSPR) and in Quantitative Structure Activity Relationships (QSAR).

Estrada et al. in 1998, invented a new topological index known as atom-bond connectivity (ABC) index [**2**]. Later in the year 2009, Bo Zhou and Nenad Trinajstić gave another topological index known as sum-connectivity index (SCI) [**7**]. Motivated by the ABC index, Furtula et al. [**3**] in 2010, put forward its modified version and named as augmented Zagreb index. Main properties and critical comparative study of these degree based topological indices are summarized in [**4**].

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DEFINITION 1.1. The sum-connectivity index of a graph G is defined as

$$SCI(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]^{-\frac{1}{2}}.$$

DEFINITION 1.2. The atom-bond connectivity index of a graph G is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) \times d_G(v)}}.$$

DEFINITION 1.3. The augmented Zagreb index of a graph G is defined as

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u) d_G(v)}{d_G(u) + d_G(v) - 2} \right)^3.$$

Transformation graphs G^{xyz}. In 2001, Wu and Meng [**8**] introduced some new graphical transformations which generalizes the concept of the total graph.

DEFINITION 1.4. Let $G(V, E)$ be a graph on $n \geqslant 3$ vertices and x, y, z be three variables taking values + or −. The transformation graph G^{xyz} is a graph having $V(G) \cup E(G)$ as a vertex set and for $\alpha, \beta \in V(G) \cup E(G)$, α and β are adjacent in G^{xyz} if and only if

(1) $\alpha, \beta \in V(G)$, α and β are adjacent in G if $x = +$ and are nonadjacent in G if $x = -$.
(2) $\alpha, \beta \in E(G)$, α and β are adjacent in G if $y = +$ and are nonadjacent in G if $y = -$.
(3) $\alpha \in V(G)$ and $\beta \in E(G)$, α and β are incident in G if $z = +$ and are not incident in G if $z = -$.

Since there are eight distinct 3-permutations of $\{+, -\}$, we obtain eight graphical transformations of G. For a given graph G, $G^{+++}, G^{++-}, G^{+-+}, G^{-++}$ and its complement $G^{---}, G^{--+}, G^{-+-}, G^{+--}$ are the eight transformation graphs. The transformation graphs are investigated in [**1, 6**].

In this paper, we have given an expression for the sum-connectivity index(SCI), atom-bond connectivity index (ABC) and augmented Zagreb index (AZI) for all the eight transformation graph of G and also obtained the same when G is a regular graph.

PROPOSITION 1.1. *Let G be a graph on n vertices and m edges. Let $u \in V(G)$ and $e \in E(G)$. Then the degrees of corresponding vertices in G^{xyz} are :*

(i) $d_{G^{+++}}(u) = 2d_G(u)$ *and* $d_{G^{+++}}(e) = d_G(u) + d_G(v)$.
(ii) $d_{G^{++-}}(u) = m$ *and* $d_{G^{++-}}(e) = d_G(u) + d_G(v) + n - 4$.
(iii) $d_{G^{+-+}}(u) = 2d_G(u)$ *and* $d_{G^{+-+}}(e) = m - d_G(u) - d_G(v) + 3$.
(iv) $d_{G^{-++}}(u) = n - 1$ *and* $d_{G^{-++}}(e) = d_G(u) + d_G(v)$.
(v) $d_{G^{---}}(u) = n + m - 2d_G(u) - 1$ *and* $d_{G^{---}}(e) = n + m - d_G(u) - d_G(v) - 1$.
(vi) $d_{G^{--+}}(u) = n - 1$ *and* $d_{G^{--+}}(e) = m - d_G(u) - d_G(v) + 3$.
(vii) $d_{G^{-+-}}(u) = n + m - 1 - 2d_G(u)$ *and* $d_{G^{-+-}}(e) = n + d_G(u) + d_G(v) - 4$.
(viii) $d_{G^{+--}}(u) = m$ *and* $d_{G^{+--}}(e) = m + n - d_G(u) - d_G(v) - 1$.

PROPOSITION 1.2. *Let G be a r-regular graph with n vertices and m edges. Let $u \in V(G)$and $e \in E(G)$. Then the degrees of corresponding vertices in G_r^{xyz} are :*

(i) $d_{G^{+++}}(u) = 2r$ *and* $d_{G^{+++}}(e) = 2r$.
(ii) $d_{G^{++-}}(u) = m$ *and* $d_{G^{++-}}(e) = 2r + n - 4$.
(iii) $d_{G^{+-+}}(u) = 2r$ *and* $d_{G^{+-+}}(e) = m - 2r + 3$.
(iv) $d_{G^{-++}}(u) = n - 1$ *and* $d_{G^{-++}}(e) = 2r$.
(v) $d_{G^{---}}(u) = n + m - 2r - 1$ *and* $d_{G^{---}}(e) = n + m - 2r - 1$.
(vi) $d_{G^{--+}}(u) = n - 1$ *and* $d_{G^{--+}}(e) = m - 2r + 3$.
(vii) $d_{G^{-+-}}(u) = n + m - 2r - 1$ *and* $d_{G^{-+-}}(e) = n + 2r - 4$.
(viii) $d_{G^{+--}}(u) = m$ *and* $d_{G^{+--}}(e) = m + n - 2r - 1$.

