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NEW TYPE OF FUZZY SOFT HYPERIDEALS OF ORDERED Γ-HYPERSEMIGROUPS

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ABSTRACT. In this paper, we introduce the concept of interval valued fuzzy soft Γ-hyperideals in ordered Γ-hypersemigroups. We define interval valued fuzzy soft left (right, bi, interior) hyperideals of ordered Γ-hypersemigroups and discuss some properties. Examples are provided illustrate our results.

1. Introduction

Zadeh [**17**] introduced the concept of fuzzy sets in 1965. Γ-semigroups was introduced by Sen and Saha [**14**]. Ordered Γ-semigroup was proposed by Keyhopulu [**7**]. Algebraic hyper structures represent a natural extension of classical algebraic structures, and they were originally proposed in 1934 by Marty [**11**]. One of the main reasons which attract researchers towards hyperstructures is its unique property that in hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an element. Soft set theory was introduced by Molodtsov [**12**] in 1999, and its a new mathematical model for dealing with uncertainty from a parameterization point of view. P. K. Maji, R. Biswas and R. Roy [**9**] studied the some new operations on fuzzy soft sets. The aim of M. Akram, J. Kavikumar and A. B. Khamis in their paper [**1**] was to apply the concept of fuzzy soft sets on Γ-semigroups. About gamma-semigroups and their phase-aspect, an interested reader can look, for example, in the following texts [**14, 4**]. K. Arulmozhi and V. Chinnadurai [**3**] discussed new type of fuzzy soft hyperideals and homomorphism of Γ-hypersemigroups. Characterization of fuzzy ideals in ordered gamma semigroups discussed by V. Chinnadurai and K. Arulmozhi [**5**]. In this paper, we define a new notion of interval valued fuzzy soft ordered Γ- hypersemigroups and investigate some of its properties with examples.

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2. Preliminaries

DEFINITION 2.1. ([**14**]) Let $S = \{a, b, c, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets. Then S is called a Γ-semigroup if it satisfies the conditions
(i) $a\alpha b \in S$,
(ii) $(a\beta b)\gamma c = a\beta(b\gamma c)$ $\forall$ $a, b, c \in S$ and $\alpha, \beta, \gamma \in \Gamma$.

DEFINITION 2.2. ([**14**]) An ordered Γ-semigroup (shortly po-Γ-semigroup) is a Γ-semigroup S together with an order relation $\leqslant$ such that $a \leqslant b$ implies $a\gamma c \leqslant b\gamma c$ and $c\gamma a \leqslant c\gamma b$ for all $c \in S$ and $\gamma \in \Gamma$.

DEFINITION 2.3. ([**14**]) Let A and B be two non empty subsets of a Γ- semigroup S. We denote
(i) $(A] = \{t \in S \,|\, t \leqslant h \text{ for some } h \in A\}$,
(ii) $A\Gamma B = \{a\alpha b : a \in A, b \in B \text{ and } \alpha \in \Gamma\}$,
(iii) $A_x = \{(y, z) \in S \times S \,|\, x \leqslant y\alpha z\}$.

DEFINITION 2.4. A map $\circ : H \times H \to P^*(H)$ is called a hyper operation or join operation on the set H, where H is a non-empty set and $P^*(H) = P(H) \setminus \{\phi\}$ denotes the set of all non-empty subset of H. A hyper groupoid is a set H together with a (binary) hyper operation.

DEFINITION 2.5. ([**6**]) A hypergroupoid $(H, \circ)$, which is associative, that is $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$, is called a hyper semigroup. Let A and B be two non-empty subsets of H. Then we define

$$A \circ B = \begin{cases} \bigcup\limits_{a \in A, b \in B} a \circ b, \quad a \circ B = \{a\} \circ B \\ A \circ b = A \circ \{b\} \end{cases}$$

DEFINITION 2.6. ([**2**]) Let S and Γ be two non-empty sets. S is called a Γ-hypersemigroup if every $\gamma \in \Gamma$ is a hyperoperation on S that is $x\gamma y \subseteq S$ for every $x, y \in S$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in H$ we have $x\alpha(y\beta z) = (x\alpha y)\beta z$. If every $\gamma \in \Gamma$ is a hyper operation, then S is a Γ-semigroup. If (S, γ) is a hypergroup for every $\gamma \in \Gamma$, then S is called a Γ-hypergroup. Let A and B be two non-empty subsets of S and $\gamma \in \Gamma$. We define

$$A\gamma B = \bigcup \{a\gamma b | a \in A, b \in B \text{ } and \text{ } \gamma \in \Gamma\}.$$

Also

$$A\Gamma B = \bigcup \{a\gamma b | a \in A, b \in B \text{ } and \text{ } \gamma \in \Gamma\} = \bigcup_{A\gamma B}.$$

DEFINITION 2.7. ([**12**]) Let U be an universel set and E be the set of parameters. $P(U)$ denote the power set of U. Let A be a non empty subset of E then the pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

DEFINITION 2.8. ([**10**]) Let (F, A) and (G, B) be two soft sets then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ is defined as $(F, A) \wedge (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$.

Definition 2.9. ([**10**]) Let (F, A) and (G, B) be two soft sets then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$ is defined as $(F, A) \vee (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$.

