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Γ - GROUP

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Abstract. We introduce the notion of $\Gamma-$group, regular $\Gamma-$group and study their properties.We prove that M is a $\Gamma-$group if and only if M is an integral, simple commutative $\Gamma-$semigroup.

1. Introduction

As a generalization of ring, the notion of a $\Gamma-$ring was introduced by Nobusawa [**12**] in 1964. The notion of a ternary algebraic system was introduced by Lehmer [**1**] in 1932. Lister [**2**] introduced ternary rings. In 1995, Murali Krishna Rao [**3, 4, 5, 6, 7**] introduced the notion of a $\Gamma-$semiring as a generalization of $\Gamma-$ring, ring, ternary semiring and semiring. Murali Krishna Rao [**14, 15**] introduced the notion of field $\Gamma-$semiring and $\Gamma-$field. Semigroup, as the basic algebraic structure was used in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages. The formal study of semigroups begin in the early 20th century. In 1981, Sen [**19**] introduced the notion of a $\Gamma-$semigroup as a generalization of semigroup. Murali Krishna Rao [**7, 8, 9, 10**] studied ideals of $\Gamma-$semirings, semirings, semigroups and $\Gamma-$semigroups. V. Neumann [**11**] studied regular rings.

In this paper,we introduce the notion of a $\Gamma-$group as a generalization of group. We study some of the properties of a $\Gamma-$group.

2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier.

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Definition 2.1. A semigroup is an algebraic system $(M, \cdot)$ consisting of a non-empty set M together with an associative binary operation $' \cdot '$.

Definition 2.2. An algebraic system $(M, \cdot)$ consisting of a non-empty set M together with an associative binary operation $' \cdot '$ is called a group if it satisfies the following

(i) there exists $e \in M$, such that $x \cdot e = e \cdot x = x$, for all $x \in M$.
(ii) if for each $x \in M$ there exists $b \in M$, such that $x \cdot b = b \cdot x = e$.

Definition 2.3. A subsemigroup T of a semigroup M is a non-empty subset T of M such that $TT \subseteq T$.

Definition 2.4. A non-empty subset T of a semigroup M is called a left (right) ideal of M if $MT \subseteq T$ $(TM \subseteq T)$.

Definition 2.5. A non-empty subset T of a semigroup M is called an ideal of M if it is both a left ideal and a right ideal of M.

Definition 2.6. An element a of a semigroup M is called a regular element if there exists an element b of M such that $a = aba$.

Definition 2.7. A semigroup M is called a regular semigroup if every element of M is a regular element.

Definition 2.8. Let M and Γ be non-empty sets. Then we call M a $\Gamma-$semigroup if there exists a mapping $M \times \Gamma \times M \to M$ (images of (x, α, y) will be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$), such that it satisfies

$$x\alpha(y\beta z) = (x\alpha y)\beta z \text{ for all } x, y, z \in M \text{ and } \alpha, \beta \in \Gamma.$$

Definition 2.9. A non-empty subset A of a $\Gamma-$semigroup M is called

(i) a $\Gamma-$subsemigroup of M if $A\Gamma A \subseteq A$.
(ii) a quasi ideal of M if $A\Gamma M \cap M\Gamma A \subseteq A$.
(iii) a bi-ideal of M if $A\Gamma M\Gamma A \subseteq A$.
(iv) an interior ideal of M if $M\Gamma A\Gamma M \subseteq A$.
(v) a left (right) ideal of M if $M\Gamma A \subseteq A$ $(A\Gamma M \subseteq A)$.
(vi) an ideal if $A\Gamma M \subseteq A$ and $M\Gamma A \subseteq A$.

Definition 2.10. A $\Gamma-$semigroup M is said to be commutative if $a\alpha b = b\alpha a$, for all $a, b \in M$, for all $\alpha \in \Gamma$.

Definition 2.11. Let M be a $\Gamma-$semigroup. An element $a \in M$ is said to be an idempotent of M if there exist $\alpha \in \Gamma$, such that $a = a\alpha a$ and a is also said to be α idempotent.

Definition 2.12. Let M be a $\Gamma-$semigroup. If every element of M is an idempotent of M, then $\Gamma-$semigroup M is said to be band.

