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EFFICIENT DOMINATION IN MYCIELSKI'S GRAPHS

M. Anitha and S. Balamurugan

Abstract. Given a graph G and any integer $m \geqslant 0$, Mycielski constructed a graph $\mu(G)$ and one can transform G into a generalized mycielskian of G, $\mu_m(G)$. This paper investigate the diameter of $\mu_m(G)$. A subset $S \subseteq V(G)$ for which $|N[v] \cap S| = 1$ for every $v \in V(G)$ is called a *perfect code.* Efficient domination is a generalization of a perfect code. A perfect code S for a graph G is also called an *efficient dominating set.* We say that G is *efficiently dominatable*. We show: For a graph G without isolated vertices, $\mu(G)$ and $\mu_m(G)$ are not efficiently dominatable whenever G is efficiently dominatable.

1. Introduction

Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E. For graph theoretic terminology, we refer to [**1**] and [**2**]. The *distance* between two vertices u and v in a graph G, denoted by $d_G(u, v)$ is the length of a shortest path between them.

For a set S of vertices, we define $d_G(u, S) = min\,\{d_G(u, v) \,|\, for \;\; all\; v \in S\}$. The *diameter* of G, denoted by $diam(G)$, is the greatest distance between two vertices of G. The *open neighborhood* of $v \in V$ is $N(v) = \{u \in V \,|\, uv \in E\}$ and *closed neighborhood* of $v \in V$ is $N[v] = N(v) \cup \{v\}$. For a set S of vertices, we define the open neighborhood $N(S) = \bigcup\limits_{v \in S} N(v)$ and the closed neighborhood $N[S] = N(S) \cup S$. A *packing* is a set of vertices whose closed neighborhoods are disjoint. The *packing number*, $\eta(G)$ of a graph G is the maximum order of a packing of G.

T. W. Haynes et al [**4**] introduced efficient domination as a generalization of a perfect code. A subset $S \subseteq V(G)$ for which $|N[v] \cap S| = 1$ for every $v \in V(G)$ is

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called a *perfect code*. We note that if $S \subseteq V(G)$ is a perfect code for G, then for every pair $u, v \in S$ we must have $d(u,v) \geqslant 3$ which implies that S is a packing. For any subset $S \subseteq V(G)$, let $I(S) = \sum\limits_{u \in S} (1 + deg(u))$ denote the *influence* of S. The *efficient domination number* of G, $F(G)$ is defined by the maximum number of vertices that can be efficiently dominated by a packing. That is

$$F(G) = max\left\{|N[S]| \mid S \text{ is a packing}\right\} = max\left\{I(S) \mid S \text{ is a packing}\right\}.$$

When $F(G) = |V(G)|$, T.W. Haynes et al says that G is *efficiently dominatable.* A perfect code S for a graph G is also called an *efficient dominating set.* We says that there exists a packing, S with $I(S) = |V(G)|$ whenever G is efficiently dominatable and the cardinality of $N[S]$ is also called as influence of S, for a packing S. That is every vertex in G is either in S or in $N(S)$.

2. The Mycielski Construction

In 1955, Mycielski, [**7**] introduced a admirable construction to increase the chromatic number of triangle free graphs without increasing a clique number. W. Lin et al [**5**] call this mycielski's graph as mycielskian of G. The Mycielskian $\mu(G)$ of a graph G is defined as follows:

Let G be a graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and edge set E. Let V^1 be a copy of the vertex set and u be a single vertex. Then the Mycielskian $\mu(G)$ has the vertex set $V^0 \cup V^1 \cup \{u\}$. The edge set of $\mu(G)$ is the set

$$\left\{v_i^0 v_j^0 : v_i v_j \in E\right\} \cup \left\{v_i^0 v_j^1 : v_i v_j \in E\right\} \cup \left\{v_j^1 u : \forall v_j^1 \in V^1\right\}.$$