Definition 2.10. Let X be a non-empty set. A mapping $\widetilde{\mu} : X \to D[0, 1]$ is called an interval valued fuzzy subsets of X. $D[0, 1]$ denotes the set of all closed subintervals of $[0, 1]$. For any $x \in X$, $\widetilde{\mu}(x) = [\,\mu^-(x), \mu^+(x)]$, where μ^- and μ^+ are fuzzy subsets of X such that $\mu^-(x) \leqslant \mu^+(x)$.

Definition 2.11. ([**15**]) By an interval number $\widetilde{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leqslant a^- \leqslant a^+ \leqslant 1$, where a^- and a^+ are the lower and upper limits of $\widetilde{a}$ respectively. We also identify the interval $[a, a]$ by the number $a \in [0, 1]$. For any interval numbers $\widetilde{a}_j = [a_j^-, a_j^+], \widetilde{b}_j = [b_j^-, b_j^+] \in D[0, 1], j \in \Omega$ where Ω is an index set.

$$\max{}^i \left\{ \widetilde{a}_j, \widetilde{b}_j \right\} = \left[\max \left\{ a_j^-, b_j^- \right\}, \max \left\{ a_j^+, b_j^+ \right\} \right],$$

$$\min{}^i \left\{ \widetilde{a}_j, \widetilde{b}_j \right\} = \left[\min \left\{ a_j^-, b_j^- \right\}, \min \left\{ a_j^+, b_j^+ \right\} \right],$$

$$\inf \widetilde{a}_j = \left[\bigcap_{j\in\Omega} a_j^-, \bigcap_{j\in\Omega} a_j^+ \right], \sup \widetilde{a}_j = \left[\bigcup_{j\in\Omega} a_j^-, \bigcup_{i\in\Omega} a_j^+ \right].$$

Let

(i) $\widetilde{a} \leqslant \widetilde{b} \Longleftrightarrow a^- \leqslant b^-$ and $a^+ \leqslant b^+$,

(ii) $\widetilde{a} = \widetilde{b} \Longleftrightarrow a^- = b^-$ and $a^+ = b^+$.

(iii) $\widetilde{a} < \widetilde{b} \Longleftrightarrow \widetilde{a} \leqslant \widetilde{b}$ and $\widetilde{a} \neq \widetilde{b}$,

(iv) $k\widetilde{a} = [ka^-, ka^+]$, whenever $0 \leqslant k \leqslant 1$.

Definition 2.12. ([**8**]) Let $\phi : H_1 \to H_2$ and $h : E_1 \to E_2$ be two maps, $A \subseteq E_1$ and $B \subseteq E_2$, where E_1 and E_2 are sets of parameters viewed on H_1 and H_2, respectively. The pair (ϕ, h) is called a fuzzy soft map from H_1 to H_2. If ϕ is a hypergroup homomorphism, then (ϕ, h) is called a fuzzy soft homomorphism from H_1 to H_2.

3. IVFS ordered Γ-hyperideals

In this section, S denotes ordered Γ- hypersemigroups.

Definition 3.1. ([**16**]) Let U be an initial universe and E a set of parameters. $P(U)$ denotes the set of all interval valued fuzzy sets of U. Let $A \subseteq E$. A pair $(\tilde{F}, A)$ is an interval valued fuzzy soft set over U, where $\tilde{F}$ is a mapping given by $\tilde{F} : A \to \tilde{P}(U)$.

The symbol $\vartheta_{\tilde{F}(a)}(x)$ is called the membership degree of x in the interval valued fuzzy soft set $(\tilde{F}, A)$ and $\inf\limits_{x \in y\alpha z} \vartheta_{\tilde{F}(a)}(x)$ is called the infimum of the membership function $\vartheta_{\tilde{F}(a)}(x)$.

Definition 3.2. Let (F, A) be a soft set and $(\tilde{F}, A)$ be a interval valued fuzzy soft(briefly IVFS) subset of S for $x, y, z \in S$, $\alpha, \beta \in \Gamma, a \in A$ and $x \leqslant y \Rightarrow \vartheta_{\tilde{F}(a)}(x) \geqslant \vartheta_{\tilde{F}(a)}(y)$.

(i) $(\tilde{F}, A)$ is called a IVFS Γ-hyper subsemigroup of S if

$$\inf_{x\in y\alpha z} \vartheta_{\tilde{F}(a)}(x) \geqslant \min\{\vartheta_{\tilde{F}(a)}(y), \vartheta_{\tilde{F}(a)}(z)\}.$$

(ii) $(\tilde{F}, A)$ is called a IVFS Γ-hyper left ideal of S if $\inf_{x\in y\alpha z} \vartheta_{\tilde{F}(a)}(x) \geqslant \vartheta_{\tilde{F}(a)}(z)$.

(iii) $(\tilde{F}, A)$ is called a IVFS Γ-hyper right ideal of S if $\inf_{x\in y\alpha z} \vartheta_{\tilde{F}(a)}(x) \geqslant \vartheta_{\tilde{F}(a)}(y)$.