PROPOSITION 1.3. *Edge set of G^{xyz} can be partitioned in E_x, E_y and E_z where $E_x = \{uv \mid u, v \in V(G)\}$, $E_y = \{st \mid s, t \in E(G)\}$ and $E_z = \{ue \mid u \in V(G), e \in E(G)\}$. For a r- regular graph with n vertices and m edges*

(i) $\mid E_{x=+} \mid = m$ *and* $\mid E_{x=-} \mid = \binom{n}{2} - m$.
(ii) $\mid E_{y=+} \mid = m(r-1)$ *and* $\mid E_{y=-} \mid = \frac{m}{2}(m - 2r + 1)$.
(iii) $\mid E_{z=+} \mid = 2m$ *and* $\mid E_{z=-} \mid = m(n-2)$.

2. Sum-connectivity index of transformation graphs G^{xyz}

THEOREM 2.1. *Let G be a connected graph on n vertices having m edges. Then*

$$SCI(G^{+++}) = \frac{1}{\sqrt{2}} SCI(G) + \sum_{st \in E_y} [d_G(a) + 2d_G(b) + d_G(c)]^{-\frac{1}{2}} + \sum_{eu \in E_z} [3d_G(u) + d_G(v)]^{-\frac{1}{2}}$$

where $s = ab, t = bc \in E(G), e = uv \in E(G)$; $u, v, a, b, c \in V(G)$ and are distinct.

PROOF.

$$\begin{aligned} SCI(G^{+++}) &= \sum_{uv \in E(G^{+++})} [d_{G^{+++}}(u) + d_{G^{+++}}(v)]^{-\frac{1}{2}} \\ &= \sum_{uv \in E_x} [d_{G^{+++}}(u) + d_{G^{+++}}(v)]^{-\frac{1}{2}} + \sum_{st \in E_y} [d_{G^{+++}}(s) + d_{G^{+++}}(t)]^{-\frac{1}{2}} \\ &+ \sum_{eu \in E_z} [d_{G^{+++}}(u) + d_{G^{+++}}(e)]^{-\frac{1}{2}} \\ &= \sum_{uv \in E_x} [2d_G(u) + 2d_G(v)]^{-\frac{1}{2}} + \sum_{st \in E_y} [d_G(a) + 2d_G(b) + d_G(c)]^{-\frac{1}{2}} \\ &+ \sum_{eu \in E_z} [2d_G(u) + d_G(u) + d_G(v)]^{-\frac{1}{2}}. \end{aligned}$$

Therefore

$$SCI(G^{+++}) = \frac{1}{\sqrt{2}}SCI(G) + \sum_{st\in E_y}[d_G(a)+2d_G(b)+d_G(c)]^{-\frac{1}{2}}$$
$$+\sum_{eu\in E_z}[3d_G(u)+d_G(v)]^{-\frac{1}{2}}$$

□

Corollary 2.1. *Let G be a r-regular graph on n vertices having m edges. Then*

$$SCI(G_r^{+++}) = \frac{m(r+2)}{2\sqrt{r}}$$

Proof.

$$SCI(G_r^{+++}) = \frac{1}{\sqrt{2}}\sum_{uv\in E_x}[2r]^{-\frac{1}{2}} + \sum_{st\in E_y}[4r]^{-\frac{1}{2}} + \sum_{eu\in E_z}[4r]^{-\frac{1}{2}}$$
$$= \frac{m}{\sqrt{4r}} + \frac{m(r-1)}{\sqrt{4r}} + \frac{2m}{\sqrt{4r}} = \frac{m(r+2)}{2\sqrt{r}}$$

□

Theorem 2.2. *Let G be a connected graph on n vertices having m edges. Then*

$$SCI(G^{++-}) = \sqrt{\frac{m}{2}} + \sum_{st\in E_y}[d_G(a)+2d_G(b)+d_G(c)+2(n-4)]^{-\frac{1}{2}}$$
$$+\sum_{eu\in E_z}[m+d_G(v)+d_G(w)+n-4]^{-\frac{1}{2}}$$

where $s = ab, t = bc \in E(G)$, $e = vw \in E(G)$; $u, v, w, a, b, c \in V(G)$ and are distinct.