The generalized Mycielskian $\mu_m(G)$ of a graph G is defined as follows: Let G be a graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and edge set E and let m be any positive integer. For each integer $k (0 \leqslant k \leqslant m)$, let V^k be a copy of vertices in V, that is $V^k = \left\{v_1^k, v_2^k, \cdots, v_n^k\right\}$. The m - mycielskian $\mu_m(G)$ has the vertex set $V^0 \cup V^1 \cup \cdots \cup V^m \cup \{u\}$ where u is a single vertex. The edge set of $\mu_m(G)$ is the set

$$\left\{v_i^0 v_j^0 : v_i v_j \in E\right\} \cup \left(\bigcup_{k=0}^{m-1} \left\{v_i^k v_j^{k+1} : v_i v_j \in E\right\}\right) \cup \left\{v_j^m u : \forall v_j^m \in V^m\right\}.$$

W. Lin et al [**5**] define $\mu_0(G)$ to be the graph obtained from G by adding a universal vertex u.

We observe that every vertex v_i^k in V^k is adjacent to the vertices v_j^{k+1} in V^{k+1} and v_j^{k-1} in V^{k-1}, $k = 1, 2, \cdots, m-1$ if v_i is adjacent to v_j in G. No two vertices in V^k are adjacent to each other except $k = 0$ and v_i^k and v_i^l are not adjacent, for all i, k, l.

3. Diameter

THEOREM 3.1. *For any graph G without isolated vertices,* $diam\left(\mu_m(G)\right) = min\left\{max\left\{diam(G), m+1\right\}, 2m+2\right\}$

Proof. Let G be a given graph with a vertex set $\{v_1, v_2, \cdots, v_n\}$. Given an integer $m \geqslant 1$, Let $\mu_m(G)$ be m - Mycielskian of G with the vertex set $V^0 \cup V^1 \cup \cdots \cup V^m \cup \{u\}$, where $V^k = \{v_i^k \mid v_i \in V\}$ is the k^{th} distinct copy of V, for $k = 0, 1, 2, \cdots, m$. Clearly, $d(u, v^0) = m + 1$ and $d(u, v^k) = m + 1 - k$ for all $k = 1, 2, \cdots, m$, since u is adjacent to each vertex v_i^m in V^m and each vertex v_i^k in V^k is adjacent to v_j^{k-1} in V^{k-1}, if v_i is adjacent to v_j in G. Next we have to calculate the length of the shortest path between v_i^k and v_j^l.

If $k = l = 0$, then $d(v_i^0, v_j^0)$ is either equal to $d(v_i, v_j)$, since the sub graph induced by V^0 is isomorphic to G, or the length of the shortest $v_i^0 - v_j^0$ path containing the vertex u. Hence $d(v_i^0, v_j^0) = min\{d(v_i, v_j), 2(m+1)\}$, for all i, j.

If any one of the integer of k, l is zero and other not a zero, suppose that $k \neq 0$ and $l = 0$, then $d(v_i^0, v_j^k)$ is the length of the shortest $v_i^0 - v_j^k$ path containing the vertex u or not containing u. Hence, $d(v_i^0, v_j^k) \leqslant 2m + 2 - k$. Now if $d(v_i, v_j) < k$, then the length of the shortest $v_i^0 - v_j^k$ path not containing the vertex u is k, when both $d(v_i, v_j)$ and k are even (or odd) and $k + 1$, otherwise. Also if $d(v_i, v_j) \geqslant k$, then $d(v_i^0, v_j^k) = d(v_i, v_j)$, for all i, j. Hence, $d(v_i^0, v_j^k) = min\{max\{d(v_i, v_j), k+1\}, 2m + 2 - k\}$.

Otherwise, the shortest $v_i^k - v_j^l$ path is in any one of the following form

(i) the path containing the vertex u
(ii) the path not containing the vertex u but containing the vertex v_r^0 in V^0, for some r
(iii) the path does not contain a vertex, v_r^0 in V^0 and u