(iv) $(\tilde{F}, A)$ is called a IVFS Γ-hyperideal of S if

$$\inf_{x\in y\alpha z} \vartheta_{\tilde{F}(a)}(x) \geqslant \max\{\vartheta_{\tilde{F}(a)}(y), \vartheta_{\tilde{F}(a)}(z)\}.$$

(v) $(\tilde{F}, A)$ is called a IVFS Γ-hyperbi-ideal of S if

$$\inf_{p\in x\alpha y\beta z} \vartheta_{\tilde{F}(a)}(p) \geqslant \min\{\vartheta_{\tilde{F}(a)}(x), \vartheta_{\tilde{F}(a)}(z)\}.$$

(vi) $(\tilde{F}, A)$ is called a IVFS Γ-hyper interior ideal of S if

$$\inf_{p\in x\alpha y\beta z} \vartheta_{\tilde{F}(a)}(p) \geqslant \vartheta_{\tilde{F}(a)}(y).$$

Theorem 3.1. *Let $(\tilde{F_1}, A)$ and $(\tilde{F_2}, B)$ be two IVFS ordered Γ-hypersub- semigroups of S, then $(\tilde{F_1}, A) \wedge (\tilde{F_2}, B)$ and $(\tilde{F_1}, A) \vee (\tilde{F_2}, B)$ are IVFS Γ- hypersubsemigroup of S.*

Proof. Let $(\tilde{F_1}, A)$ and $(\tilde{F_2}, B)$ be two IVFS ordered Γ-hypersubsemigroups of S defined as $(\tilde{F_1}, A) \wedge (\tilde{F_2}, B)$, where $C = A \times B$ and $\tilde{M}(a, b) = \tilde{F_1}(a) \cap \tilde{F_2}(b)$, for all $(a, b) \in C = A \times B, x, y, z \in S$ and $\gamma \in \Gamma$.

$$\begin{aligned}
\inf_{z\in x\gamma y}\{\vartheta_{\tilde{M}(a,b)}(z)\} &= \inf_{z\in x\gamma y}\{\min\{\vartheta_{\tilde{F_1}(a)}(z), \vartheta_{\tilde{F_2}(b)}(z)\}\} \\
&= \min\{\inf_{z\in x\gamma y} \vartheta_{\tilde{F_1}(a)}(z), \inf_{z\in x\gamma y} \vartheta_{\tilde{F_2}(b)}(z)\} \\
&\geqslant \min\{\min\{\vartheta_{\tilde{F_1}(a)}(x), \vartheta_{\tilde{F_1}(a)}(y)\}, \min\{\vartheta_{\tilde{F_2}(b)}(x), \vartheta_{\tilde{F_2}(b)}(y)\}\} \\
&= \min\{(\vartheta_{\tilde{F_1}(a)} \cap \vartheta_{\tilde{F_2}(b)})(x), (\vartheta_{\tilde{F_1}(a)} \cap \vartheta_{\tilde{F_2}(b)})(y)\} \\
&= \min\{\vartheta_{\tilde{M}(a,b)}(x), \vartheta_{\tilde{M}(a,b)}(y)\}.
\end{aligned}$$

Similarly $(\tilde{F_1}, A) \vee (\tilde{F_2}, B)$ is a IVFS ordered Γ-hypersubsemigroup of S. □

Theorem 3.2. *Let $(\tilde{F_1}, A)$ and $(\tilde{F_2}, B)$ be two IVFS ordered Γ-hyperleft (right) ideals of S, then $(\tilde{F_1}, A) \wedge (\tilde{F_2}, B)$ and $(\tilde{F_1}, A) \vee (\tilde{F_2}, B)$ is IVFS ordered Γ-hyperleft (right) ideal of S.*

Proof. Let $(\tilde{F_1}, A)$ and $(\tilde{F_2}, B)$ be two IVFS ordered Γ-hyperleftideals of S defined as $(\tilde{F_1}, A) \wedge (\tilde{F_2}, B)$ where $C = A \times B$ and $\tilde{M}(a, b) = \tilde{F_1}(a) \cap \tilde{F_2}(b)$, for all $(a, b) \in C = A \times B$. For $x, y, z \in S$ and $\gamma \in \Gamma$.

$$\begin{aligned}
\inf_{z\in x\gamma y}\{\vartheta_{\tilde{M}(a,b)}(z)\} &= \inf_{z\in x\gamma y}\{\min\{\vartheta_{\tilde{F_1}(a)}(z), \vartheta_{\tilde{F_2}(b)}(z)\}\} \\
&= \min\{\inf_{z\in x\gamma y} \vartheta_{\tilde{F_1}(a)}(z), \inf_{z\in x\gamma y} \vartheta_{\tilde{F_2}(b)}(z)\} \\
&= \min\{\min\{\vartheta_{\tilde{F_1}(a)}(y), \vartheta_{\tilde{F_2}(b)}(y)\}\} \\
&= \vartheta_{\tilde{M}(a,b)}(y).
\end{aligned}$$

Similarly $(\tilde{F_1}, A) \vee (\tilde{F_2}, B)$ are IVFS ordered Γ-left hyperideals of S. □