Proof.

$$SCI(G^{++-}) = \sum_{uv\in E(G^{++-})}[d_{G^{++-}}(u)+d_{G^{++-}}(v)]^{-\frac{1}{2}}$$
$$= \sum_{uv\in E_x}[d_{G^{++-}}(u)+d_{G^{++-}}(v)]^{-\frac{1}{2}}$$
$$+\sum_{st\in E_y}[d_{G^{++-}}(s)+d_{G^{++-}}(t)]^{-\frac{1}{2}} + \sum_{eu\in E_z}[d_{G^{++-}}(u)+d_{G^{++-}}(e)]^{-\frac{1}{2}}$$
$$= \sum_{uv\in E_x}[2m]^{-\frac{1}{2}}$$
$$+\sum_{st\in E_y}[d_G(a)+d_G(b)+n-4+d_G(b)+d_G(c)+n-4]^{-\frac{1}{2}}$$
$$+\sum_{eu\in E_z}[m+d_G(v)+d_G(w)+n-4]^{-\frac{1}{2}}.$$

Therefore

$$SCI(G^{++-}) = \sqrt{\frac{m}{2}} + \sum_{st\in E_y} [d_G(a) + 2d_G(b) + d_G(c) + 2(n-4)]^{-\frac{1}{2}} + \sum_{eu\in E_z} [m + d_G(v) + d_G(w) + n - 4]^{-\frac{1}{2}}$$

□

COROLLARY 2.2. *Let G be a r-regular graph on n vertices having m edges. Then*

$$SCI(G_r^{++-}) = \sqrt{\frac{m}{2}} + \frac{m(r-1)}{\sqrt{2(k_1-4)}} + \frac{m(n-2)}{\sqrt{k_1-4+m}}$$

Where $k_1 = n + 2r$

PROOF.

$$SCI(G_r^{++-}) = \sqrt{\frac{m}{2}} + \sum_{st\in E_y} [4r + 2(n-4)]^{-\frac{1}{2}} + \sum_{eu\in E_z} [m + 2r + n - 4]^{-\frac{1}{2}}.$$

Therefore

$$SCI(G_r^{++-}) = \sqrt{\frac{m}{2}} + \frac{m(r-1)}{\sqrt{2(k_1-4)}} + \frac{m(n-2)}{\sqrt{k_1-4+m}}.$$

□

THEOREM 2.3. *Let G be a connected graph on n vertices having m edges. Then*

$$SCI(G^{+-+}) = \frac{1}{\sqrt{2}} SCI(G) + \sum_{st\in E_y} [2(m+3) - (d_G(a) + d_G(b) + d_G(c) + d_G(d))]^{-\frac{1}{2}} + \sum_{eu\in E_z} [m + d_G(u) - d_G(v) + 3]^{-\frac{1}{2}}$$

where $s = ab, t = cd \in E(G)$, $e = uv \in E(G)$; $u, v, a, b, c, d \in V(G)$ and are distinct.

PROOF.

$$SCI(G^{+-+}) = \sum_{uv\in E(G^{+-+})} [d_{G^{+-+}}(u) + d_{G^{+-+}}(v)]^{-\frac{1}{2}}$$

$$= \sum_{uv\in E_x} [d_{G^{+-+}}(u) + d_{G^{+-+}}(v)]^{-\frac{1}{2}} + \sum_{st\in E_y} [d_{G^{+-+}}(s) + d_{G^{+-+}}(t)]^{-\frac{1}{2}} + \sum_{eu\in E_z} [d_{G^{+-+}}(u) + d_{G^{+-+}}(e)]^{-\frac{1}{2}}$$

and

$$\begin{aligned}SCI(G^{+-+}) = &\sum_{uv\in E_x}[2d_G(u)+2d_G(v)]^{-\frac{1}{2}}\\ &+\sum_{st\in E_y}[m-d_G(a)-d_G(b)+3+m-d_G(c)-d_G(d)+3]^{-\frac{1}{2}}\\ &+\sum_{eu\in E_z}[2d_G(u)+m-d_G(u)-d_G(v)+3]^{-\frac{1}{2}},\end{aligned}$$

Therefore

$$\begin{aligned}SCI(G^{+-+}) = &\frac{1}{\sqrt{2}}SCI(G)\\ &+\sum_{st\in E_y}[2(m+3)-(d_G(a)+d_G(b)+d_G(c)+d_G(d))]^{-\frac{1}{2}}\\ &+\sum_{eu\in E_z}[m+d_G(u)-d_G(v)+3]^{-\frac{1}{2}}\end{aligned}$$

□

COROLLARY 2.3. *Let G be a r-regular graph on n vertices having m edges. Then*

$$SCI(G_r^{+-+}) = \frac{m}{2}\left[\frac{1}{\sqrt{r}}+\frac{(m-2r+1)}{\sqrt{2(m-2r+3)}}+\frac{4}{\sqrt{m+3}}\right]$$

PROOF.

$$SCI(G_r^{+-+}) = \frac{1}{\sqrt{2}}\sum_{uv\in E_x}[2r]^{-\frac{1}{2}}+\sum_{st\in E_y}[2(m+3)-4r]^{-\frac{1}{2}}+\sum_{eu\in E_z}[m+3]^{-\frac{1}{2}}.$$

Therefore

$$SCI(G_r^{+-+}) = \frac{m}{2}\left[\frac{1}{\sqrt{r}}+\frac{(m-2r+1)}{\sqrt{2(m-2r+3)}}+\frac{4}{\sqrt{m+3}}\right].$$

□

THEOREM 2.4. *Let G be a connected graph on n vertices having m edges. Then*

$$\begin{aligned}SCI(G^{-++}) = &\frac{\binom{n}{2}-m}{\sqrt{2(n-1)}}+\sum_{st\in E_y}[d_G(a)+2d_G(b)+d_G(c)]^{-\frac{1}{2}}\\ &+\sum_{eu\in E_z}[n-1+d_G(u)+d_G(v)]^{-\frac{1}{2}}\end{aligned}$$

where $s=ab, t=bc\in E(G),\ e=uv\in E(G);\ u,v,a,b,c\in V(G)$ and are distinct.