It is clear that (iii) is the required shortest path if both $|k - l|$ and $d(v_i, v_j)$ is even or both $|k - l|$ and $d(v_i, v_j)$ is odd with $i \neq j$. In these cases, $d(v_i^k, v_j^l) = min\{max\{d(v_i, v_j), |k - l|\}, 2m + 2 - k - l\}$. Otherwise, the required shortest path must be in any one of the form (i) and (ii). Let j_1 and j_2 be any two distinct indices such that $d(v_{j_1}, v_{j_2})$ is minimum in G. Clearly, $d(v_{j_1}, v_{j_2}) \geqslant 1$. Hence the required shortest path must contains the section P_1, from v_i^k to $v_{j_1}^0$ and the section P_2, from $v_{j_2}^0$ to v_j^l or the section Q_1, from v_i^k to u and the section Q_2, from u to v_j^l. Since $d(v_i^k, v_{j_1}^0) = k$ for some j_1 and $d(v_j^l, v_{j_2}^0) = l$ for some j_2, the shortest path between $v_{j_1}^0$ and $v_{j_2}^0$ together with P_1 and P_2 form a path between v_i^k and v_j^l through a vertex v_r^0 in V^0, for some r. Hence the length of this path is $d(v_{j_1}, v_{j_2}) + k + l$. On the other hand, $Q_1 \cup Q_2$ is a path between v_i^k and v_j^l through a vertex u, then the length of a path $Q_1 \cup Q_2$ is $2m + 2 - k - l$. So that $d(v_i^k, v_j^l) = min\{d(v_{j_1}, v_{j_2}) + k + l, 2m + 2 - k - l\}$.

Since $0 \leqslant k \leqslant m$ and $0 \leqslant l \leqslant m$, all the distance discussed above are less than $2m + 2$. Hence $diam\,(\mu_m(G)) \leqslant 2m + 2$. Also $diam\,(\mu_m(G)) = diam(G)$ if $diam(G) \geqslant |k - l|$ and $diam\,(\mu_m(G)) = |k - l|$ if $diam(G) < |k - l|$. Hence,

$$diam\,(\mu_m(G)) = min\{max\{diam(G), m+1\}, 2m+2\}.$$

□

COROLLARY 3.1. [**3**] *For any graph G without isolated vertices,*

$$diam\left(\mu(G)\right) = min\left\{max\left\{diam(G), 2\right\}, 4\right\}.$$

In the above theorem, j_1 and j_2 are considered as distinct indices. Suppose if $j_1 = j_2$, we can replace the section $v^1_{j_1-1}v^0_{j_1}v^1_{j_1+1}$ by $v^1_{j_1-1}v^2_{j_1}v^1_{j_1+1}$ in the $v^k_i - v^l_j$ path. Hence there exists a $v^k_i - v^l_j$ path not containing the vertex u and v^0_r in V^0. Which is contradiction to the case assumption. Hence $d(v_{j_1}, v_{j_2}) \geqslant 1$. If the shortest $v^k_i - v^l_j$ path must be in any one of the form (i) and (ii), then $2m+2-k-l \geqslant d(v_{j_1}, v_{j_2}) + k + l + 2$ if and only if $m \geqslant k + l$, since $d(v_{j_1}, v_{j_2}) \geqslant 1$.
Hence,

$$d(v^k_i, v^l_j) = \begin{cases} d(v_{j_1}, v_{j_2}) + k + l + 1, & m \geqslant k + l \\ 2m + 2 - k - l, & m < k + l \end{cases}.$$

4. The Efficient Domination

THEOREM 4.1 ([**4**]). *The following are equivalent:*

a) $S = \{v_1, v_2, \cdots, v_k\}$ *is a perfect code for G.*

b) $\{N[v_1], N[v_2], \cdots, N[v_k]\}$ *is a partition of $V(G)$*

c) S *is a packing and* $\sum_{v \in S} (1 + deg(v)) = |V(G)|$

THEOREM 4.2 ([**3**]). *For a graph G without isolated vertices, $\eta\left(\mu(G)\right) = \eta(G)$.*

THEOREM 4.3. *For any graph G, $\mu_0(G)$ is efficiently dominatable with an efficient dominating set $\{u\}$.*

THEOREM 4.4. *For a graph G without isolated vertices, $\mu(G)$ is not efficiently dominatable whenever G is efficiently dominatable.*