THEOREM 3.3. *Let* $(\tilde{F_1}, A)$ *and* $(\tilde{F_2}, B)$ *be two IVFS ordered* Γ*-hyperbi-ideals of* S*, then* $(\tilde{F_1}, A) \wedge (\tilde{F_2}, B)$ *is IVFS ordered* Γ*-hyperbi-ideal of* S*.*

PROOF. Let $(\tilde{F_1}, A)$ and $(\tilde{F_2}, B)$ be two IVFS of S defined as $(\tilde{F_1}, A) \wedge (\tilde{F_2}, B)$ where $C = A \times B$, $\tilde{M}(a, b) = \tilde{F_1}(a) \cap \tilde{F_2}(b)$, where $(a, b) \in C = A \times B$, $p, x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

$$\begin{aligned}
\inf_{p \in x\alpha y\beta z}\{\vartheta_{\tilde{M}(a,b)}(z)\} &= \inf_{p \in x\alpha y\beta z}\{\min\{\vartheta_{\tilde{F_1}(a)}(p), \vartheta_{\tilde{F_2}(b)}(p)\}\} \\
&= \min\{\inf_{p \in x\alpha y\beta z} \vartheta_{\tilde{F_1}(a)}(p), \inf_{p \in x\alpha y\beta z} \vartheta_{\tilde{F_2}(b)}(p)\} \\
&\geqslant \min\{\min\{\vartheta_{\tilde{F_1}(a)}(x), \vartheta_{\tilde{F_1}(a)}(z)\}, \min\{\vartheta_{\tilde{F_2}(b)}(x), \vartheta_{\tilde{F_2}(b)}(z)\}\} \\
&= \min\{(\vartheta_{\tilde{F_1}(a)} \cap \vartheta_{\tilde{F_2}(b)})(x), (\vartheta_{\tilde{F_1}(a)} \cap \vartheta_{\tilde{F_2}(b)})(z)\} \\
&= \min\{\vartheta_{\tilde{M}(a,b)}(x), \vartheta_{\tilde{M}(a,b)}(z)\}.
\end{aligned}$$

Hence $(\tilde{F_1}, A) \wedge (\tilde{F_2}, B)$ is a IVFS ordered Γ-hyperbi-ideal of S. □

THEOREM 3.4. *Let* $(\tilde{F_1}, A)$ *and* $(\tilde{F_2}, B)$ *be two IVFS ordered* Γ*-hypersubsemigroup (ideal, interior-ideal) of* S*, then* $(\tilde{F_1}, A) \cap_\epsilon (\tilde{F_2}, B)[(\tilde{F_1}, A) \cup_\epsilon (\tilde{F_2}, B)]$ *is a IVFS ordered* Γ*-hypersubsemigroup (ideal,interior-ideal) of* S*.*

PROOF. Let $(\tilde{F_1}, A)$ and $(\tilde{F_2}, B)$ be two IVFS ordered Γ-hypersubsemigroup of S as defined $(\tilde{F_1}, A) \cap_\epsilon (\tilde{F_2}, B) = (\tilde{M}, C)$ where $C = A \cup B$.

$$\tilde{M}(c) = \begin{cases} \tilde{F_1}(c) & \text{if } c \in A \setminus B \\ \tilde{F_2}(c) & \text{if } c \in B \setminus A \\ \tilde{F_1}(c) \cap \tilde{F_2}(c) & \text{if } c \in A \cap B. \end{cases}$$

Case (i) Let $c \in A \setminus B$ and $\gamma \in \Gamma$, $\inf_{z \in x\gamma y}\{\vartheta_{\tilde{M}(c)}(z)\} \geqslant \min\{\vartheta_{\tilde{M}(c)}(x), \vartheta_{\tilde{M}(c)}(y)\}$.

Case (ii) Let $c \in B \setminus A$ and $\gamma \in \Gamma$, $\inf_{z \in x\gamma y}\{\vartheta_{\tilde{M}(c)}(z)\} \geqslant \min\{\vartheta_{\tilde{M}(c)}(x), \vartheta_{\tilde{M}(c)}(y)\}$.

Case (iii) Let $c \in A \cap B$, $\gamma \in \Gamma$ and $\tilde{M}(c) = \tilde{F_1}(c) \cap \tilde{F_2}(c)$. By Theorem 3.1, $\inf_{z \in x\gamma y}\{\vartheta_{\tilde{M}(c)}(z)\} \geqslant \inf_{z \in x\gamma y}\{\vartheta_{\tilde{M}(c)}(x), \vartheta_{\tilde{M}(c)}(y)\} = \min\{\vartheta_{\tilde{M}(c)}(x), \vartheta_{\tilde{M}(c)}(y)\}$. □

THEOREM 3.5. *Let* $(\tilde{F_1}, A)$ *and* $(\tilde{F_2}, B)$ *be two IVFS ordered* Γ*- hypersubsemigroup(ideal, interior-ideal) of* S*, then* $(\tilde{F_1}, A) \cap_R (\tilde{F_2}, B)[(\tilde{F_1}, A) \cup_R (\tilde{F_2}, B)]$ *is a IVFS ordered* Γ*- hypersubsemigroup (ideal, interior-ideal) of* S*.*