PROOF.

$$SCI(G^{-++}) = \sum_{uv\in E(G^{-++})} [d_{G^{-++}}(u) + d_{G^{-++}}(v)]^{-\frac{1}{2}}$$
$$= \sum_{uv\in E_x} [d_{G^{-++}}(u) + d_{G^{-++}}(v)]^{-\frac{1}{2}}$$
$$+ \sum_{st\in E_y} [d_{G^{-++}}(s) + d_{G^{-++}}(t)]^{-\frac{1}{2}}$$
$$+ \sum_{eu\in E_z} [d_{G^{-++}}(u) + d_{G^{-++}}(e)]^{-\frac{1}{2}}.$$

and

$$SCI(G^{-++}) = \sum_{uv\in E_x} [2(n-1)]^{-\frac{1}{2}} + \sum_{st\in E_y} [d_G(a) + 2d_G(b) + d_G(c)]^{-\frac{1}{2}}$$
$$+ \sum_{eu\in E_z} [n-1+d_G(u)+d_G(v)]^{-\frac{1}{2}}.$$

Therefore

$$SCI(G^{-++}) = \frac{\binom{n}{2} - m}{\sqrt{2(n-1)}} + \sum_{st\in E_y} [d_G(a) + 2d_G(b) + d_G(c)]^{-\frac{1}{2}}$$
$$+ \sum_{eu\in E_z} [n-1+d_G(u)+d_G(v)]^{-\frac{1}{2}}.$$

□

COROLLARY 2.4. *Let G be a r-regular graph on n vertices having m edges. Then*

$$SCI(G_r^{-++}) = \frac{\binom{n}{2} - m}{\sqrt{2(n-1)}} + \frac{m}{2}\left[\frac{(r-1)}{\sqrt{r}} + \frac{4}{\sqrt{n+2r-1}}\right]$$

PROOF.

$$SCI(G_r^{-++}) = \frac{\binom{n}{2} - m}{\sqrt{2(n-1)}} + \sum_{st\in E_y} [4r]^{-\frac{1}{2}} + \sum_{eu\in E_z} [n-1+2r]^{-\frac{1}{2}}.$$

Therefore

$$SCI(G_r^{-++}) = \frac{\binom{n}{2} - m}{\sqrt{2(n-1)}} + \frac{m}{2}\left[\frac{(r-1)}{\sqrt{r}} + \frac{4}{\sqrt{n+2r-1}}\right].$$

□

Analogous to above we obtained the following Theorems:

THEOREM 2.5. *Let G be a connected graph on n vertices having m edges. Then*

(i) $SCI(G^{---}) = \sum_{uv\in E_x} \left[2\big(k-d_G(v)-d_G(u)\big)\right]^{-\frac{1}{2}} + \sum_{st\in E_y} \left[2k-\big(d_G(a)+d_G(b)+d_G(c)+d_G(d)\big)\right]^{-\frac{1}{2}} + \sum_{ue\in E_z} \left[2k-\big(2d_G(u)+d_G(v)+d_G(w)\big)\right]^{-\frac{1}{2}}$
where $k = n+m-1;\ s = ab, t = cd \in E(G),\ e = vw \in E(G);$
$u, v, w, a, b, c, d \in V(G)$ *and are distinct.*

(ii) $SCI(G^{--+}) = \dfrac{\binom{n}{2}-m}{\sqrt{2(n-1)}} + \sum_{st\in E_y}[2(m+3)-(d_G(a)+d_G(b)+d_G(c)+d_G(d))]^{-\frac{1}{2}} + \sum_{ue\in E_z}[n+m-d_G(u)-d_G(v)+2]^{-\frac{1}{2}}$
where $s = ab, t = cd \in E(G),\ e = uv \in E(G);\ u, v, a, b, c, d \in V(G)$ *and are distinct.*

(iii) $SCI(G^{-+-}) = \sum_{uv\in E_x}[2(k-d_G(u)-d_G(v))]^{-\frac{1}{2}} + \sum_{st\in E_y}[2(n-4)+d_G(a)+2d_G(b)+d_G(c)]^{-\frac{1}{2}} + \sum_{ue\in E_z}[(k+n-2d_G(u)+d_G(v)+d_G(w)-4)]^{-\frac{1}{2}}$
where $k = n+m-1;\ s = ab, t = bc \in E(G),\ e = vw \in E(G);$
$u, v, w, a, b, c \in V(G)$ *and are distinct.*