PROOF. Given a graph G is efficiently dominatable. Let $S = \{v_1, v_2, \cdots, v_m\}$ be a perfect code of G. Let $|V(G)| = n$. Then, S is a packing of G and $I(S) = n$ (By theorem, 4.1). Since S is a packing of G, for all $i = 1, 2, \cdots, m; j = 1, 2, \cdots, m$ and $i \neq j$

$$d(v_i, v_j) \geqslant 3 \tag{4.1}$$

Now, $m + \sum_{s \in S} deg\ s = \sum_{s \in S} (1 + degs) = I(S) = n$. This implies that

$$\sum_{s \in S} degs = n - m \tag{4.2}$$

Let $S' = \left\{v^0_1, v^0_2, \cdots, v^0_m\right\}$ and $S'' = \left\{v^0_1, v^0_2, \cdots, v^0_{i-1}, v^1_i, v^0_{i+1}, \cdots, v^0_m\right\}$. From (4.1), $d(v^0_i, v^0_j) \geqslant 3$, for all $i = 1, 2, \cdots, m; j = 1, 2, \cdots, m$ and $i \neq j$ then S' is a packing of $\mu(G)$. We have to prove that $d(v^0_i, v^1_k) \geqslant 3$, for all $i \neq k; i = 1, 2, \cdots, m$. Suppose $d(v^0_i, v^1_k) \leqslant 2$, then $d(v^0_i, v^0_k) \leqslant 2$. Which leads to the contradiction. Hence $d(v^0_i, v^1_k) \geqslant 3$, for all $i \neq k; i = 1, 2, \cdots, m$; implies that S'' is a packing of $\mu(G)$. Since $\eta\left(\mu(G)\right) = \eta(G)$ (By theorem, 4.2) and $|S| = |S'| = |S''| = m$, S' and S'' are the maximum packing of $\mu(G)$. Now

$$I(S') = \sum_{v_i^0 \in S'} \left(1 + degv_i^0\right)$$

$$= m + \sum_{v_i^0 \in S'} degv_i^0 = m + 2 \sum_{v_i \in S} degv_i = m + 2(n-m) = 2n - m < 2n + 1.$$

Hence

$$I(S') < |V(\mu(G))| \tag{4.3}$$

Also,

$$I(S'') = \sum_{s \in S''} (1 + degs) = \sum_{v_i^0 \in S'; i \neq k} \left(1 + degv_i^0\right) + (1 + degv_k^1)$$

$$< \sum_{v_i^0 \in S'; i \neq k} \left(degv_i^0\right) + m + (degv_k^0) = m + 2 \sum_{v_i \in S} degv_i$$

$$= m + 2(n-m) = 2n - m < 2n + 1.$$

Hence

$$I(S'') < |V(\mu(G))| \tag{4.4}$$

From (4.3) and (4.4), we get $F(\mu(G)) \neq |V(\mu(G))|$. This implies that $\mu(G)$ is not efficiently dominatable, since $\mu(G)$ has no perfect code. □

For example, K_2 is efficiently dominatable but $\mu(K_2) = C_5$ not. In general, we can say that $\mu_m(K_2) = C_{2m+3}$ is efficiently dominatable if and only if $m = 3k, k = 0, 1, 2, \cdots$.

Theorem 4.5. *For a connected graph $G \neq K_2$ without isolated vertices, $\mu_m(G)$ is not efficiently dominatable whenever G is efficiently dominatable.*

Proof. Given a graph G is efficiently dominatable. For a positive integer $m > 1$, $\mu_m(G)$ be a generalized mysielskian graph. Let S be a maximum packing of $\mu_m(G)$ and $T = N(S)$. Suppose that $\mu_m(G)$ is efficiently dominatable. Then $F(G) = N[S] = V(\mu_m(G))$.

Case: 1 If $u \in S$, then $v_i^m \in T$, for all i and no vertex of V^m and V^{m-1} belongs to S, since $d(v_i^m, u) = 1$ and $d(v_i^{m-1}, u) = 2$. Let x and y be any two vertices in G. If $N(x) = N(y)$, then clearly, x and y are nonadjacent vertices in G. Since $d(x^{m-2}, y^{m-2}) = 2$, either x^{m-2} or y^{m-2} is in S. Hence either $x^{m-2} \notin N[S]$ or $y^{m-2} \notin N[S]$.

If $N(x) \supset N(y)$, then clearly x and y are non adjacent vertices in G. Since $d(x^{m-2}, y^{m-2}) = 2$, either x^{m-2} or y^{m-2} is in S. Suppose if $x^{m-2} \in S$, $y^{m-2} \notin N[S]$. Suppose if $y^{m-2} \in S$ then $x^{m-2} \notin N[S]$. Let $z \in N(x) \backslash N(y)$. For $x^{m-2} \in T$, $z^{m-3} \in S$, then clearly $z^{m-1} \notin N[S]$, since $d(z^{m-3}, z^{m-1}) = 2$.