PROOF. Let $(\tilde{F_1}, A)$ and $(\tilde{F_2}, B)$ be two IVFS ordered Γ- hypersubsemigroup of S, then $(\tilde{F_1}, A) \cap_R (\tilde{F_2}, B) = (\tilde{M}, C)$ where $C = A \cap B$ and $\tilde{M}(c) = \tilde{F_1}(c) \cap \tilde{F_2}(c)$

for all $c \in C$.

$$\begin{aligned}\inf_{z\in x\gamma y}\vartheta_{\tilde{M}(c)}(z) &= \inf_{z\in x\gamma y}\{\min\{\vartheta_{\tilde{F_1}(c)}(z),\vartheta_{\tilde{F_2}(c)}(z)\}\}\\ &= \min\{\inf_{z\in x\gamma y}\vartheta_{\tilde{F_1}(c)}(z),\inf_{z\in x\gamma y}\vartheta_{\tilde{F_2}(c)}(z)\}\\ &\geqslant \min\{\min\{\vartheta_{\tilde{F_1}(c)}(x),\vartheta_{\tilde{F_1}(c)}(y)\},\min\{\vartheta_{\tilde{F_2}(c)}(x),\vartheta_{\tilde{F_2}(c)}(y)\}\}\\ &= \min\{(\vartheta_{\tilde{F_1}(c)}\cap\vartheta_{\tilde{F_2}(c)})(x),(\vartheta_{\tilde{F_1}(c)}\cap\vartheta_{\tilde{F_2}(c)})(y)\}\\ &= \min\{\vartheta_{\tilde{M}(c)}(x),\vartheta_{\tilde{M}(c)}(y)\}.\end{aligned}$$

Hence $(\tilde{F_1},A)\cap_R(\tilde{F_2},B)$ is a IVFS ordered Γ- hypersubsemigroup of S. □

REMARK 3.1. Every IVFS ordered Γ-hyperideal is a IVFS ordered Γ− hypersubsemigroup but converse need not be true as in the exm.

EXAMPLE 3.1. Let $S=\{a,b,c,d\}$ and $(S,\leqslant)$ is a ordered Γ-semihypergroup.

γ	a	b	c	d
a	a	$\{b,d\}$	c	d
b	{b,d}	b	{b,d}	d
c	c	{b,d}	a	d
d	d	d	d	d

$\leqslant$: $\{(a,a),(b,b),(c,c),(d,d),(b,d)\}$. The covering relation is given by $\prec=\{(b,d)\}$. Let $E=\{x,y,u,v\}$ and $A=\{x,y\}$. Define the IVFS set $(\tilde{F},A)=\{\tilde{F}(x),\tilde{F}(y)\}$, where

$$\tilde{F}(x)=\left\{\frac{a}{[0.5,0.7]},\frac{b}{[0.7,0.8]},\frac{c}{[0.3,0.5]},\frac{d}{[0.7,0.9]}\right\},$$
$$\tilde{F}(y)=\left\{\frac{a}{[0.4,0.6]},\frac{b}{[0.6,0.7]},\frac{c}{[0.1,0.5]},\frac{d}{[0.6,0.8]}\right\}$$

Hence $(\tilde{F},A)$ is a IVFS sub Γ- hypersemigroups but not a IVFS hyperideal, since

$$\inf_{x\in a\gamma c}\vartheta_{\tilde{F}(e)}(x)=\inf_{x\in a\gamma c}\vartheta_{\tilde{F}(e)}(c)=[0.3,0.5]\ngeq[0.5,0.7]$$
$$=\max\{[0.5,0.7],[0.3,0.5]\}=\max\{\vartheta_{\tilde{F}(e)}(a),\vartheta_{\tilde{F}(e)}(c)\}$$

REMARK 3.2. Every IVFS ordered Γ-hyperideal is IVFS ordered Γ-hyperbi-ideals but converse need not be true as in the exm.

EXAMPLE 3.2. Let $S=\{a,b,c,d\}$ and $\Gamma=\{\alpha,\beta\}$ then S is Γ-hypersemigroup.

α	a	b	c	d	β	a	b	c	d
a	a	$\{a,b\}$	{c, d}	d	a	a	$\{a,b\}$	{c, d}	d
b	{a, b}	b	{c, d}	d	b	{a, b}	$\{a,b\}$	{c, d}	d
c	{c, d}	{c, d}	c	d	c	{c, d}	{c, d}	c	d
d	d	d	d	d	d	d	d	d	d

$\leqslant$: $\{(a,a),(a,b),(b,b),(c,b),(c,c),(c,d),(d,b),(d,d)\}$. The covering relation is given by $\prec=\{(a,b),(c,d),(d,b)\}$. Let $E=\{x,y,u,v\}$ and $A=\{x,y\}$. Define the IVFS set $(\tilde{F},A)$, where

$$\tilde{F}(x) = \left\{ \frac{a}{[0.2,0.4]}, \frac{b}{[0.2,0.5]}, \frac{c}{[0.6,0.8]}, \frac{d}{[0.5,0.7]} \right\},$$
$$\tilde{F}(y) = \left\{ \frac{a}{[0.4,0.6]}, \frac{b}{[0.4,0.7]}, \frac{c}{[0.8,0.9]}, \frac{d}{[0.7,0.8]} \right\}$$

Hence $(\tilde{F}, A)$ is a IVFS ordered Γ-hyperbi-ideal but not a IVFS ordered Γ-hyperideal. Since

$$\inf_{x \in b\alpha c} \vartheta_{\tilde{F}(e)}(x) = \inf_{x \in b\alpha c} \vartheta_{\tilde{F}(e)}(d) = [0.5, 0.7] \ngeq [0.6, 0.8]$$
$$= \max\{[0.2, 0.5], [0.6, 0.8]\} = \max\{\vartheta_{\tilde{F}(e)}(b), \vartheta_{\tilde{F}(e)}(c)\}$$

REMARK 3.3. Every IVFS Γ−hyperideal is IVFS Γ− hyper interior-ideal but converse not true.