Definition 2.13. Let M be a $\Gamma-$semigroup. An element $a \in M$ is said to be regular element of M if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 2.14. Let M be a $\Gamma-$semigroup. Every element of M is a regular element of M then M is said to be a regular $\Gamma-$semigroup M.

DEFINITION 2.15. A $\Gamma-$semigroup M is said to be left (right) singular if for each $a \in M$ there exist $\alpha \in \Gamma, b \in M$ such that $a\alpha b = a(a\alpha b = b)$.

DEFINITION 2.16. Let M be a $\Gamma-$semigroup. An element $a \in M$ is said to be regular element of M if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

DEFINITION 2.17. Let M be a $\Gamma-$semigroup. Every element of M is a regular element of M then M is said to be a regular $\Gamma-$semigroup M.

DEFINITION 2.18. Let M be a $\Gamma-$semigroup and $\alpha \in \Gamma$. Define a binary operation $*$ on M by $a * b = a\alpha b$, for all $a, b \in M$. Then $(M, *)$ is a semigroup. It is denoted by M_α.

DEFINITION 2.19. ([**19**]) A $\Gamma-$semigroup M is called a $\Gamma-$ group, if M_j is a group for some (hence for all) $j \in \Gamma$.

3. $\Gamma-$group

In this section, we introduce the notion of a unity element of a $\Gamma-$ semigroup, an inverse element of a $\Gamma-$semigroup, $\Gamma-$group, simple $\Gamma-$group and we study the properties of $\Gamma-$group.

DEFINITION 3.1. Let M be a $\Gamma-$semigroup. An element $1 \in M$ is said to be unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$.

DEFINITION 3.2. In a $\Gamma-$semigroup M with unity 1, an element $a \in M$ is said to be left invertible (right invertible) if there exist $b \in M, \alpha \in \Gamma$ such that $b\alpha a = 1(a\alpha b = 1)$.

DEFINITION 3.3. In a $\Gamma-$semigroup M, an element $u \in M$ is said to be unit if there exist $a \in M$ and $\alpha \in \Gamma$, such that $a\alpha u = 1 = u\alpha a$.

DEFINITION 3.4. A $\Gamma-$semigroup M is said to be simple $\Gamma-$semigroup if it has no proper ideals of M.

DEFINITION 3.5. A non-zero element a in a $\Gamma-$semigroup M is said to be zero divisor if there exist a non zero element $b \in M, \alpha \in \Gamma$ such that $a\alpha b = b\alpha a = 0$.

DEFINITION 3.6. A $\Gamma-$semigroup M with unity 1 and zero element 0 is called an integral $\Gamma-$semigroup if it has no zero divisors.

DEFINITION 3.7. A $\Gamma-$semigroup M with zero element 0 is said to be hold cancellation laws if $a \neq 0$, $a\alpha b = a\alpha c$, $b\alpha a = c\alpha a$, where $a, b, c \in M, \alpha \in \Gamma$ then $b = c$.

DEFINITION 3.8. Let M be a $\Gamma-$semigroup with unity 1 and zero element 0 is called a pre -integral $\Gamma-$semigroup if M holds cancellation laws.

DEFINITION 3.9. A $\Gamma-$semigroup M is said to be $\Gamma-$group if it satisfies the following

(i) if there exists $1 \in M$ and for each $x \in M$ there exists $\alpha \in \Gamma$, such that $x\alpha 1 = 1\alpha x = x$.

(ii) if for each element $0 \neq a \in M$ there exist $b \in M, \alpha \in \Gamma$ such that $a\alpha b = b\alpha a = 1$.

Every group M is a $\Gamma-$group if Γ =M and ternary operation is $x\gamma y$ defined as the binary operation of the group.

Example 3.1. Let M and Γ be the set of all rational numbers and the set of all natural numbers respectively. Define the ternary operation $M \times \Gamma \times M \to M$ by $(a, \alpha, b) \to a\alpha b$, using the usual multiplication. Then M is a $\Gamma-$group by the Definition 3.9.

Remark 3.1. In the Example 3.1, M is a $\Gamma-$group by the Definition 2.19 and M is also a $\Gamma-$group by the Definition 3.9.