If $N(x)$ and $N(y)$ are not comparable. Suppose if $N(x) \bigcap N(y) = \phi$. If x and y are adjacent in G, then the subgraph, H induced by the vertices $x, y, N(x)$ and $N(y)$ of a graph G is either P_3 or a double star. Then, the copy of any two adjacent vertices of H in V^{m-2} belongs to S and the copy of the remaining vertices in V^{m-2}, say r^{m-2}, are not in $N[S]$.

If $N[x] \cup N[y] = V(G)$, $N[S] \subset V(\mu_m(G))$, since $r^{m-2} \notin N[S]$.

Otherwise, for $r^{m-2} \in T$, if exists $q^{m-3} \in S$, where r is adjacent to $q \in V(G) \setminus (N[x] \bigcup N[y])$ in G, then $q^{m-1} \notin N[S]$. If x and y are non adjacent in G, then there exists a shortest path between x and y through a vertex of $N(x)$ and $N(y)$, since G is connected.

Let $N(x) = \{x_i \mid 1 \leqslant i \leqslant r\}$ and $N(y) = \{y_j \mid 1 \leqslant j \leqslant s\}$. Let $P : p_0 p_1 \cdots p_l$ be such a path in G where $p_0 = x$, $p_1 = x_i$, $p_{l-1} = y_j$ and $p_l = y$ We have to prove that there exists a vertex, v in $\mu_m(G)$ such that $v \notin N[S]$. We use induction on l. If $l = 3$, then the copy of any two adjacent vertices of the path P in V^{m-2} belongs to S and the copy of the remaining vertices in V^{m-2}, say r^{m-2}, are not in $N[S]$.

If $N[x] \cup N[y] = V(G)$, $N[S] \subset V(\mu_m(G))$, since $r^{m-2} \notin N[S]$.

Otherwise, for $r^{m-2} \in T$, if exists $q^{m-3} \in S$, where r is adjacent to $q \in V(G) \setminus (N[x] \bigcup N[y])$ in G, then $q^{m-1} \notin N[S]$.

Assume that the result is true for all path of length less than l. Let P be the path of length l. By induction hypothesis, there exists a vertex z_1^{m-2} such that $z_1^{m-2} \notin N[S]$ where z_1 is the vertex in the path of length $l-1$. Add a new vertex, z_2 to this path such that the resultant is a path of length l. If $d(z_1, z_2)$ is odd in G, then clearly, $z_1^{m-2} \notin N[S]$. If $d(z_1, z_2) = 4k+2, k = 0, 1, 2 \cdots$, then either $z_1^{m-2} \notin N[S]$ or $z_2^{m-2} \notin N[S]$. If $d(z_1, z_2) = 4k, k = 0, 1, 2 \cdots$, then $d(z_2^{m-2}, S) = 2$, since $d(z_1^{m-2}, S) = 2$. Hence neither z_1^{m-2} nor z_2^{m-2} belongs to S. Therefore there exists a vertex, v in $\mu_m(G)$ such that $v \notin N[S]$.

If $N(x) \bigcap N(y) \neq \phi$, then either x^{m-2} or y^{m-2} are in S. Without loss of generality, Let $x^{m-2} \in S$, Let $z \in N(y) \setminus N(x)$. For $y^{m-2} \in T$, $z^{m-3} \in S$, then clearly $z^{m-1} \notin N[S]$, since $d(z^{m-3}, z^{m-1}) = 2$.

Case: 2 Let $u \notin S$. For $u \in T$, there exists at most one vertex x^m in V^m belongs to S, since $d(x^m, y^m) = 2$ for all $x^m, y^m \in V^m$.