(iv) $SCI(G^{+--}) = \sqrt{\dfrac{m}{2}} + \sum_{st\in E_y} \left[2k-\big(d_G(a)+d_G(b)+d_G(c)+d_G(d)\big)\right]^{-\frac{1}{2}} + \sum_{ue\in E_z}[m+k-d_G(v)-d_G(w)]^{-\frac{1}{2}}$
Where $k = n+m-1;\ s = ab, t = cd \in E(G),\ e = vw \in E(G);$
$u, v, w, a, b, c, d \in V(G)$ *and are distinct.*

COROLLARY 2.5. *Let* G *be a* r*-regular graph on* n *vertices having* m *edges. Then*

(i) $SCI(G_r^{---}) = \dfrac{1}{2\sqrt{2(k-2r)}}\left[n(n-1)+m\big(2(n-r)+m-5\big)\right]$

(ii) $SCI(G_r^{--+}) = \dfrac{\binom{n}{2}-m}{\sqrt{2(n-1)}} + \dfrac{m}{2}\left[\dfrac{(m-2r+1)}{\sqrt{2(m-2r+3)}} + \dfrac{4}{\sqrt{n+m+2(1-r)}}\right]$

(iii) $SCI(G_r^{-+-}) = \dfrac{\binom{n}{2}-m}{\sqrt{2(k-2r)}} + m\left[\dfrac{(r-1)}{\sqrt{2(n-4+2r)}} + \dfrac{(n-2)}{\sqrt{k+n-4}}\right]$

(iv) $SCI(G_r^{+--}) = \dfrac{m}{\sqrt{2}}\left[\dfrac{1}{\sqrt{m}} + \dfrac{(m-2r+1)}{2\sqrt{(k-2r)}} + (n-2)\sqrt{\dfrac{2}{k+m-2r}}\right].$

3. Atom-bond connectivity index of transformation graphs G^{xyz}

THEOREM 3.1. *Let* G *be a connected graph on* n *vertices having* m *edges. Then*

(i) $ABC(G^{+++}) = \sum_{st\in E_y} \sqrt{\dfrac{d_G(a)+2d_G(b)+d_G(c)-2}{[d_G(a)+d_G(b)][d_G(b)+d_G(c)]}} + \dfrac{1}{\sqrt{2}}\left[\sum_{uv\in E_x} \sqrt{\dfrac{d_G(u)+d_G(v)-1}{d_G(u)d_G(v)}} + \sum_{eu\in E_z} \sqrt{\dfrac{3d_G(u)+d_G(v)-2}{d_G(u)[d_G(u)+d_G(v)]}}\right]$
where $s = ab, t = bc \in E(G), e = uv \in E(G);\ u, v, a, b, c \in V(G)$ *and are distinct.*

(ii) $ABC(G^{++-}) = \sum_{eu\in E_z} \sqrt{\dfrac{n+m+d_G(v)+d_G(w)-6}{m[d_G(v)+d_G(w)+n-4]}}$

$$+\sqrt{2(m-1)}+\sum_{st\in E_y}\sqrt{\frac{d_G(a)+2d_G(b)+d_G(c)+2n-10}{[d_G(a)+d_G(b)+n-4][d_G(b)+d_G(c)+n-4]}}$$

where $s=ab, t=bc\in E(G),\ e=vw\in E(G);\ u,v,w,a,b,c\in V(G)$ *and are distinct.*

(iii) $$ABC(G^{+-+})=\frac{1}{\sqrt{2}}\Bigg[\sum_{uv\in E_x}\sqrt{\frac{d_G(u)+d_G(v)-1}{d_G(u)d_G(v)}}$$
$$+\sum_{eu\in E_z}\sqrt{\frac{m+d_G(u)-d_G(v)+1}{d_G(u)[m-d_G(u)-d_G(v)+3]}}\Bigg]$$
$$+\sum_{st\in E_y}\sqrt{\frac{2m-d_G(a)-d_G(b)-d_G(c)-d_G(d)+4}{[m-d_G(a)-d_G(b)+3][m-d_G(c)-d_G(d)+3]}}$$

where $s=ab, t=cd\in E(G),\ e=uv\in E(G);\ u,v,a,b,c,d\in V(G)$ *and are distinct.*

(iv) $$ABC(G^{-++})=\sum_{st\in E_y}\sqrt{\frac{d_G(a)+2d_G(b)+d_G(c)-2}{[d_G(a)+d_G(b)][d_G(b)+d_G(c)]}}$$
$$+\sum_{eu\in E_z}\sqrt{\frac{n+d_G(u)+d_G(v)-3}{(n-1)[d_G(u)+d_G(v)]}}+\left(\frac{n}{2}-\frac{m}{n-1}\right)\sqrt{2(n-2)}$$

where $s=ab, t=bc\in E(G),\ e=uv\in E(G);\ u,v,a,b,c\in V(G)$ *and are distinct.*