Example 3.2. Let $M = \{0, 1\}$ and $\Gamma = \{\alpha, \beta\}$. We define operations with the following tables:

α	0	1
0	0	0
1	1	1

β	0	1
0	0	0
1	0	1

.

Then M is a $\Gamma-$semigroup and M is a $\Gamma-$group.

Example 3.3. Let $M = \{0, 1\}$ and $\Gamma = \{\alpha, \beta\}$. We define operations with the following tables:

α	0	1
0	0	0
1	0	1

β	0	1
0	0	1
1	1	0

.

Then M is a $\Gamma-$group by the Definition 3.9.

Remark 3.2. In the example 3.3, M is not a $\Gamma-$group by the Definition 2.19, since M_α and M_β are not groups.

Theorem 3.1. *Let M be a $\Gamma-$semigroup with unity. A $\Gamma-$semigroup M with unity is a $\Gamma-$group if and only if for any non-zero elements $a, b \in M$ and for each $\alpha \in \Gamma$ there exmist $x \in M, \beta \in \Gamma$such that $a\alpha x\beta b = b$.*

Proof. Let M be a $\Gamma-$group and a, b be non-zero elements of M. Then there exists $\beta \in \Gamma$, such that $1\beta b = b$ and there exist $\alpha \in \Gamma$, $x \in M$ such that $a\alpha x = 1$. Then $a\alpha x = 1$. Thus $a\alpha x\beta b = 1\beta b$ and $a\alpha x\beta b = b$.

Conversely suppose that for any non-zero elements $a, b \in M$ and for each $\alpha \in \Gamma$ there exist $x \in M, \beta \in \Gamma$ such that $a\alpha x\beta b = b$. Let $0 \neq a, 1 \in M, \alpha \in \Gamma$ then there exists $x \in M, \beta \in \Gamma$ such that $a\alpha x\beta 1 = 1$ implies $a\alpha(x\beta 1) = 1$. Hence M is a $\Gamma-$group. □

Theorem 3.2. *Let M be a $\Gamma-$semigroup with unity* 1. *If $a, b \in M, \delta, \beta \in \Gamma$ such that $a\delta b$ is $\beta-$ idempotent and a is left invertible. Then b is a regular element.*

Proof. Let $a, b \in M$ and a be left invertible. This means that there exist $d \in M, \delta, \gamma \in \Gamma$ such that $1\delta b = b$ and $d\gamma a = 1$. Then $d\gamma a = 1$ implies $d\gamma a\delta b = 1\delta b$ and $d\gamma a\delta b = b$. Suppose $a\delta b$ is β-idempotent. Thus $a\delta b\beta a\delta b = a\delta b$ and $d\gamma a\delta b\beta a\delta b = d\gamma a\delta b$. So, $b\beta a\delta b = b$. Therefore, b is a regular element. □

The proof of the following theorem is similar to above theorem.

THEOREM 3.3. *Let M be a $\Gamma-$semigroup with unity* 1. *If $a, b \in M, \delta, \beta \in \Gamma$ such that $a\delta b$ is $\beta-$ idempotent and b is right invertible, then a is regular.*

THEOREM 3.4. *If M is a $\Gamma-$semigroup with unity and $a \in M$ is left invertible, then a is a regular.*

PROOF. Let M be a $\Gamma-$semigroup with unity 1. Suppose $a \in M$ is left invertible, there exist $b \in M, \alpha \in \Gamma$, such that $b\alpha a = 1$. Since 1 is unity, there exists $\delta \in \Gamma$ such that $a\delta 1 = 1\delta a = a$. Then from $a\delta 1 = a$ implies $a\delta(b\alpha a) = a.$. Thus $a\delta b\alpha a = a$. So, a is a regular element. □

COROLLARY 3.1. *If M is a $\Gamma-$semigroup with unity and $a \in M$ is invertible, then a is regular.*

THEOREM 3.5. *If M is a $\Gamma-$group, then M is a regular $\Gamma-$group.*

PROOF. Let M be a $\Gamma-$group. Then each non zero element is invertible. By Corollary 3.1, each element is a regular. Therefore M is a regular $\Gamma-$group. □