EXAMPLE 3.3. Let $S = \{a, b, c, d, e\}$, $\Gamma = \{\alpha, \beta\}$ then S is Γ−hypersemigroup

α	a	b	c	d	e
a	$\{a, b\}$	$\{b, e\}$	c	{c, d}	e
b	{b, e}	e	c	{c, d}	e
c	c	c	c	c	c
d	{c, d}	{c,d}	c	d	{c, d}
e	e	e	c	{c, d}	e

β	a	b	c	d	e
a	$\{b, e\}$	e	c	{c, d}	e
b	e	e	c	{c, d}	e
c	c	c	c	c	c
d	{c, d}	{c,d}	c	d	{c, d}
e	e	e	c	{c, d}	e

$\leqslant: \{(a, a), (b, a), (b, b), (c, c), (c, e), (d, d), (e, b), (e, e)\}$. The covering relation is given by $\prec = \{(b, a), (c, e), (e, b)\}$. Define the IVFS set $(\tilde{F}, A) = \{\tilde{F}(w), \tilde{F}(x), \tilde{F}(y)\}$, where

$$\tilde{F}(w) = \{(a, [0.2, 0.4]), (b, [0.5, 0.7]), (c, [0.8, 0.9]), (d, [0.1, 0.2]), (e, [0.7, 0.8])\}$$
$$\tilde{F}(x) = \{(a, [0.5, 0.6]), (b, [0.6, 0.7]), (c, [0.9, 1]), (d, [0.3, 0.5]), (e, [0.7, 0.8])\}$$
$$\tilde{F}(y) = \{(a, [0.2, 0.4]), (b, [0.4, 0.6]), (c, [0.6, 0.8]), (d, [0.1, 0.3]), (e, [0.5, 0.7])\}$$

Hence $(\tilde{F}, A)$ is a IVFS Γ−hyperinterior ideal but not a IVFS Γ−hyperideal as

$$\inf_{x \in b\alpha d} \vartheta_{\tilde{F}(e)}(x) = \inf_{x \in b\alpha d} \vartheta_{\tilde{F}(e)}(d) = [0.3, 0.5] \ngeq [0.6, 0.7] =$$
$$\max\{[0.6, 0.7], [0.3, 0.5]\} = \max\{\vartheta_{\tilde{F}(e)}(b), \vartheta_{\tilde{F}(e)}(d)\}.$$

DEFINITION 3.3. Let $(\tilde{F}, A)$ be a interval valued fuzzy soft set over S. For each $t \in [0, 1]$, the set $(\tilde{F}, A)^t$ is called an t-level set of $(\tilde{F}, A)$ and is defined as $\{x \in S | \tilde{F}(e)(x) \geqslant t\}$ for each $e \in A$. Obviously, $(\tilde{F}, A)^t$ is a soft set over S.

THEOREM 3.6. *Let $(\tilde{F}, A)$ be a IVFS set of S, $(\tilde{F}, A)$ is a IVFS ordered Γ-hypersemigroup if and only if $(\tilde{F}, A)^t$ is a soft ordered Γ-hypersemigroup of S for each $t \in D[0, 1]$.*

PROOF. Assume that $(\tilde{F}, A)^t$ is a soft ordered Γ-hypersemigroup of S for each $t \in D[0, 1]$. For each $x_1, x_2 \in S$ and $a \in A$, let $t = \min\{\vartheta_{\tilde{F}(a)}(x_1), \vartheta_{\tilde{F}(a)}(x_2)\}$ then $x_1\gamma x_2 \in \vartheta^t_{\tilde{F}(a)}$. Since $\vartheta^t_{\tilde{F}(a)}$ is a Γ- hypersubsemigroup of S, then $x_1\gamma x_2 \in \vartheta^t_{\tilde{F}(a)}$. That is $\vartheta_{\tilde{F}(a)}(x_1\gamma x_2) \geqslant t = \min\{\vartheta_{\tilde{F}(a)}(x_1), \vartheta_{\tilde{F}(a)}(x_2)\}$. This shows that $\vartheta_{F(a)}$ is IVF ordered Γ-hypersubsemigroup of S. Conversely, assume that $(\tilde{F}, A)$ is a IVFS in S. For each $a \in A, t \in D[0, 1]$ and $x_1, x_2 \in \vartheta^t_{\tilde{F}(a)}$, we have $\vartheta_{\tilde{F}(a)}(x_1) \geqslant$