(v) $$ABC(G^{---})=\sum_{uv\in E_x}\sqrt{\frac{2(k-d_G(u)-d_G(v)-1)}{[k-2d_G(u)][k-2d_G(v)]}}$$
$$+\sum_{st\in E_y}\sqrt{\frac{2(k-1)-[d_G(a)+d_G(b)+d_G(c)+d_G(d)]}{[k-d_G(a)-d_G(b)][k-d_G(c)-d_G(d)]}}$$
$$+\sum_{eu\in E_z}\sqrt{\frac{2(k-1)-[2d_G(u)+d_G(v)+d_G(w)]}{[k-2d_G(u)][k-d_G(v)-d_G(w)]}}$$

where $k=n+m-1;\ s=ab, t=cd\in E(G),\ e=vw\in E(G);$
$u,v,w,a,b,c,d\in V(G)$ *and are distinct.*

(vi) $$ABC(G^{--+})=\left(\frac{n}{2}-\frac{m}{n-1}\right)\sqrt{2(n-2)}$$
$$+\sum_{st\in E_y}\sqrt{\frac{2(m+2)-[d_G(a)+d_G(b)+d_G(c)+d_G(d)]}{[m-d_G(a)-d_G(b)+3][m-d_G(c)-d_G(d)+3]}}$$
$$+\sum_{eu\in E_z}\sqrt{\frac{n+m-[d_G(u)+d_G(v)]}{(n-1)[m-d_G(u)-d_G(v)+3]}}$$

where $s=ab, t=cd\in E(G),\ e=uv\in E(G);\ u,v,a,b,c,d\in V(G)$ *and are distinct.*

(vii) $$ABC(G^{-+-})=\sum_{uv\in E_x}\sqrt{\frac{2(k-d_G(u)-d_G(v)-1)}{[k-2d_G(u)][k-2d_G(v)]}}$$

$$+\sum_{st\in E_y}\sqrt{\frac{2(n-5)+d_G(a)+2d_G(b)+d_G(c)}{[n+d_G(a)+d_G(b)-4][n+d_G(b)+d_G(c)-4]}}$$

$$+\sum_{eu\in E_z}\sqrt{\frac{k+n-2d_G(u)+d_G(v)+d_G(w)-6}{[k-2d_G(u)][n+d_G(v)+d_G(w)-4]}}$$

where $k=n+m-1;\ s=ab, t=bc\in E(G),\ e=vw\in E(G);$ $u,v,w,a,b,c\in V(G)$ *and are distinct.*

(viii) $ABC(G^{+--})=\sqrt{2(m-1)}$

$$+\sum_{st\in E_y}\sqrt{\frac{2(k-1)-[d_G(a)+d_G(b)+d_G(c)+d_G(d)]}{[k-d_G(a)-d_G(b)][k-d_G(c)+d_G(d)]}}$$

$$+\sum_{eu\in E_z}\sqrt{\frac{k+m-2-d_G(v)-d_G(w)}{m(k-d_G(v)-d_G(w))}}$$

where $k=n+m-1;\ s=ab, t=cd\in E(G),\ e=vw\in E(G);$ $u,v,w,a,b,c,d\in V(G)$ *and are distinct.*

COROLLARY 3.1. *Let* G *be a* r*-regular graph on* n *vertices having* m *edges. Then*

(i) $ABC({G_r}^{+++})=\dfrac{m(r+2)\sqrt{2r-1}}{r\sqrt{2}}$

(ii) $ABC({G_r}^{++-})=\sqrt{2(m-1)}+\dfrac{m(r-1)}{2r+n-4}\sqrt{2(2r+n-5)}$
$+(n-2)\sqrt{\dfrac{m(n+m+2r-6)}{2r+n-4}}$

(iii) $ABC({G_r}^{+-+})=\dfrac{m}{\sqrt{2}}\Bigg[\dfrac{1}{r}\sqrt{2r-1}+\sqrt{\dfrac{4(m+1)}{r(m-2r+3)}}$
$+\left(\dfrac{m-2r+1}{m-2r+3}\right)\sqrt{m+2(1-r)}\Bigg]$

(iv) $ABC({G_r}^{-++})=\left(\dfrac{n}{2}-\dfrac{m}{n-1}\right)\sqrt{2(n-2)}$
$+2m\left[\dfrac{r-1}{r}\sqrt{\dfrac{2r-1}{8}}+\sqrt{\dfrac{n+2r-3}{2r(n-1)}}\right]$

(v) $ABC({G_r}^{---})=\dfrac{1}{k-2r}\sqrt{\dfrac{k-2r-1}{2}}\Big(n(n-1)+m(2n+m-2r-5)\Big)$