THEOREM 3.6. *A $\Gamma-$group is a pre-integral $\Gamma-$ semigroup.*

PROOF. Let M be a $\Gamma-$group. Suppose $a \neq 0$ and $a\alpha b = a\alpha c$, where $a, b, c \in M, \alpha \in \Gamma$. Then there exist $\delta, \beta \in \Gamma$ such that $1\delta b = b$ and $1\beta c = 1$. Thus $a\alpha(1\delta b) = a\alpha\,(1\beta c)$ and $(a\alpha 1)\,\delta b = (a\alpha 1)\,\beta c$. Since $a\alpha 1 \neq 0$, there exist $\gamma \in \Gamma, d \in M$ such that $d\gamma(a\alpha 1) = 1$. From this follows $d\gamma(a\alpha 1)\,\delta b = d\gamma(a\alpha 1)\,\beta c$. Further on, we have $1\delta b = 1\beta c$ and $b = c$. Hence $\Gamma-$group M is a pre-integral $\Gamma-$semigroup. □

THEOREM 3.7. *A $\Gamma-$ group is an integral $\Gamma-$semigroup.*

PROOF. Let M be al $\Gamma-$group. Then $\Gamma-$group M is a pre-integral $\Gamma-$ semigroup by Theorem 3.6. Suppose $a, b \in M$, $\alpha \in \Gamma$, $a\alpha b = 0$, $b \neq 0$. Thus $a\alpha b = 0\alpha b$ and $a = 0$. Hence M is an integral $\Gamma-$semigroup. □

THEOREM 3.8. *If M is a $\Gamma-$group, then the equation $a\alpha x = b$ has a unique solution for any non-zero elements $a, b \in M$ and $\alpha \in \Gamma$.*

PROOF. Let M be a $\Gamma-$group and the equation $a\alpha x = b$ for any non-zero elements $a, b \in M$ and $\alpha \in \Gamma$. Then there exist $c \in M$, $\beta \in \Gamma$, such that $1\beta b = b$ and $a\alpha c = 1$. Now, from $a\alpha c = 1$ follows $a\alpha c\beta b = 1\beta b$ and $a\alpha(c\beta b) = b$. Suppose there exist $x, y \in M$, such that $a\alpha x = b$ and $a\alpha y = b$. Then $a\alpha x = a\alpha y$. Therefore $x = y$ by Theorem 3.21. This completes the proof. □

COROLLARY 3.2. *If M is a $\Gamma-$group, then the equation $x\alpha a = b$ has a unique solution for any non-zero elements $a, b \in M$ and $\alpha \in \Gamma$.*

THEOREM 3.9. *Any commutative finite pre-integral $\Gamma-$semigroup M is a $\Gamma-$group M.*

Proof. Let $M = \{a_1, a_2, \cdots, a_n\}$ and $0 \neq a \in M$, $\alpha \in \Gamma$. We consider the n products $a\alpha a_1, a\alpha a_2, \cdots, a\alpha a_n$. These products are all distinct. Since $a\alpha a_i = a\alpha a_j$ we have $a_i = a_j$. Since $1 \in M$, there exists $a_i \in M$ such that $a\alpha a_i = 1$. Therefore a has inverse. Hence any commutative finite pre-integral $\Gamma-$semigroup M is a $\Gamma-$group. □

Theorem 3.10. *Let M be a $\Gamma-$group with zero element 0. If I is an ideal of $\Gamma-$group M containing a unit element then $I = M$.*

Proof. Let I be an ideal of the $\Gamma-$group M containing a unit element u. Let $x \in M$. Then there exists $\alpha \in \Gamma$ such that $x\alpha 1 = x$ and $x\alpha u \in I$, since I is an ideal. Since u is a unit element, there exist $\delta \in \Gamma$, $t \in M$ such that $u\delta t = 1 \Rightarrow \; x\alpha u\delta t = x\alpha 1 = x \in I$. Hence $I = M$. □

Theorem 3.11. *Every $\Gamma-$group with zero element 0. is an integral $\Gamma-$semigroup.*

Proof. Let $a, b \in M$ and $a\alpha b = 0, \alpha \in \Gamma$ and $a \neq 0$. Since $a \neq 0$ there exists $\beta \in \Gamma$ such that $a^{-1}\beta a = 1$. Thus $a\alpha b = 0$ implies $a^{-1}\beta(a\alpha b) = a^{-1}\beta 0$ and $(a^{-1}\beta a)\,\alpha b = 0$. So, $1\alpha b = 0 = 1\alpha 0$. Therefore $b = 0$. Hence M is an integral $\Gamma-$semigroup. □