$t, \vartheta_{\tilde{F}(a)}(x_2) \geqslant t$. Therefore $\vartheta_{F(a)}$ is a IVF ordered Γ-hypersubsemigroup of S. Thus $\gamma \in \Gamma$ there exists $z \in x_1\gamma x_2$ such that $\inf\limits_{z \in x_1\gamma x_2}(z) \geqslant \min\{\vartheta_{\tilde{F}(a)}(x_1), \vartheta_{\tilde{F}(a)}(x_2)\} \geqslant t$. Therefore for all $z \in x_1\gamma x_2$ we have $z \in \vartheta^t_{\tilde{F}(a)}$, this implies that $x_1\gamma x_2 \in \vartheta^t_{\tilde{F}(a)}$, that is $\vartheta^t_{\tilde{F}(a)}$ is hyper Γ-subsemigroup of S. Therefore $(\tilde{F}, A)^t$ is a soft ordered Γ-hypersemigroup of S for each $t \in D[0,1]$. □

THEOREM 3.7. *Let $(\tilde{F}, A)$ be a IVFS set of S, $(\tilde{F}, A)$ is a IVFS ordered Γ-hyperleft(right, bi-ideal)ideal if and only if $(\tilde{F}, A)^t$ is a soft ordered Γ-hyper left(right, bi-ideal) ideal of S for each $t \in D[0,1]$.*

PROOF. Suppose that $(\tilde{F}, A)^t$ is a soft ordered Γ-hyper left ideal of S for each $t \in D[0,1]$ and $a \in A, \gamma \in \Gamma$. For each $x_1 \in S$, let $t = \vartheta_{\tilde{F}(a)}(x_1)$, then $x_1 \in \vartheta^t_{\tilde{F}(a)}$. Since $\vartheta^t_{\tilde{F}(a)}$ is a Γ- hyper left ideal of S, then $x\gamma x_1 \in \vartheta^t_{\tilde{F}(a)}$, for each $x \in S$. Hence $\vartheta_{\tilde{F}(a)}(x\gamma x_1) \geqslant t = \vartheta_{\tilde{F}(a)}(x_1)$. This shows that $\vartheta_{F(a)}$ is IVF ordered Γ-hyper left ideal of S. Thus, $(\tilde{F}, A)$ is a IVFS ordered Γ-hyper left ideal of S.

Conversely, assume that $(\tilde{F}, A)$ is a IVFS ordered Γ-hyper left ideal of S. For each $a \in A, t \in D[0,1]$ and $x_1 \in \vartheta^t_{\tilde{F}(a)}$, we have $\vartheta_{\tilde{F}(a)}(x_1) \geqslant t$ and hence $\vartheta_{\tilde{F}(a)}$ is a IVF Γ-hyper left ideal of S. Thus for $\gamma \in \Gamma$ there exists $z \in x\gamma x_1$ such that $\inf\limits_{z \in x\gamma x_1}(z) \geqslant \vartheta_{\tilde{F}(a)}(x_1) \geqslant t$. Therefore for all $z \in x\gamma x_1$. We have $z \in \vartheta^t_{\tilde{F}(a)}$, that is $\vartheta^t_{\tilde{F}(a)}$ is hyper ordered Γ-left ideal of S. Therefore $(\tilde{F}, A)^t$ is a soft ordered Γ-hyper left ideal of S for each $t \in D[0,1]$. Similar proof holds for other case. □

THEOREM 3.8. *Let $(\tilde{F}, A)$ be a IVFS set of S, $(\tilde{F}, A)$ is a IVFS ordered Γ-hyper ideal if and only if $(\tilde{F}, A)^t$ is a soft ordered Γ-hyper ideal of S for each $t \in D[0,1]$.*

4. IVFS image and inverse image of $\Gamma-$ hypersemigroups

DEFINITION 4.1. Let $\rho : H_1 \to H_2$ and $\sigma : A \to B$ be two functions, A and B be two parameter sets for the crisp sets H_1 and H_2,respectively. Then the pair (ρ, σ) is called a IVFS function from H_1 to H_1.

DEFINITION 4.2. Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two IVFS sets of the sets H_1 and H_2, respectively, and (ρ, σ) be a IVFS map from H_1 to H_2. (i) The image of $(\tilde{F}, A)$ under (ρ, σ) denoted by $(\rho, \sigma)(\tilde{F}, A)$, is a IVFS set of H_2 defined by $(\rho, \sigma)(\tilde{F}, A) = (\rho(F), \sigma(A))$, where for all $b \in \sigma(A)$ and for all $y \in H_2$,

$$\vartheta_{\rho_{\tilde{F}(b)}}(y) = \begin{cases} \sup\limits_{\rho(x)=y} \sup\limits_{\sigma(a)=b} \vartheta_{\tilde{F}(a)}(x), \; if \; \rho^{-1}(y) \neq \phi \\ 0 \quad \text{otherwise} \end{cases}$$

(ii) The inverse image of $(\tilde{G}, B)$ under (ρ, σ) denoted by $(\rho, \sigma)^{-1}(\tilde{G}, B)$ is a IVFS set of H_1 defined by $(\rho, \sigma)^{-1}(\tilde{G}, B) = (\rho^{-1}(G), \sigma^{-1}(B))$, for all $a \in \sigma^{-1}(B)$ and $x \in H_1$, $\vartheta_{\rho^{-1}_{\tilde{G}(a)}}(y) = \vartheta_{\tilde{G}_{\sigma(a)}}(\rho(x))$