(vi) $ABC({G_r}^{--+})=\left(\dfrac{n}{2}-\dfrac{m}{n-1}\right)\sqrt{2(n-2)}$
$+2m\left[\dfrac{m-2r+1}{m-2r+3}\sqrt{\dfrac{m+2(1-r)}{8}}+\sqrt{\dfrac{n+m-2r}{(n-1)(m-2r+3)}}\right]$

(vii) $ABC({G_r}^{-+-})=\dfrac{\binom{n}{2}-m}{k-2r}\sqrt{2(k-2r-1)}$
$+m\left[\dfrac{r-1}{n+2r-4}\sqrt{2(n-5+2r)}+(n-2)\sqrt{\dfrac{k+n-6}{(k-2r)(n+2r-4)}}\right]$

(viii) $ABC({G_r}^{+--}) = \sqrt{2(m-1)}$
$+\, m\Big[\dfrac{m-2r+1}{k-2r}\sqrt{\dfrac{k-2r-1}{2}} + (n-2)\sqrt{\dfrac{k+m-2(1+r)}{m(k-2r)}}\Big]$

4. Augmented Zagreb index of transformation graphs G^{xyz}

THEOREM 4.1. *Let G be a connected graph on n vertices having m edges. Then*

(i) $AZI(G^{+++}) = \sum_{st\in E_y}\left(\dfrac{[d_G(a)+d_G(b)][d_G(b)+d_G(c)]}{d_G(a)+2d_G(b)+d_G(c)-2}\right)^3$
$+\sum_{eu\in E_z}\left(\dfrac{2d_G(u)[d_G(u)+d_G(v)]}{3d_G(u)+d_G(v)-2}\right)^3 + \sum_{uv\in E_x}\left(\dfrac{2d_G(u)d_G(v)}{d_G(u)+d_G(v)-1}\right)^3$
where $s = ab, t = bc \in E(G), e = uv \in E(G)$; $u, v, a, b, c \in V(G)$ and are distinct.

(ii) $AZI(G^{++-}) = m\left(\dfrac{m^2}{2(m-1)}\right)^3$
$+\sum_{st\in E_y}\left(\dfrac{[d_G(a)+d_G(b)+n-4][d_G(b)+d_G(c)+n-4]}{d_G(a)+2d_G(b)+d_G(c)+2n-10}\right)^3$
$+\sum_{eu\in E_z}\left(\dfrac{m[d_G(v)+d_G(w)+n-4]}{n+m+d_G(v)+d_G(w)-6}\right)^3$
where $s = ab, t = bc \in E(G),\ e = vw \in E(G)$; $u, v, w, a, b, c \in V(G)$ and are distinct.

(iii) $AZI(G^{+-+}) = \sum_{uv\in E_x}\left(\dfrac{2d_G(u)d_G(v)}{d_G(u)+d_G(v)-1}\right)^3$
$+\sum_{st\in E_y}\left(\dfrac{[m-d_G(a)-d_G(b)+3][m-d_G(c)-d_G(d)+3]}{2(m+2)-(d_G(a)+d_G(b)+d_G(c)+d_G(d))}\right)^3$
$+\sum_{eu\in E_z}\left(\dfrac{2d_G(u)[m-d_G(u)-d_G(v)+3]}{m+d_G(u)-d_G(v)+1}\right)^3$
where $s = ab, t = cd \in E(G),\ e = uv \in E(G)$; $u, v, a, b, c, d \in V(G)$ and are distinct.

(iv) $AZI(G^{-++}) = \sum_{st\in E_y}\left(\dfrac{[d_G(a)+d_G(b)][d_G(b)+d_G(c)]}{d_G(a)+2d_G(b)+d_G(c)-2}\right)^3$
$+\sum_{eu\in E_z}\left(\dfrac{(n-1)[d_G(u)+d_G(v)]}{n+d_G(u)+d_G(v)-3}\right)^3 + \left(\binom{n}{2}-m\right)\left(\dfrac{(n-1)^2}{2(n-2)}\right)^3$
where $s = ab, t = bc \in E(G),\ e = uv \in E(G)$; $u, v, a, b, c \in V(G)$ and are distinct.

(v) $AZI(G^{---}) = \sum_{uv\in E_x}\left(\dfrac{[k-2d_G(u)][k-2d_G(v)]}{2(k-d_G(u)-d_G(v)-1)}\right)^3$
$+\sum_{st\in E_y}\left(\dfrac{[k-d_G(a)-d_G(b)][k-d_G(c)-d_G(d)]}{2(k-1)-[d_G(a)+d_G(b)+d_G(c)+d_G(d)]}\right)^3$

$+\sum_{eu\in E_z}\left(\frac{[k-2d_G(u)][k-d_G(v)-d_G(w)]}{2(k-1)-[2d_G(u)+d_G(v)+d_G(w)]}\right)^3$

where $k=n+m-1;\ s=ab, t=cd\in E(G),\ e=vw\in E(G);$
$u,v,w,a,b,c,d\in V(G)$ *and are distinct.*

(vi) $AZI(G^{--+})=\left(\binom{n}{2}-m\right)\left(\frac{(n-1)^2}{2(n-2)}\right)^3$