Theorem 3.12. *M is a $\Gamma-$group if and only if M is an integral, simple commutative $\Gamma-$semigroup.*

Proof. Let M be a $\Gamma-$group. Let I be a proper ideal of $\Gamma-$group M. Every non-zero element of M is a unit. By Theorem 3.10, we have $I = M$. Therefore $\Gamma-$group M contains no proper ideals. Hence $\Gamma-$group is a simple $\Gamma-$ semigroup. By Theorem 3.11, M is an integral $\Gamma-$ semigroup.

Conversely suppose that M is an integral, simple commutative $\Gamma-$semigroup. Let $0 \neq a \in M, \alpha \in \Gamma$. Consider $a\alpha M, a\alpha M \neq \{0\}$, since M is an integral $\Gamma-$semigroup. Clearly $a\alpha M$ is a proper ideal of $M \;\Rightarrow\; a\alpha M = M$, since M is a simple $\Gamma-$semigroup. Therefore, there exmists $b \in M$ such that $a\alpha b = 1$. Hence the theorem. □

Theorem 3.13. *Let M be a commutative $\Gamma-$semgroup. M satisfies the condition,for each, $0 \neq a \in M, \alpha \in \Gamma$ and $d \in M$. Then there exist $b \in M, \beta \in \Gamma$ such that $a\alpha b\beta d = dl$ if and only if M is a $\Gamma-$group.*

Proof. Let M be a commutative $\Gamma-$semgroup. Suppose M is a $\Gamma-$group, $0 \neq a \in M$ and $c \in M$. Since M is a $\Gamma-$group, there exist $b \in M, \alpha \in \Gamma$ such that $a\alpha b = 1$. Since 1 is the unity element, there exists $\beta \in \Gamma$ such that $1\beta c = c$. Therefore $a\alpha b\beta c = 1\beta c \Rightarrow a\alpha b\beta c = c$. Finally, M is a $\Gamma-$group.

Conversely suppose that M is a commutative $\Gamma-$semigroup, satisfies the condition,for each $0 \neq a \in M, \alpha \in \Gamma$ then there exmist $b \in M, \beta \in \Gamma$ such that $a\alpha b\beta d = d$, for all $d \in M$.. Let $0 \neq a \in M, \alpha \in \Gamma$ and $d \in M$. Then there exists $\beta \in \Gamma$ such that $a\alpha b\beta d = d$. Therefore $a\alpha b = 1$. Hence every non-zero element of M has inverse. Thus M is a $\Gamma-$group. □

THEOREM 3.14. *Let M be a $\Gamma-$semigroup with zero element. Then M is a $\Gamma-$group if and only if $\Gamma-$group M is zero divisors free and $\Gamma-$semigroup $M \setminus \{0\}$ has no proper ideals.*

PROOF. Suppose M is a $\Gamma-$group. By Theorem 3.11, M is zero divisors free. Let I be an ideal of the $\Gamma-$group $M \setminus \{0\}$ and $a \in I$. Since $0 \neq a \in I$, there exist $\alpha \in \Gamma, x \in M$ such that $a\alpha x = 1$. Therefore $1 \in I$. Let $x \in M \setminus \{0\}$. Then $x\alpha 1 \in I$, for all $\alpha \in \Gamma \Rightarrow x \in I$. Therefore $M \setminus \{0\} = I$. Thus $\Gamma-$group $M \setminus \{0\}$ has no proper ideals.

Conversely suppose that $\Gamma-$group M is zero divisors free and $\Gamma-$group $M \setminus \{0\}$ has no proper ideals. Let $0 \neq a \in M, \alpha \in \Gamma$. Consider $a\alpha M \neq \{0\}$. Then $a\alpha M = M$. Therefore there exmists $b \in M$ such that $a\alpha b = 1$. Hence M is a $\Gamma-$group. □

THEOREM 3.15. *M is a $\Gamma-$group if and only if M_α is a group for some $\alpha \in \Gamma$, then M_β is a group for all $\beta \in \Gamma$.*

PROOF. Let M be a $\Gamma-$group.Suppose M_α is a group for some $\alpha \in \Gamma$,and $a \in M \setminus \{0\}$ and $\alpha \in \Gamma$. Suppose $b \in M \setminus \{0\}, \beta \in \Gamma$, Then $a\beta b \neq 0$. By definition of the group, we have $(a\beta b)\alpha c = 1, c \in M$ and $a\beta(b\alpha c) = 1$. Hence M_β is a group.