THEOREM 4.1. *Let $\rho : H_1 \to H_2$ be a homomorphism of S. If $(\tilde{G}, B)$ is a IVFS $\Gamma-$hypersubsemigroup of H_2, then $(\rho,\sigma)^{-1}(\tilde{G}, B)$ is a IVFS $\Gamma-$hypersubsemigroup of H_1.*

PROOF. Let $(\tilde{G}, B)$ is a IVFS $\Gamma-$hypersubsemigroup of H_2. Let $x, y, z \in H_1$, $\alpha \in \Gamma_1$. We have

$$\begin{aligned}\inf_{z\in x\alpha y}\left\{\vartheta_{\rho^{-1}_{\tilde{G}(a)}}(z)\right\} &= \inf_{z\in x\alpha y}\left\{\vartheta_{\tilde{g}_{\sigma(a)}}(\rho(z))\right\}\\ &= \inf_{\rho(z)\in\rho(x\alpha y)}\left\{\vartheta_{\tilde{g}_{\sigma(a)}}(\rho(z))\right\}\\ &\geqslant \min\left\{\vartheta_{\tilde{g}_{\sigma(a)}}\rho(x), \vartheta_{\tilde{g}_{\sigma(a)}}\rho(y)\right\}\\ &= \min\left\{\vartheta_{\rho^{-1}_{\tilde{G}(a)}}(x), \vartheta_{\rho^{-1}_{\tilde{G}(a)}}(y)\right\}\end{aligned}$$

Therefore $(\rho,\sigma)^{-1}(\tilde{G}, B)$ is a IVFS $\Gamma-$hypersubsemigroup of H_1. □

THEOREM 4.2. *Let $\rho : H_1 \to H_2$ be a homomorphism of S. If $(\tilde{G}, B)$ is a IVFS $\Gamma-$hyperleft(right, bi-ideal, interior) of H_2, then $(\rho,\sigma)^{-1}(\tilde{G}, B)$ is a IVFS $\Gamma-$hyperleft(right, bi-ideal, interior)ideal of H_1.*

THEOREM 4.3. *Let $\rho : H_1 \to H_2$ be a homomorphism of S. If $(\tilde{F}, A)$ is a IVFS $\Gamma-$hypersubsemigroup of H_1, then $(\rho,\sigma)(\tilde{F}, A)$ is a IVFS $\Gamma-$hypersubsemigroup of H_2.*

PROOF. Let $(\tilde{F}, A)$ is a IVFS $\Gamma-$hypersubsemigroup of H_1.
Let $x_1, y_1,\ z_1 \in H_2$, $\beta \in \Gamma_2$.

$$\begin{aligned}\inf_{z_1\in x_1\beta y_1}\left\{\vartheta_{\rho_{\tilde{F}(b)}}(z_1)\right\} &= \inf_{z_1\in x_1\beta y_1}\left\{\sup_{t\in\rho^{-1}(z_1)}\sup_{\sigma(a)=b}\vartheta_{\tilde{F}(a)}(t)\right\}\\ &\geqslant \inf_{z\in x_1\beta y_1}\left\{\sup_{\sigma(a)=b}\vartheta_{\tilde{F}(b)}(z_1)\right\}\\ &= \inf_{\rho(z)\in\rho(x\beta y)}\left\{\sup_{\sigma(a)=b}\vartheta_{\tilde{F}(b)}(z)\right\}\\ &\geqslant \sup_{\sigma(a)=b}\min\left\{\vartheta_{\tilde{F}(b)}(x), \vartheta_{\tilde{F}(b)}(y)\right\}\\ &\geqslant \sup_{x\beta y\subseteq\rho^{-1}(x_1)h^{-1}(\beta)\rho^{-1}(y_1)}\left\{\sup_{\sigma(a)=b}\min\left\{\vartheta_{\tilde{F}(b)}(x), \vartheta_{\tilde{F}(b)}(y)\right\}\right\}\\ &= \min\left\{\sup_{\rho(x)=y}\sup_{\sigma(a)=b}\vartheta_{\tilde{F}(a)}(x),\ \sup_{\rho(x)=y}\sup_{\sigma(a)=b}\vartheta_{\tilde{F}(a)}(y)\right\}\\ &\geqslant \min\left\{\vartheta_{\rho_{\tilde{F}(b)}}(x_1), \vartheta_{\rho_{\tilde{F}(b)}}(y_1)\right\}\end{aligned}$$

Therefore $(\rho,\sigma)(\tilde{F}, A)$ is a IVFS $\Gamma-$hyper subsemigroup of H_2. □

THEOREM 4.4. *Let* $\rho : H_1 \to H_2$ *be a homomorphism of* S. *If* $(\tilde{F}, A)$ *is a IVFS* $\Gamma-$*hyperleft(right, bi-ideal, interior)ideal of* H_1, *then* $(\rho, \sigma)(\tilde{F}, A)$ *is a IVFS* $\Gamma-$*hyperleft(right, bi-ideal, interior)ideal of* H_2.

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