$+\sum_{st\in E_y}\left(\frac{[m-d_G(a)-d_G(b)+3][m-d_G(c)-d_G(d)+3]}{2(m+2)-[d_G(a)+d_G(b)+d_G(c)+d_G(d)]}\right)^3$

$+\sum_{eu\in E_z}\left(\frac{(n-1)[m-d_G(u)-d_G(v)+3]}{n+m-d_G(u)-d_G(v)}\right)^3$

where $s=ab, t=cd\in E(G),\ e=uv\in E(G);\ u,v,a,b,c,d\in V(G)$ *and are distinct.*

(vii) $AZI(G^{-+-})=\sum_{uv\in E_x}\left(\frac{[k-2d_G(u)][k-2d_G(v)]}{2(k-d_G(u)-d_G(v)-1)}\right)^3$

$+\sum_{st\in E_y}\left(\frac{[n+d_G(a)+d_G(b)-4][n+d_G(b)+d_G(c)-4]}{2(n-5)+d_G(a)+2d_G(b)+d_G(c)}\right)^3$

$+\sum_{eu\in E_z}\left(\frac{[k-2d_G(u)][n+d_G(v)+d_G(w)-4]}{k+n-2d_G(u)+d_G(v)+d_G(w)-6}\right)^3$

where $k=n+m-1;\ s=ab, t=bc\in E(G),\ e=vw\in E(G);$
$u,v,w,a,b,c\in V(G)$ *and are distinct.*

(viii) $AZI(G^{+--})=\sum_{st\in E_y}\left(\frac{[k-d_G(a)-d_G(b)][k-d_G(c)+d_G(d)]}{2(k-1)-[d_G(a)+d_G(b)+d_G(c)+d_G(d)]}\right)^3$

$+\sum_{eu\in E_z}\left(\frac{m(k-d_G(v)-d_G(w))}{k+m-2-d_G(v)-d_G(w)}\right)^3+m\left(\frac{m^2}{2(m-1)}\right)^3$

where $k=n+m-1;\ s=ab, t=cd\in E(G),\ e=vw\in E(G);$
$u,v,w,a,b,c,d\in V(G)$ *and are distinct.*

COROLLARY 4.1. *Let G be a r-regular graph on n vertices having m edges. Then*

(i) $AZI({G_r}^{+++})=m(r+2)\left(\frac{2r^2}{2r-1}\right)^3$

(ii) $AZI({G_r}^{++-})=m\left[\left(\frac{m^2}{2(m-1)}\right)^3+(r-1)\left(\frac{(2r+n-4)^2}{2(2r+n-5)}\right)^3\right.$
$\left.+(n-2)\left(\frac{m(2r+n-4)}{n+m+2(r-3)}\right)^3\right]$

(iii) $AZI({G_r}^{+-+})=m\left[\left(\frac{2r^2}{2r-1}\right)^3+\frac{(m-2r+1)}{2}\left(\frac{(m-2r+3)^2}{2(m-2r+2)}\right)^3\right.$
$\left.+2\left(\frac{2r(m-2r+3)}{m+1}\right)^3\right]$

(iv) $AZI(G_r{}^{-++}) = \left(\binom{n}{2} - m\right)\left(\frac{(n-1)^2}{2(n-2)}\right)^3$
$+ m\left[(r-1)\left(\frac{2r^2}{2r-1}\right)^3 + 2\left(\frac{2r(n-1)}{n+2r-3}\right)^3\right]$

(v) $AZI(G_r{}^{---}) = \left(\frac{(k-2r)^2}{2(k-1-2r)}\right)^3\left(\binom{n}{2} + \frac{m}{2}(m+2n-2r-5)\right)$

(vi) $AZI(G_r{}^{--+}) = \left(\binom{n}{2} - m\right)\left(\frac{(n-1)^2}{2(n-2)}\right)^3$
$+ m\left[\left(\frac{m-2r+1}{2}\right)\left(\frac{(m-2r+3)^2}{2(m+2-2r)}\right)^3 + 2\left(\frac{(n-1)(m-2r+3)}{n+m-2r}\right)^3\right]$

(vii) $AZI(G_r{}^{-+-}) = \left(\binom{n}{2} - m\right)\left(\frac{(k-2r)^2}{2(k-2r-1)}\right)^3$
$+ m\left[(r-1)\left(\frac{(n+2r-4)^2}{2(n-5+2r)}\right)^3 + (n-2)\left(\frac{(k-2r)(n+2r-4)}{k+n-6}\right)^3\right]$

(viii) $AZI(G_r{}^{+--}) = m\left[\left(\frac{m^2}{2(m-1)}\right)^3 + \frac{1}{2}(m-2r+1)\left(\frac{(k-2r)^2}{2(k-2r-1)}\right)^3 +\right.$
$\left.(n-2)\left(\frac{m(k-2r)}{k+m-2(1+r)}\right)^3\right]$

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