Converse is obvious. □

4. Conclusion

We introduced the notion of $\Gamma-$group, regular $\Gamma-$group, integral $\Gamma-$semigroup, simple $\Gamma-$semigroup and pre integral $\Gamma-$semigroup and studied their properties and relations between them. We proved that M is a $\Gamma-$group if and only if M is an integral, simple and commutative $\Gamma-$semigroup and if M is a $\Gamma-$group, then the equation $a\alpha x = b$ has a unique solution for any non-zero elements $a, b \in M$ and for $\alpha \in \Gamma$.

References

[1] H. Lehmer. A ternary analogue of abelian groups. *Am. J. Math.*, **54**(2)(1932), 329–338

[2] W. G. Lister. Ternary rings. *Tran. Am. Math. Soc.*, **154** (1971), 37–55.

[3] M. M. Krishna Rao. $\Gamma-$semirings-I. *Southeast Asian Bull. Math.*, **19**(1)(1995), 49–54.

[4] M. M. Krishna Rao. Γ-semirings-II. *Southeast Asian Bull. Math.*, **21**(3)(1997), 281–287.

[5] M. M. Krishna Rao. The Jacobson radical of Γ-semiring. Southeast Asian Bull. Math., **23**(1)(1999), 127–134.

[6] M. M. Krishna Rao. $\Gamma-$ semiring with identity. *Discuss. Math. General Al. Appl.*, **37**(2)(2017), 189-207.

[7] M. M. Krishna Rao. Ideals in ordered $\Gamma-$ semirings. *Discuss. Math. General Al. Appl.*, **38**(1)(2018), 47–68.

[8] M. M. Krishna Rao. Bi-interior ideals in semigroups. *Discuss. Math. General Al. Appl.*, **38**(1)(2018), 69–78.

[9] M. M. Krishna Rao and B. Venkateswarlu. Bi-interior Ideals in Γ-semirings. *Discuss. Math. General Al. Appl.*, **38**(2)(2018), 239-244. doi:10.7151/dmgaa.1258

[10] M.M. Krishna Rao. Bi-quasi-ideals and fuzzy bi-quasiideals of Γsemigroups. *Bull. Int. Math. Virtual Inst.*, **7**(2)(2017), 231-242.

[11] M. M. Krishna Rao, B. Venkateswarlu and N.Rafi. Left bi-quasi-ideals of Γ-semirings. *Asia Pacific J. Math.*, **4**(2)(2017), 144–153.

[12] M. M. Krishna Rao. Left bi-quasi ideals of semirings. *Bull. Int. Math. Virtual Inst.*, **8**(1)(2018), 45–53.

[13] M. M. Krishna Rao. Left bi-quasi ideals of semirings. *Bull. Int. Math. Virtual Inst.*, **8**(1)(2018), 45-53.

[14] M. M. Krishna Rao. $\Gamma-$field. *Discuss. Math. General Al. Appl.*, **39**(1)(2019), 125-133.

[15] M. M. Krishna Rao and B. Venkateswarlu. Regular $\Gamma-$incline and field $\Gamma-$semiring, *Novi Sad J. Math.*, **45**(2)(2015), 155–171.

[16] V. Neumann. On regular rings. *Proc. Nat. Acad. Sci.*, **22**(12)(1936), 707–713.

[17] N. Nobusawa. On a generalization of the ring theory. *Osaka. J.Math.*, **1**(1)(1964), 81–89.

[18] M. K. Sen. On $\Gamma-$semigroup. IN: *Procedings of International conference of algebra and its application* (New Delhi, 1981) (pp. 301–308). Decker Publicaiton, New York, 1984.

[19] M. K. Sen and N. K. Saha, On -semigroup-I. Bull. Cal.Math.Soc. **78**(3)(1986), 180–186.

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