Suppose, Let x_1 be a unique vertex adjacent to x in G. Clearly, $x_1^m \notin N[S]$. Since $G \neq K_2$, there exists a vertex, x_2 in $V(G) \setminus \{x, x_1\}$, adjacent to x_1. For $x_1^m \in T$, either $x^{m-1} \in S$ or $x_2^{m-1} \in S$. If $x_2^{m-1} \in S$, then $x^{m-1} \notin N[S]$. For $x^{m-1} \in T$ there exists no vertex, v in V^{m-3} such that $d(v, S) \geqslant 3$. If $x^{m-1} \in S$ then x_2^m and x_2^{m-1} are not in S. For $x_2^m \in T$, if there exists a vertex, x_3 in $V(G) \setminus \{x, x_1, x_2\}$, adjacent to x_2 such that $x_3 \notin N(\{x, x_1\})$, Then $x_3^{m-1} \in S$ but $x_3^m \notin N[S]$. For $x_3^m \in T$, if there exists a vertex, x_4 in $V(G) \setminus \{x, x_1, x_2, x_3\}$, adjacent to x_3 such that $x_4 \notin N(\{x, x_1, x_2\})$, Then $x_4^{m-1} \in S$ but still $x_2^{m-1} \notin N[S]$. For $x_2^{m-1} \in T$, if there exists a vertex, x_5 in $V(G) \setminus \{x, x_1, x_2, x_3, x_4\}$, adjacent to x_2 such that $x_5 \notin N(\{x, x_1, x_3, x_4\})$, Then $x_5^{m-2} \in S$ but $x_5^m \notin N[S]$. Proceeding in this way, we concluded that there exists a vertex, v either in V^m or in V^{m-1} such that $v \notin N[S]$.

Otherwise, Let $N(x) = \{x_1, x_2, \cdots x_t\}$ in G. If $x^{m-1} \in S$ then the copy of all the vertices of $N(x)$ in V^m, V^{m-1} and V^{m-2} belongs to T. Since $x^{m-2}, x^{m-3} \notin S$, If possible, for $x^{m-2} \in T$ and $x^{m-3} \in T$, the copy of $x_i \in N(x)$, for some unique i, in V^{m-3} and V^{m-4} belongs to S respectively. Hence the copy of the vertices of $N(x) \setminus \{x_i\}$ in V^{m-3} and V^{m-4} does not belongs to $N[S]$. Suppose if there exists a vertex, $v_1 \in V(G) \setminus N[x]$ is adjacent to any vertex, $x_j (i \neq j)$ in $N(x)$, then $v_1^m \notin N[S]$ and $v_1^{m-1} \notin N[S]$. For $v_1^m \in T$, Suppose if there exists a vertex,

$v_2 \in V(G) \setminus (\{v_1\} \cup N[x])$ is adjacent to v_1, then $v_2^{m-1} \in S$, but $v_2^m \notin N[S]$. For $v_2^m \in T$, Suppose if there exists a vertex, $v_3 \in V(G) \setminus (N[x] \cup \{v_1, v_2\})$ is adjacent to v_2, then $v_3^{m-1} \in S$, but still $v_1^{m-1} \notin N[S]$ For $v_1^{m-1} \in T$, Suppose if there exists a vertex, $v_4 \in V(G) \setminus (N[x] \cup \{v_1, v_2, v_3\})$ is adjacent to v_1, then $v_4^{m-2} \in S$, but still $v_4^m \notin N[S]$. Proceeding in this way, we concluded that there exists a vertex, v either in V^m or in V^{m-1} such that $v \notin N[S]$.

Both cases show that there must be exists a vertex, v in $V(\mu_m(G))$ (in particular, v is in V^m or V^{m-1}) such that $v \notin N[S]$.

Hence $N[S] \subset V(\mu_m(G))$. This implies that $F(\mu_m(G)) \neq |V(\mu_m(G))|$. Which leads to the contradiction. Therefore $\mu_m(G)$ is not efficiently dominatable. □

COROLLARY 4.1. *For a graph $G \neq K_2$ without isolated vertices $\mu_m(G)$ is not efficiently dominatable whenever G is efficiently dominatable.*

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DEPARTMENT OF MATHEMATICS, SYED AMMAL ARTS AND SCIENCE COLLEGE, RAMANATHAPURAM, TAMILNADU, INDIA

E-mail address: ani.thania@gmail.com

PG DEPARTMENT OF MATHEMATICS, GOVERNMENT ARTS COLLEGE, MADURAI, TAMILNADU, INDIA

E-mail address: balapoojaa2009@gmail.com